INTRODUCTION TO PRACTICAL RADIO

DURWARD J. TUCKER, B.S. in E.E., E.E.
Chief Radio Engineer, WRR, KVP, KVPA
Member, American Institute of Electrical Engineers
Senior Member, Institute of Radio Engineers
Member, National Society of Professional Engineers

COLLEG

1945

THE MACMILLAN COMPANY · NEW YORK

COPYRIGHT 1945 BY THE MACMILLAN COMPANY

All rights reserved—no part of this book may be reproduced in any form without permission in writing from the publisher, except by a reviewer who wishes to quote brief passages in connection with a review written for inclusion in magazine or newspaper.

PRINTED IN THE UNITED STATES OF AMERICA

Preface

Radio is such an extensive subject that it would require many volumes to cover it thoroughly. Such an undertaking would be a tremendous task and no such set of books has been written, although there are many covering its various phases.

The author has long felt the need for a single text dealing with the basic fundamentals of radio that could be used as a steppingstone to more advanced books on the subject. This text was written with that thought in mind. No attempt has been made to cover the whole subject of radio but rather to limit the text to the initial principles that any student must first learn before he takes up the more advanced phases.

By confining the text material to first principles, the author has been able to cover each step adequately. In addition, the necessary mathematical tools are supplied as needed, in the belief that they will be of material aid to those lacking the prerequisites necessary to study most radio texts.

It is hoped that more advanced students will find the book an extensive reference and an invigorating "refresher" in relearning the many fundamental details.

The author wishes to express his appreciation to Dean E. H. Flath of the Southern Methodist University Engineering School, to Walter J. E. Schiebel, Director of the Dallas Public Evening School, and to many others, too numerous to mention, for their constant encouragement and many helpful suggestions. He is likewise indebted to his many students and other friends for the answers to the problem exercises and for detailed work in checking several chapters of the manuscript.

The author is deeply indebted and grateful to his wife for typing the manuscript and for her many suggested improvements.

DURWARD J. TUCKER

Contents

Chapter I. Introduction

1. Relation to Electricity 1
2. Building Stones of Radio 1
3. Mathematics 2
4. Examples and Problems 2
5. Conclusion 2

Chapter II. Elementary Electricity

6. Introduction 4
7. Molecule and Atom 4
8. Electron 4
9. Electrical Charges 5
10. Electric Current 5
11. Voltage 6
12. Resistance 6
13. Resistance Properties of Materials 6
14. Ohm 7
15. Conductors and Insulators 10
16. Electrical Circuits 11
17. Resistance Devices 11
18. Electrical Symbols 12
19. Slide Rule 12

Chapter III. Ohm's Law

20. Arithmetic 15
21. Algebra 15
22. Mathematical Symbols 15
23. Coefficients 16
24. Positive and Negative Numbers 16
25. Laws of Positive and Negative Numbers 17
26. Zero 17

27. Equal Signs 18
28. Equations 18
29. Equation Rules 18
30. Ohm's Law 21
31. Law of Signs 24
32. Grouping Signs 24
33. Exponents 25
34. Law of Exponents for Multiplication 25
35. Law of Exponents for Division 26
36. Radicals 28
37. Addition and Subtraction of Radicals 28
38. Multiplication of Radicals 28
39. Division of Radicals 29
40. Fractions 29
41. Fractional Exponents 30
42. Powers of Ten 31

Chapter IV. Resistance Circuits

43. Division of Current 35
44. Currents in Resistance Branches 37
45. Voltage Drop 38
46. Series Circuits 40
47. Series Parallel Circuits 42
48. Parallel Connected Resistors 43
49. Conductance 49

Chapter V. Direct-current Power

50. Work 54
51. Power 54
52. Heating Effect of Current 55
53. Other Expressions of Power 56
54. Efficiency 57
55. Power Used by Resistors 58
56. Variable Resistors 59
57. Sources of D-c Power 60
58. Batteries 60
59. Series Connected Batteries 62
60. Parallel Connected Batteries 63
61. Motor Generators and Vibrators 63
62. Alternating-current Rectifiers 64

63. Internal Resistance of Power Source 66
64. Voltage Regulation 66

Chapter VI. Equipment Wiring

65. Point-to-point Wiring 68
66. Form Wiring 72
67. Simple Circuits 73
68. Advanced Circuits 75
69. Summary 81

Chapter VII. Kirchhoff's Laws

70. First Law 82
71. Second Law 83
72. Further Studies of Mathematics 84
73. Addition and Subtraction 85
74. Multiplication 86
75. Division 87
76. Factoring 89
77. Simultaneous Equations 90
78. Graphical Solution 90
79. Addition and Subtraction Solution 90
80. Substitution, Comparison, Determinants 92
81. Simple Applications of Kirchhoff's Laws 94
82. Determination of Direction of Current Flow 97
83. Extended Applications of Kirchhoff's Laws 100
84. Superposition Solution of Circuits 106

Chapter VIII. Magnetism

85. Magnetic Field 111
86. Magnetic Properties of Materials 112
87. Magnetic Materials 112
88. Theory of Magnetism 113
89. Magnetic Poles 113
90. Magnetic Lines of Force 114
91. Magnetic Fields of Magnets 115
92. Practical Uses of Magnets 115
93. Logarithms 116
94. Different Bases 117
95. Logarithm Tables 117

96. Characteristic and Mantissa 117
97. To Find the Logarithm of a Given Number 119
98. Antilogarithm 121
99. Multiplication by Logarithms 121
100. Division by Logarithms 122
101. Cologarithms 123
102. Logarithm of Powers and Roots 123
103. Change of Logarithm Base 124

Chapter IX. Electromagnetism

104. Magnetic Field Around a Conductor 126
105. Direction of Current Flow 127
106. Left-hand Rule 127
107. Electromagnetism 128
108. Permeability 129
109. Headphones 130
110. Speakers 131
111. Relays 132
112. Magnetomotive Force 133
113. Magnetizing Force 135
114. Reluctance 137
115. Ohm's Law for Magnetic Circuits 139
116. Flux Density 141
117. Core Saturation 141
118. Magnetic Shielding 143
119. Residual Magnetism 145
120. Hysteresis 146
121. Hysteresis Losses 147
122. Magnetic-circuit Calculations 149

Chapter X. Alternating Current

123. Alternating Current 152
124. Alternation and Cycle 152
125. Frequency 153
126. Angles 154
127. Triangles 155
128. Trigonometry 155
129. Right Triangles 155
130. Relation of Sides to Angles 157
131. Trigonometric Functions 158

132. Table of Trigonometric Functions 160
133. Given the Angle—To Find the Function 161
134. Given the Function—To Find the Angle 163
135. Signs in Different Quadrants 164
136. Sine, Cosine, and Tangent of Angle in Different Quadrants 164
137. Radian 165
138. Sine Wave Graph 167
139. Waves 168
140. Alternating-current Wave 168
141. Alternating-current Voltage Wave 169
142. Wave Additions 170
143. Complex Waves 174
144. Generation of Alternating Current 175
145. Effect of Alternating Current and Voltage 176
146. Ohm's Law for R in an A-c Circuit 179
147. Alternating-current Power in Resistances 179
148. Vectors 181
149. Addition of Vectors 182
150. Graphical Addition of Vectors 182
151. Vector Components 184
152. Addition of Vector Components 184
153. Current and Voltage Vectors 187
154. Phase and A-c Power 187

Chapter XI. Electrical Instruments

155. Electrical Measurements 192
156. D'Arsonval Movement 192
157. Current Measuring D-c Instruments 195
158. Voltage Measuring D-c Instruments 195
159. Direct-current Instrument Movement Types 196
160. Direct-current Milliammeters 196
161. Shunts 197
162. Direct-current Voltmeter 198
163. Multipliers 199
164. Care of D'Arsonval Type Instruments 201
165. Galvanometer 201
166. Measurement of Resistance 202
167. Wheatstone Bridge 203
168. Ohmmeter 204
169. Volt-ohmmeter 206

170. Measurement of Power 206
171. Alternating-current Measurements 206
172. The Effect of Frequency 208
173. Electrodynamometer Type Instrument 208
174. Iron-vane Type Instruments 210
175. Hot-wire Type Instrument 212
176. Thermocouple Type Instrument 213
177. Rectifier Type Instrument 214
178. Alternating-current Instrument Scales 214
179. Conclusions 216
180. Other Electrical Measuring Equipment 216

Chapter XII. Inductance

181. Lenz's Law 218
182. Inductance 219
183. Self-inductance 219
184. Determination of Self-inductance 219
185. Unit of Inductance 220
186. Mutual Inductance 222
187. Determination of Mutual Inductance 223
188. The Transformer 224
189. Transformer Properties 224
190. Transformer Losses 227
191. Inductances Connected in Series-parallel 227
192. Rate of Change of Current 228
193. Phase Relation of E and I for Inductance 229
194. Inductive Reactance 230
195. Power in an Inductance 231

Chapter XIII. Capacitance

196. The Condenser 234
197. The Electric Field 234
198. Dielectric Constant 238
199. Dielectric Materials 238
200. Energy-storing Properties of a Condenser 239
201. Unit of Capacitance 240
202. Capacitance of a Condenser 240
203. Condensers Connected in Series-parallel 241
204. Rate of Change of Voltage 242
205. Phase Relation of E and I for a Condenser 242

206. Capacitive Reactance 243
207. Power in a Condenser 244
208. Condenser Properties 245
209. Variable Condensers 246
210. Mica Condensers 246
211. Paper Condensers 246
212. Electrolytic Condensers 247

Chapter XIV. Impedance

213. Determination of Impedance 250
214. Alternating-current Voltage and Current in Impedance Circuits 251
215. Phase of E-I for Series, Parallel Circuits 251
216. Resistance and Inductance in Series 251
217. Resistance and Capacitance in Series 253
218. Resistance, Inductance, and Capacitance in Series 255
219. Series Resonance 256
220. Effect of Resistance on Series Resonance 257
221. Resistance and Inductance in Parallel 259
222. Resistance and Capacitance in Parallel 261
223. Resistance, Inductance, and Capacitance in Parallel 262
224. Parallel Resonance 263
225. Effect of Resistance on Parallel Resonance 264
226. Series, Parallel A-c Circuits 265
227. Impedance Matching 265

Appendix

Greek Alphabet 272
Table V (Logarithms) 273
Table VI (Trigonometric Functions) 292
Table VII (Powers—Roots—Reciprocals) 294
Mathematical Glossary 312
Electrical Glossary 314

Index 319

Symbols Used in Text

A	Cross-sectional area
a-c	Alternating-current
B	Flux density
C	Capacitance
d-c	Direct-current
e	Instantaneous voltage; also, Napierian system base where $e = 2.71828\ldots\ldots$
E	Voltage
E_{eff}	Effective voltage
E_{max}	Maximum voltage
f	Frequency
F	Magnetomotive force measured in Gilberts
H	Magnetizing force in Oersteds
i	Instantaneous value of current
I	Current
I_{eff}	Effective current
I_{max}	Maximum current
k	Coefficient of coupling between two coils
K	Constant determined by the properties of the magnetic material; also, dielectric constant of a condenser dielectric
l	Length
L	Inductance
M	Mutual inductance between two coils
N	Number of turns in a coil
p	Instantaneous power
P	Electric power
P_{avg}	Average power
P_h	Hysteresis core loss in watt-seconds per cubic centimeter per cycle
P_{max}	Maximum power
R	Resistance in ohms
$\mathcal{R}$	Reluctance
t	Thickness of dielectric
W	Watts (unit of electrical power)
W	Hysteresis core loss in watt-seconds per cubic centimeter per cycle

x	Horizontal axis
X	Reactance
X_C	Capacitive reactance
X_L	Inductive reactance
y	Vertical axis
Z	Impedance
$=$	Equal sign
$\neq$	Unequal sign
[]	Brackets
{ }	Braces
$\sqrt{\ }$	Radical sign
ϕ	Total number of lines of force (Maxwells)
μ	Permeability
π	Ratio of Circumference of circle to Diameter (3.1416.........)
ρ	Resistivity of metallic conductor
∞	Infinity

INTRODUCTION TO PRACTICAL RADIO

CHAPTER I *Introduction*

1. Relation to Electricity. The first thing that the student of radio must realize is that radio is not a separate science apart from electricity. Actually, it is but a specialized subject of electricity. To understand radio fully, one must first fully master the fundamentals of electricity.

It may seem to the student just starting the study of radio that the time and space in this text devoted to the study of the elements of electricity is excessive. Especially may this be true where the student has already covered the material in other texts. So much in radio depends upon a thorough knowledge of the fundamentals of electricity, however, that this material should be an excellent review for those who have studied it before and provide a suitable foundation in electricity for those covering it for the first time.

It is a waste of time to try to learn radio without first acquiring some knowledge of the basic principles underlying electricity. These are the abc's of radio, and the student must fully understand each one before progressing to the next. No attempt has been made to "pad" the text. Only those parts of electricity that are used over and over again are included.

2. Building Stones of Radio. Many believe that radio is quite difficult to learn and beyond their ability. In most cases, this can be traced to an improper approach to the subject. Here again is an example of why *first things must come first*. Lack of knowledge on the part of the student as to what to study first and how to choose from the maze of radio texts available contributes to his confusion.

The student will soon find that radio has many building stones fashioned from three principal sources: namely, resistance, inductance, and capacitance. These three quantities may be compared to wood, brick, and cement used extensively by the building industry. One knows that many building materials are formed from these basic substances. Likewise, many circuit elements are formed from resistance, inductance, and capacitance. The student will find a generous supply of pictures throughout the book showing many typical radio parts. These should be of material assistance in helping the student familiarize himself with the many different pieces of radio apparatus.

3. Mathematics. Students are often concerned with what knowledge of mathematics will be required in the study of radio.

There is no doubt that the use of mathematics is very helpful in acquiring a full understanding of radio principles. The choice of the kind of mathematics and the extent to which it is used in a text should be governed by the text material and the student. Naturally, as the student progresses farther and farther into the subject of radio, a greater knowledge of mathematics will be required.

The author has striven to keep in mind that this is a course in radio and not one in mathematics. The author is a firm believer in setting down the material, not only in the simplest mathematical form, but also in the plainest language possible.

Only those phases of mathematics deemed absolutely necessary are included in this text. They are placed, like the workman's tools, at the student's side at the right points in the book where he will need them. No attempt has been made to make the mathematical sections complete from a mathematical standpoint. Only enough mathematical material for a proper understanding of the radio material is included.

4. Examples and Problems. Numerous examples are included in order to familiarize the student fully with the principles involved.

Problem exercises have been inserted as near the theoretical treatment of a subject as possible. The student should solve these problems and be sure that he understands each section before passing on to new material. Answers are provided at the end of each chapter for the convenience of the student in checking his work since, in many instances, the text will be self-taught.

5. Conclusion. Study and experience in radio as a professional Radio Engineer and as a teacher, have taught the author that *there is no "short cut" to learning radio*. The student will do well to profit by this experience if he is to master the subject of radio most effectively in the shortest possible time.

At this point, the student is most likely to ask: "For what will this text prepare me?" This is a pertinent question but one which is difficult to answer. Many, beginning the study of radio are surprised, confused, disappointed, and discouraged that all the secrets of radio cannot be found between the two covers of one book. Many visualize the study of radio in the same light as the study of typing, shorthand, bookkeeping, and other business courses. They visualize progress in Radio Engineering as easy as gaining speed from week to week in typing. Such thoughts are not of Radio Engineering, but are of radio mechanics or a "screw-driver and pliers" knowledge in radio. The mechanics of testing and changing tubes, setting up amplifiers, learning the technique of soldering and wiring, making replacements of charred and burned parts and other similar mechanical details can be learned within a few weeks. This is a most essential part of the radio industry but should never be confused with the Radio Engineering Profession.

The author has started at the very beginning and has striven to make the text *complete within itself* in order that the student may acquire a firm foundation in the fundamental principles of radio without having to refer to other texts for additional material. After finishing the text, the student should have acquired sufficient knowledge to take up the study of other books dealing with the more advanced phases of radio.

REVIEW QUESTIONS

1. Why is an understanding of the basic principles of electricity necessary in order to learn radio?
2. Is a thorough knowledge of mathematics helpful in the study of radio? Why?
3. What are the advantages of providing the proper mathematical tools along with the radio material?
4. Why are problem exercises helpful in the study of a subject such as radio?
5. Compare the study of radio to the study of typing, radio code, and riveting.

CHAPTER II *Elementary Electricity*

6. Introduction. The engineering development of electrical equipment, since the turn of the century, has done much to add to the comfort of man. Many new branches of electricity have been developed. Radio is one of these, and its development has added much to the present knowledge of electricity.

Despite the wealth of knowledge of the behavior and characteristics of electricity, its exact nature is not too well defined. One may be surprised to learn that electricity is a part of matter itself.

7. Molecule and Atom. One is not accustomed to think in terms of units as small as the *molecule* or *atom* or their relation to matter, but in order to understand electricity better, it is necessary to go this far and farther. Matter is best known from many common materials, such as wood, linoleum, and cotton. Each material is determined by the *character of the molecules* of which it consists. Matter can be broken up into still smaller particles. The molecule can be broken down into *atoms. Two or more atoms are required to form a molecule.*

The many thousands of types of matter are formed from less than one hundred substances called *elements. The smallest unit of an element is an atom.* The structure of the atom determines whether matter is iron, gold, silver, or some other element. If the structure of the atom could be readily changed, then one element could be made into another, such as changing iron to gold. No such set of conditions exists. The atom vigorously opposes any attempt to break it up into smaller parts.

8. Electron. Scientists have been able to find out many things about the actual structure of the atom. Principle of these is that all atoms are composed of positive and negative charges of electricity. *Positive* charges of electricity are called *protons* and *negative* charges of electricity are called *electrons.* The number of positive and negative particles and their relative positions to each other in an atom are the factors that determine what element is formed.

It is indeed most unfortunate that a positive charge in reality denotes a deficiency of electrons. Furthermore, if a body receives an excess of electrons, it becomes negatively charged.

9. Electrical Charges. One of the simplest laws of electricity is the law of electrical charges:

Like charges repel and unlike charges attract.

Two bodies with like charges will tend to repel each other, while two bodies with unlike charges will tend to attract each other.

10. Electric Current. The molecules and atoms of all matter are in *constant motion.* Likewise, the electrons that make up the atoms are also in motion within the atoms. The atoms are continually colliding. This causes electrons to be dislodged from the atoms. An atom which has lost electrons will attract other electrons that may have been dislodged from another atom. This continual exchange of electrons from atom to atom is more pronounced in some materials than in others, depending upon the ease with which electrons may be dislodged from the atoms of the material. It appears that all materials contain *free electrons* in numbers dependent upon the material. Conductors contain more than nonconductors.

Under normal conditions, electrons move within a restricted area and in no particular direction. Under other conditions, there is a general movement of the electrons in a specific direction. *The movement of electrons in a given direction constitutes an electric current.* The number of electrons present determines the quantity of electricity. The electron is so very small that it is not convenient to think of quantities of electrical charges in terms of the tremendous number of electrons that would be involved. A more convenient term is the *coulomb.* The coulomb represents an electrical charge of 6.3×10^{18} electrons, which may be compared to the gallon used to measure liquids. It has already been stated that an electric current is a movement of electrons. It is seen that there is a need for a unit representing the movement of the electrons past a given point in an electric circuit, just as we have "gallons per minute" to measure the flow of water or other liquids. There is such a unit which is called the *ampere. An ampere is 1 coulomb of electric current flowing past a given point* *in 1 second.* The radio engineer is more accustomed to the milliampere (one-thousandth of an ampere) unit for the measurement of current as a majority of the currents encountered in radio circuits are less than one ampere.

Since a positive electric charge represents a shortage or deficiency of electrons, it is obvious that the current will flow from negative to positive when a proper path is provided.

11. Voltage. A city's water-supply system not only requires water, but it also requires pressure to force the water through the mains. An electrical circuit not only needs a supply of electrons that can be moved along the circuit, but also a pressure to force the electrons along. This electrical pressure is called *electromotive force.* The unit of electromotive force is the *volt* and *is the electromotive force necessary to force 1 ampere of current through 1 ohm of resistance.*

12. Resistance. The continual motion of the molecules and atoms of a material obstructs the free passage of electrons along a conductor just as a crowd of people moving about the entrance to a building obstructs the free passage of anyone from the street into the building. It has already been shown that an electric current is dependent upon the electrons given up by the atoms of the material and the free electrons in the material. The ease with which the atoms of a material exchange electrons from atom to atom is also an important factor in determining the opposition the material offers to any attempt to force current through the material. That property of a material which gives rise to its *opposition* to an electrical current flowing in the material is called *resistance.*

13. Resistance Properties of Materials. Resistance is determined by four factors: (1) resistivity of the material used, (2) temperature of the material, (3) length of the material, (4) cross-sectional area of the material.

Some metals, such as copper, silver, aluminum, and gold, have a relatively low resistivity as compared to iron, platinum, mercury, and carbon. *Copper* is used for practically all electrical circuits and in all electrical equipment where wire conductors are used. Aluminum is used extensively for open cross-country high-voltage power-transmission lines. Silver is little used except as contact material for coil contact points and switch contact points.

Most metal materials increase in resistance with an increase in temperature. This is due to an increase in *molecular motion.* A hot electric motor has more resistance in its copper windings than it would have if it were cool.

The resistance of a conductor increases as the length increases. The increase in resistance is in *direct proportion* to the increase in length. This can be easily seen if the student will keep in mind the conditions in a material which give rise to an opposition to a flow of current. If the length of a conductor is doubled, the resistance is doubled. If the length of a conductor is decreased, the resistance is decreased accordingly. If a conductor is replaced by one with a larger cross-sectional area, the resistance is decreased for the same reason; i.e., it is easier to travel down a broad highway than a narrow highway. The resistance of a conductor is said to be *inversely proportional* to the cross-sectional area of the conductor.

14. Ohm. The discovery of the opposition or resistance of materials to the flow of current naturally gave rise to the need of some kind of a standard or unit of measure of resistance. If there were no standard, there would be no way of stating how much resistance a certain material offered to the flow of current; i. e., 100 feet of size no. 30 gauge copper wire.

It was learned in section 13 that the resistance of a material is dependent on four factors. Naturally, any standard must incorporate all four of these factors.

The unit of resistance is the ohm. International agreement defines the ohm as the resistance offered by a column of mercury of uniform cross-section, 106.3 centimeters long, at a temperature of 0° centigrade, and weighing 14.45 grams.

The resistance of a conductor may be determined from the expression:

$$R = \frac{\rho l}{A} \tag{1}$$

where

R = resistance in ohms
ρ = resistivity of metallic conductor
A = cross-sectional area of conductor
l = length of conductor

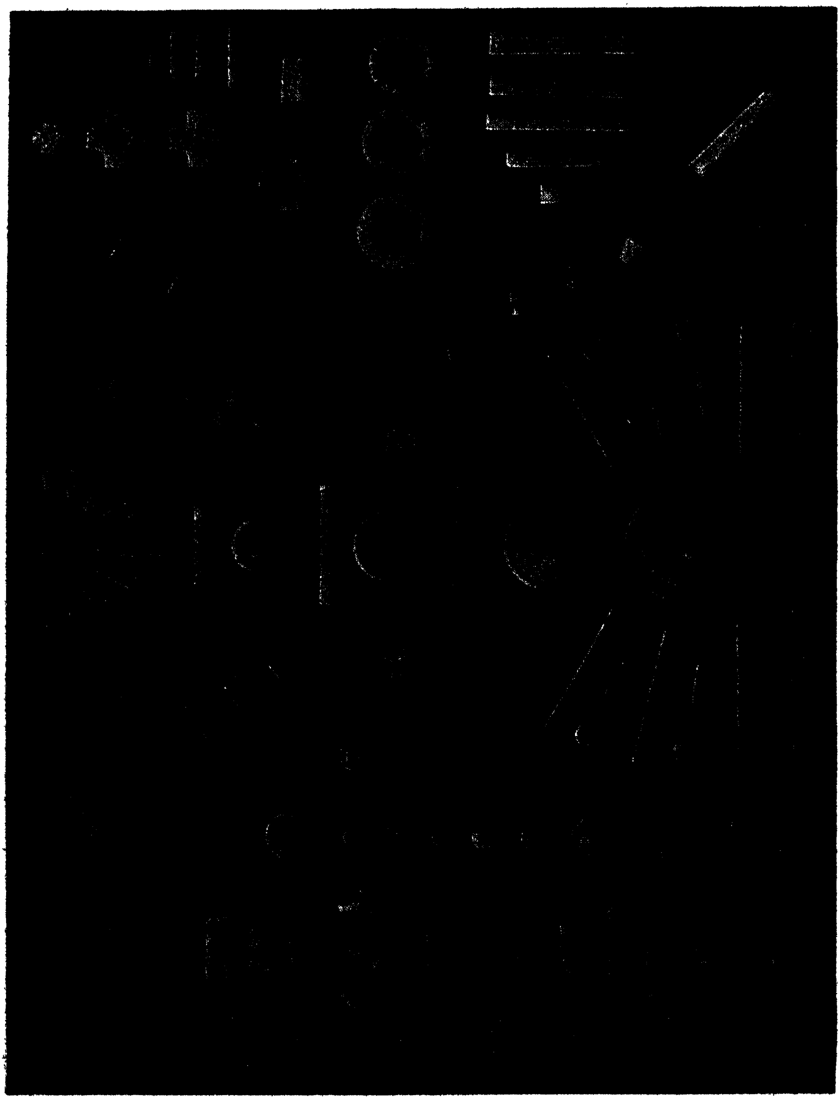


Fig. 1. Typical insulators used in the fabrication of radio receivers, transmitters, and other radio devices. (Courtesy, General Ceramics.)

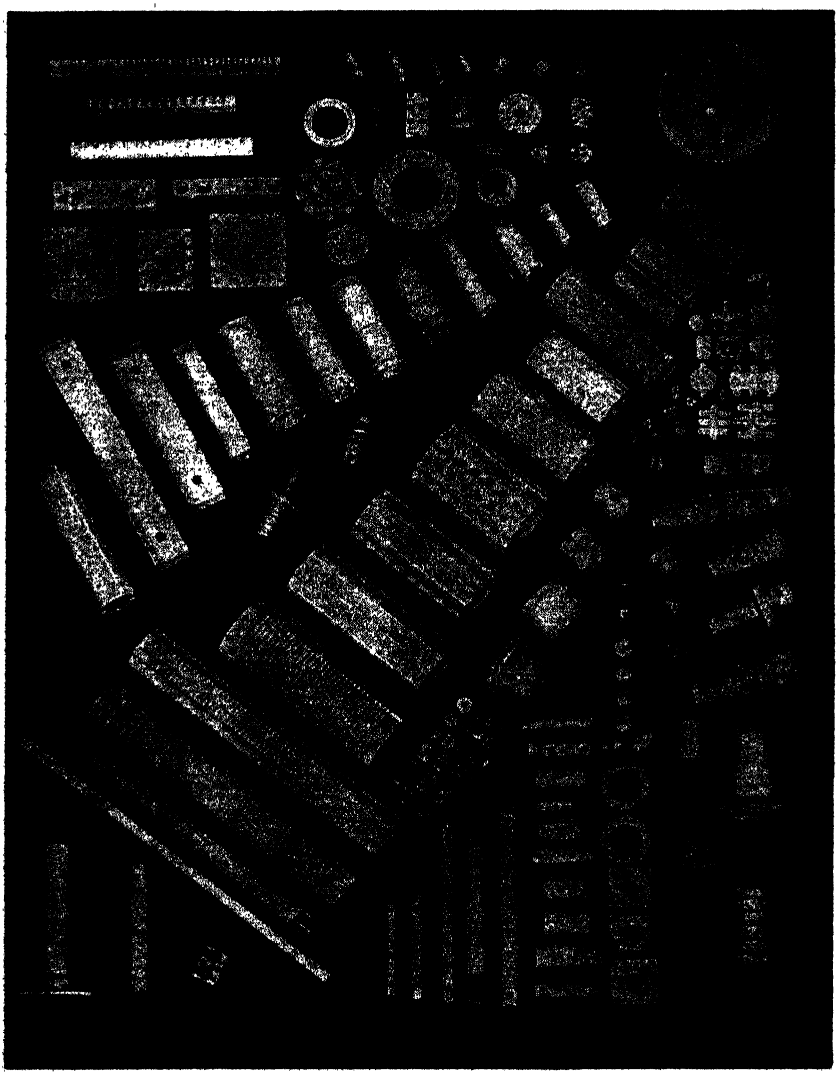


Fig. 2. Typical insulators used in the fabrication of radio receivers, transmitters, and other radio devices. (Courtesy, Stupakoff Ceramic and Manufacturing Company.)

15. Conductors and Insulators. All matter may be divided into two general classifications of *conductors* and *insulators*. A material which allows the flow of current through the material with little opposition is called a *conductor*, and a material which offers so much opposition that practically no current at all flows through the material is called an *insulator*. The resistance of a conductor is low, whereas the resistance of insulators is extremely high. Insulating materials are used to cover and support conductors in order that electric current may be directed along certain paths. There is no perfect conductor or insulator. Commercial insulating materials are readily available whose insulating properties are satisfactory for most common uses.

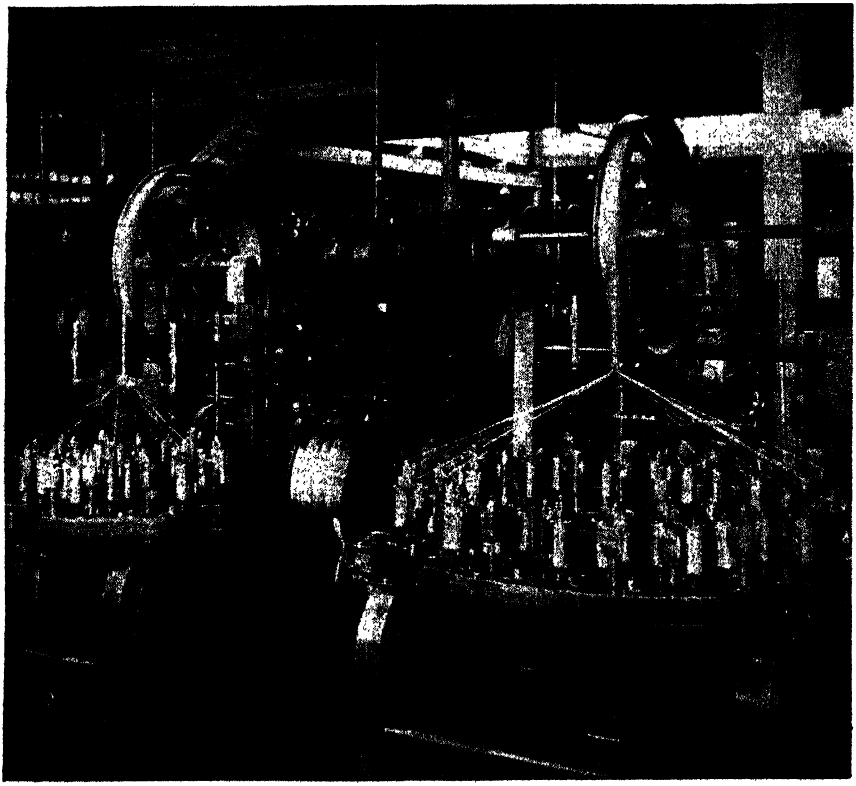


Fig. 3. Machines that braid several wires together, forming a stranded conductor. (Courtesy, Belden Manufacturing Company.)

A radio receiver or transmitter has a multitude of insulators, insulated parts, insulated conductors and noninsulated conductors. Coils of wire for inductances may be wound on an insulator coil form with insulated wire or bare wire spaced such that the turns do not touch. A few insulators used in radio work are shown in Figs. 1 and 2.

16. Electrical Circuits. The electrical paths formed by electrical conductors are called *electrical circuits.* The circuit of a radio receiver not only embraces the conductors or wires that connect the various parts together electrically but also the various electrical paths through such parts.

17. Resistance Devices. There are a multitude of electrical devices which make use of the resistance principle. Soldering irons, electric heaters, and electric-light bulbs contain conductors which have a high resistance per foot, compared to the resistance of the connecting cord or power wires to the device. Resistance wire is used in these

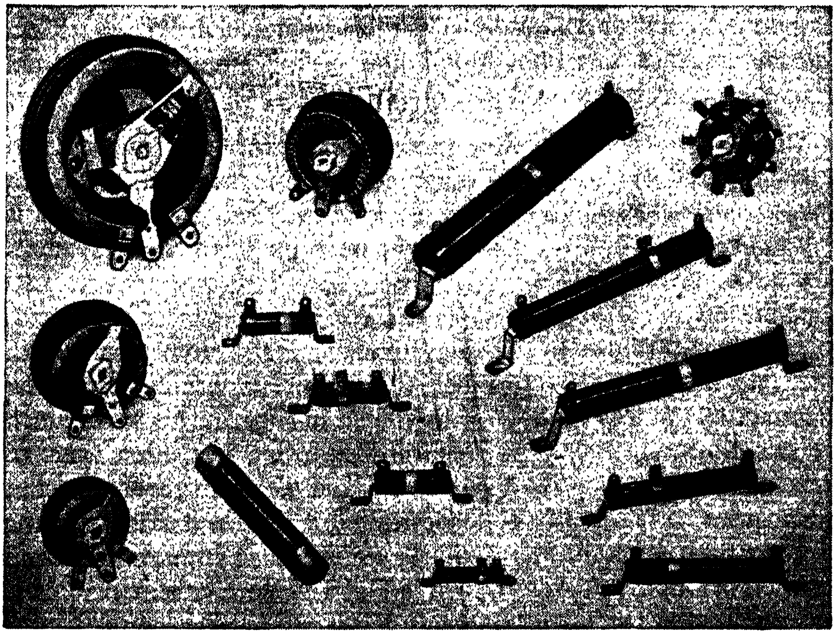


Fig. 4. Various types of variable, tapped, and special purpose wire-wound resistance units common to the radio industry. (Courtesy, Ohmite Manufacturing Company.)

devices and heats up when very much current is forced through it. Radio circuits contain many resistance elements. Commercial resistance elements are called *resistors*. Such resistors are available in almost any resistance size and current-flowing capacity desired. Some radio-circuit resistance elements are variable in resistance and are called *rheostats* and *potentiometers*. Some are designated by their function in the circuit such as the volume control.

Resistors are not always made from resistance wire. Carbon and carbon compounds are used extensively in the manufacture of small sized resistors whose current-carrying requirements are relatively low. Carbon is mixed into a compound with an insulating material which acts as a binder. The ratio of carbon and insulator material determines the resistance of the mixture. Resistors are molded from this compound and suitable connecting leads attached. Fig. 4 shows various types of variable, tapped, and special purpose wire wound resistance units common to the radio industry.

18. Electrical Symbols. Radio circuits are often quite involved and it is not practical to use block diagrams in laying them out. It is much more convenient and accurate to use *symbols*. Each circuit element is represented by a symbol. A few of the more common circuit symbols are shown in Fig. 5.

Quite often the question is raised as to which type of "crossed-wires" symbol to use. In drawings involving many crossed wires with the lines relatively close together, the practice is usually to run the lines straight across each other as shown in Fig. 5. This saves much of the draftsman's time in preparing the print, but often leads to confusion when the print becomes old and worn. In reading such a worn print, it is often hard to determine if the lines cross or if the dot has been worn off and the lines are meant to connect. The loop method of crossed wires, as shown in Fig. 5, is more satisfactory and is usually used in less involved circuit diagrams.

19. Slide Rule. One of the first things taught all engineering students early in their college curriculum is the use and operation of the slide rule. This is usually given in the freshman year and requires no particular prerequisite. In fact, many high schools now include this in the regular curriculum. The use of the slide rule is not particularly hard to learn, and is a great timesaver in performing various mathe-

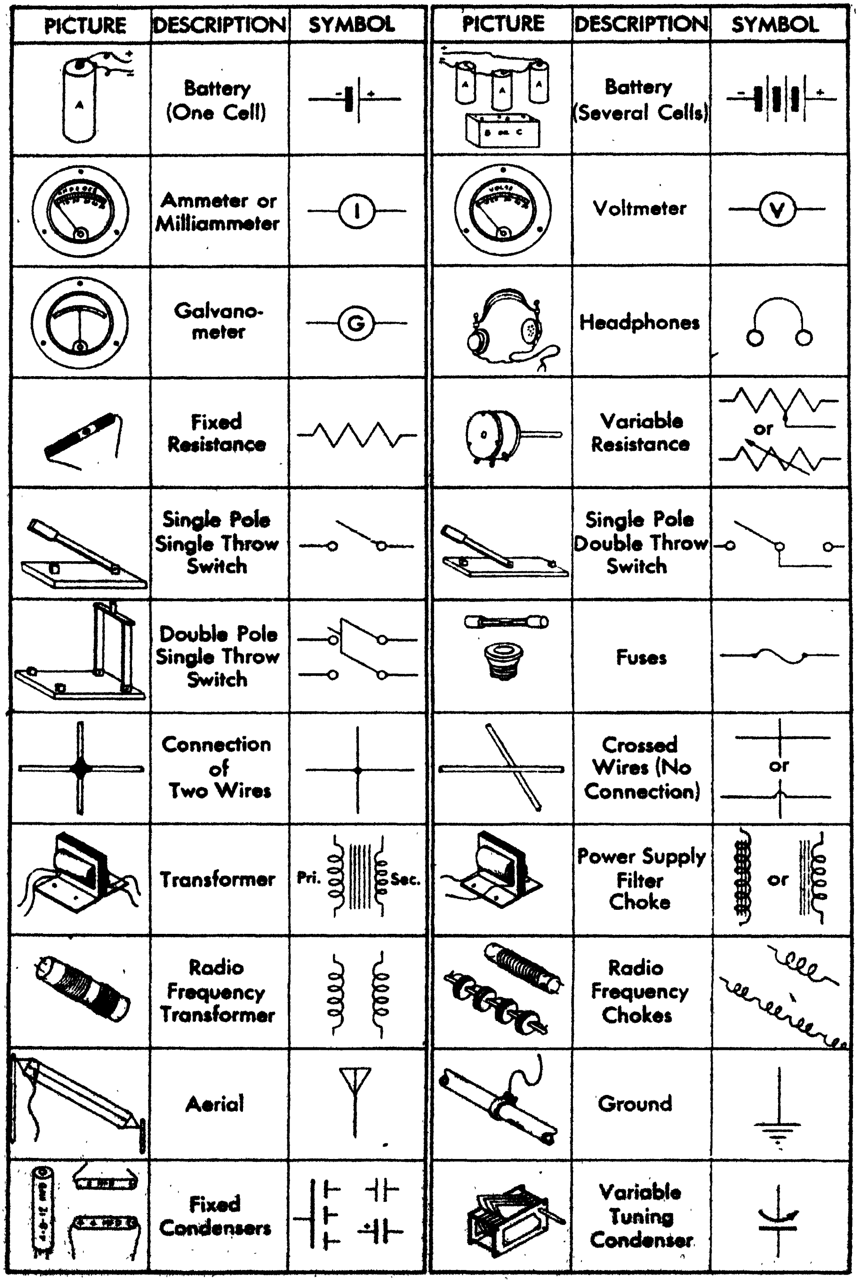


Fig. 5. A few of the more commonly used circuit symbols.

matical operations, such as multiplication, division, square roots, and cube roots.

The accuracy of the slide rule is satisfactory for most radio-engineering calculations and was used entirely in computing the answers to all problems in this book requiring multiplication, division, and roots; therefore, the student should not attempt to obtain answers that necessarily agree exactly with the given answer in every instance. An answer of 45.6 is sufficiently close if the given answer is 45.5 or 45.7. Such an answer is well within the accuracy of the slide rule and is satisfactory in most calculations of radio-circuit elements and their behavior. It is desirable that students acquire the habit of obtaining answers that coincide with practical applications.

It is beyond the scope of this book to cover the operation of the slide rule, even though its use is highly recommended. Many different types of slide rules are manufactured, most of which are accompanied by a book of instructions.

REVIEW QUESTIONS

1. What are elements?

2. Give the approximate number of known elements.

3. Define an atom.

4. Compare the atom with the molecule.

5. What is an electron?

6. Is there any relation between the electron and the many common materials, such as wood, cotton, rubber, etc.?

7. What is the law in regard to like and unlike electrical charges?

8. Discuss the positive charge of electricity.

9. Define an electrical current.

10. Name and define the unit of electric current.

11. How does current "flow"?

12. In what way may the voltage of an electric circuit be compared to pressure in a water system?

13. Explain electrical resistance.

14. Give the factors that determine the resistance of a material.

15. What is meant by the resistivity of metals?

16. Give and define the unit of resistance.

17. Discuss the properties of materials that determine whether the material is either an insulator or a conductor.

18. What is an electrical circuit?

19. What are resistors?

20. Why are electrical symbols used in radio circuits?

CHAPTER III *Ohm's Law*

In the preceding chapter, electric current was explained. Electric current flows as a result of voltage. Any electrical conductor offers opposition to the flow of current; therefore, the circuit must contain a battery or other voltage source before any current will flow.

Voltage, current, and resistance are closely associated by a mathematical relationship, covered in this chapter.

A proper understanding of the electrical fundamentals requires a certain knowledge of arithmetic and algebra. Numerous passages of mathematics are included, therefore, all of which may not be needed in the study of the electrical sections; however, all the material should be well learned as it will be needed later on.

20. Arithmetic. Arithmetic deals with numbers represented by the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Operations of arithmetic deal with multiplication, division, addition, and subtraction. *Arithmetic does not deal with negative numbers.* This fact alone limits the usefulness of arithmetic. Arithmetic does not deal with letters and symbols; therefore, the simple statement of Ohm's law in equation 2 could not be made if we were confined only to the use of arithmetic.

21. Algebra. The shortcomings of arithmetic have already been discussed. From that discussion, it is evident that more flexible mathematical tools must be used if one is to progress far in the study of radio. Algebra is one of these tools. A popular concept of algebra is that it deals with unknowns. *Algebra is a mathematical science whereby both numbers and symbols* are used and whose values may be either *positive* or *negative.*

22. Mathematical Symbols. A *symbol* is an abbreviated name representing a quantity. Examples of this are: E = voltage; I = current; R = resistance. Symbols are used for convenience in long mathematical calculations and for a statement of relations in formulas, such as $E = IR$. Numerous quantities such as voltage, current, resistance, impedance, and many others, are used so extensively in radio calculations that certain symbols have been universally adopted to represent them.

Quite often it is not only necessary to identify quantities, one from the other, but to distinguish between similar quantities, such as several resistors. R represents resistance. To distinguish one resistor from another in a circuit containing several resistors, it is necessary to use *subscripts*. Subscripts are the small numbers, or symbols, placed at the *lower right-hand side of symbols*. Examples are: R_1, R_2, R_3. These are read: *R sub one*, *R sub two*, and *R sub three*. Subscripts are not to be confused with exponents, which are placed on the upper right-hand side. Exponents will be taken up in their proper order. It should be kept in mind that subscripts have no value and are used only to *name a quantity* and are not, in any way, to indicate the value of the quantity.

23. Coefficients. The expression *ayc* indicates: *a* times *y* times *c* or $a \times y \times c$. *a*, *y*, and *c* are all factors of the product *ayc*. Likewise, *ay*, *ac*, and *yc* are also factors of the product *ayc*. *a* is a coefficient of the remaining portion of the product which is *yc*. Likewise, *y* is a coefficient of *ac* and *c* is a coefficient of *ay*.

Any factor of a product is a coefficient of the remaining factors of the product.

24. Positive and Negative Numbers. Numbers *greater* than zero are positive. Numbers *less* than zero are negative. Negative numbers are always prefixed with a negative or minus sign (−). Thus, −27; −4; −122 are negative numbers. Positive numbers may be prefixed with a positive or plus sign (+), but this sign is usually omitted, as an absence of any sign is an indication that the number is positive. Thus, 17; 8; 19; 364 are all considered to be positive, since no sign precedes the numbers.

Figure 6 is a *graphical representation* of the relation between positive and negative numbers. Plus 3 is greater than plus 2. Plus 3 is greater than any negative number, however large, such as −2; −4; −9; −2742; and so on. This should not be confused with the *absolute value* of a number which is its value without regard to sign.

It must be kept in mind that negative values are just as effective as positive values. In most instances, it is common practice for power supplies furnishing power to vacuum-tube circuits to have the negative side grounded. The other terminal, or free terminal, is positive and is well insulated and shielded from possible contact. In transmitting circuits, the power supplies that furnish grid-bias voltage to the vacuum tubes have the positive terminal grounded and the negative terminal is the free terminal. *High voltage is just as dangerous with the positive terminal grounded as with the negative terminal grounded.* In reality, there is no difference as one must make contact in such a way that both terminals are connected to the body, either by direct contact with the terminals or to conductors that connect to the terminals before a shock is obtained.

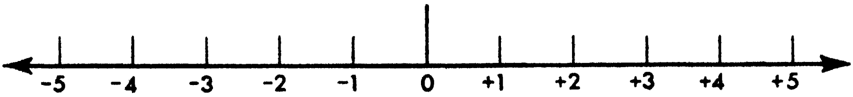


Fig. 6

25. Laws of Positive and Negative Numbers. A positive number added to a positive number gives a *plus* sum, such as 6 plus 4 equals plus 10.

A negative number added to a negative number gives a *negative* value, such as −6 plus −4 equals minus 10.

A positive number added to a negative number gives a sum whose sign is positive if the numerical value of the positive number is greater than that of the negative number. Thus, 6 plus −4 equals plus 2. If the numerical value of the positive number is less than that of the negative number, the sign of the sum is minus. Thus, 6 plus −9 equals minus 3. Likewise, $-9 + 6 = -3$.

26. Zero. *Division by zero is not allowed.* Sometimes it is said that any number divided by zero gives infinity, represented by the symbol ∞. Thus, $\frac{6}{0} = \infty$, but so would any other number divided by zero equal infinity, so such a division is meaningless. If $\frac{6}{0}$, $\frac{30}{0}$, were each equal to infinity, then we would have $\infty \times 0 = 6$ and $\infty \times 0 = 30$, and 30 equal to 6, which is not true.

Any number multiplied by zero is equal to zero.

Examples:

1. $6 \times 0 = 0$
2. $0 \times 10 = 0$
3. $1{,}628 \times 0 = 0$
4. $-3 \times 0 = 0$
5. $I \times 0 = 0$
6. $a \times 0 \times b = 0$
7. $6 \times 0 \times 4 = 0$

Zero added to or subtracted from a number does not change the value of that number in any way.

Examples:

1. $8 + 0 = 8$
2. $8 - 0 = 8$
3. $8 + (0 \times 2) = 8$
4. $8 + 0 + 2 = 10$
5. $8 - 0 + 2 = 10$
6. $8 - 2 + 0 = 6$

27. Equal Signs. The *equal sign* (=) is a symbol used to separate *two equal quantities.*

The unequal sign ($\neq$) is a symbol used to indicate that two quantities are not equal, such as: $6 \neq 4$.

28. Equations. An equation is a statement that two quantities are equal. One quantity is separated from the other quantity to which it is equal by an equal sign. The expression, $E = IR$, is a typical equation. It is possible for each side of the equation to contain several terms.

Examples:

1. $I_1 + I_2 = I_3$
2. $E_1 + E_2 = E_3 - I_4R_4$
3. $P_1 = EI - 2I_2{}^2R_2$
4. $2I(R + 2R) = E - E_R$
5. $E_1 + E_2 + E_3 - E = 0$
6. $2 - 3 + 5 - 1 + 1 - 4 = 0$
7. $I_1 - I = 0$

Inspection of the above examples shows that it is possible for all the terms on one side of an equation to be equal to 0. In such instances, the terms cancel out, leaving $0 = 0$.

29. Equation Rules. Equations are used extensively in the solution of many radio problems. Before the student attempts the solution of such problems, he should be fully familiar with the fundamental rules associated with the mathematical manipulation of the equation.

Rule 1. A term may be transferred from one side of the equation to the other by changing the sign of the term.

Examples:

1. $16y - 4x = 3z$
   $16y = 3z + 4x$

2. $2x + y - 3z = 3a - 2b$
   $2x - 3z - 3a + 2b = -y$

Rule 2. The signs of all terms on one side of the equation may be changed if all the signs of the terms on the other side of the equation are changed.

Examples:

1. $2x + 2y = 4a - 3b$
   $-2x - 2y = 3b - 4a$

2. $a - 2b - 3c = 5d - 10x - 4y$
   $2b + 3c - a = 10x - 5d + 4y$

Rule 3. A quantity or number may be added to one side of an equation if the same is added to the other side.

Examples:

1. $2x + 3y = 4z - 2a$
   $2x + 3y + 3b = 4z - 2a + 3b$

2. $R = \dfrac{R_1R_2}{R_1 + R_2}$

   $R + 2 = \dfrac{R_1R_2}{R_1 + R_2} + 2$

Rule 4. A quantity or number may be subtracted from one side of an equation if the same is subtracted from the other side.

Examples:

1. $IR_1 = E_1$
   $IR_1 - E_2 = E_1 - E_2$

2. $I_1 - I_2 = I + I_3$
   $I_1 - I_2 - I_4 = I + I_3 - I_4$

Rule 5. Both sides of an equation may be multiplied or divided by the same number and the two sides will still be equal.

Examples:

1. $I_1 = I_2$

Multiply both sides by R

$$I_1R = I_2R$$

Divide both sides by R

$$\frac{I_1R}{R} = \frac{I_2R}{R}$$

2. $E_1 = E_2$

Divide both sides by R

$$\frac{E_1}{R} = \frac{E_2}{R}$$

3. $I_2 = I_3$

Multiply both sides by $(R - 2)$

$$I_2(R - 2) = I_3(R - 2)$$

4. $R_1 + R_2 = R + R_3$

Multiply both sides by $\frac{a}{2}$

$$\frac{a(R_1 + R_2)}{2} = \frac{a(R + R_3)}{2}$$

Rule 6. The denominator on one side of an equation may be transferred across the equal sign if it is placed in the numerator of the other side.

Rule 7. The numerator on one side of an equation may be transferred across the equal sign if it is placed in the denominator of the other side.

Rules 6 and 7 pertain to what is commonly called *cross-multiplication*. These rules are particularly helpful in simplifying equations, thereby making their solution comparatively simple.

Examples:

1. $\frac{a}{b} = \frac{c}{d}$

$$a = \frac{bc}{d}$$

$$ad = bc$$

$$\frac{ad}{c} = b$$

$$d = \frac{bc}{a}$$

2. $R = \frac{R_1R_2}{R_1 + R_2}$

$$R(R_1 + R_2) = R_1R_2$$

$$\frac{R(R_1 + R_2)}{R_1} = R_2$$

$$\frac{R(R_1 + R_2)}{R_2} = R_1$$

$$R_1 + R_2 = \frac{R_1R_2}{R}$$

$$R_1 = \frac{R_1R_2}{R} - R_2$$

Exercise 1:

1. $\frac{E_1}{R_1} = \frac{E_2}{R_2}$; $E_2 = ?$
2. $\frac{a + b}{c} = 16$; $a = ?$
3. $\frac{x + y}{a - b} = 4$; $b = ?$
4. $\frac{(2x - 4y)}{2a} = \frac{1}{(b - 1)}$; $b = ?$
5. $R = \frac{R_1 R_2}{R_1 + R_2}$; $R_1 = ?$
   $R_2 = ?$

30. Ohm's Law. Early experimenters found that certain laws govern electrical circuits and circuit elements. From these laws the behavior of voltage and current can be predetermined for a circuit. One of the early experimenters was Simon Ohm, who made extensive studies of the resistive properties of materials. For this reason, the unit of resistance was named *ohm* in his honor.

One of the most important laws that govern electrical circuits is *Ohm's law*. Ohm's law is the relation between resistance, current, and voltage,

$$E = IR \tag{2}$$

where E is voltage in *volts*, I is current in *amperes*, and R is resistance in *ohms*. Quite often, currents in radio circuits are less than 1 ampere and are measured in milliamperes. Any current in milliamperes must be converted into its respective fractional part of an ampere before it can be used in the above formula. Divide milliamperes by 1,000 in order to convert them to amperes.

It may not always be convenient to use Ohm's law as expressed in equation 2. Other expressions derived from this equation by means of simple algebra are:

$$I = \frac{E}{R} \tag{3}$$

$$R = \frac{E}{I} \tag{4}$$

Equation 2 states that in Fig. 7, E is the voltage necessary to force a current, I, through a resistance, R. Suppose it is desired to know what battery voltage is necessary to force a current of 2 amperes through a resistance of 7 ohms.

E—to be determined
I—given as 2 amperes
R—given as 7 ohms

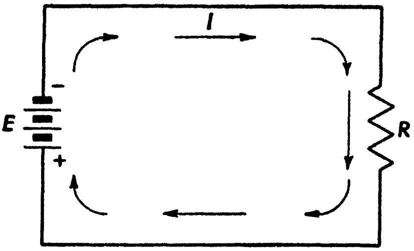


Fig. 7

The circuit is shown in Fig. 8.

$$E = IR$$
$$E = 2 \times 7$$
$$E = 14 \text{ volts}$$

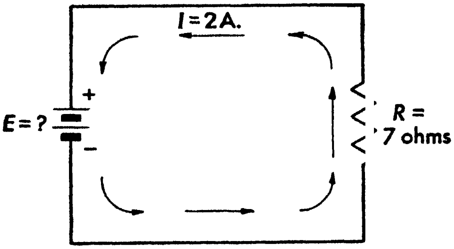


Fig. 8

Equation 3 states that the current flowing in the circuit of Fig. 7 is determined by dividing the voltage by the resistance. From Fig. 9,

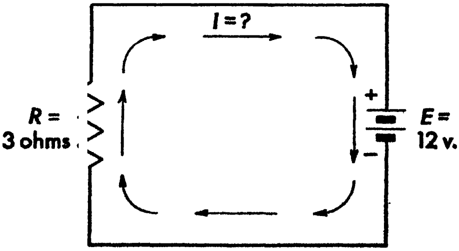


Fig. 9

what current will be forced through the 3-ohm resistor by the battery, whose voltage is 12 volts?

I—to be determined
E—given as 12 volts
R—given as 3 ohms

$$I = \frac{E}{R}$$

$$I = \frac{12}{3}$$

$$I = 4 \text{ amperes}$$

Equation 4 states that the resistance of the circuit is determined by dividing the voltage by the current. Determine, from Fig. 10, what resistance will be required to limit the current flowing in the circuit to 4 amperes if a voltage of 20 volts is forcing the current through the resistance.

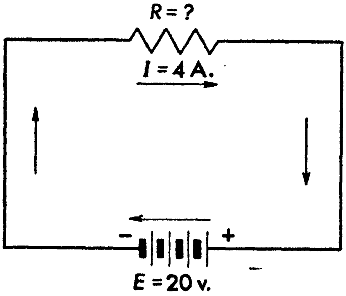


Fig. 10

R—to be determined
I—given as 4 amperes
E—given as 20 volts

$$R = \frac{E}{I}$$

$$R = \frac{20}{4}$$

$$R = 5 \text{ ohms}$$

Exercise 2:

1. Find the voltage necessary to force 15 amperes of current through a resistance of 30 ohms.
2. Find the resistance of a circuit if the current is 2 amperes and the voltage is 15 volts.
3. What current will flow through a resistor of 50 ohms, connected to a battery of 20 volts?
4. A light bulb, having a resistance of 1.5 ohms, is connected in series with a 6-volt storage battery. Find the current.

5. The filament terminals of a vacuum tube are connected across the terminals of a 6-volt storage battery. The current through the filament is 300 milliamperes. What is the resistance of the filament?

31. Law of Signs. It is possible to change the sign of the numerator of a fraction if either the sign in front of the fraction or the sign of the denominator is changed.

Example:

$$-\frac{-a}{-b} = \frac{a}{-b} \; or \; -\frac{a}{b}$$

Likewise, the sign of the denominator must be changed if the sign in front of the fraction or the sign of the numerator is changed.

Example:

$$-\frac{-c}{-d} = \frac{-c}{d} \; or \; -\frac{c}{d}$$

The sign in front of a fraction may be changed if either the sign of the numerator or the sign of the denominator is changed.

$$-\frac{-x}{-y} = \frac{x}{-y} \; or \; \frac{-x}{y}$$

32. Grouping Signs. The most common signs of grouping are the parentheses ( ), the brackets [ ], and the braces { }. These signs are used to group together portions of an equation which are affected by the same operation, such as the multiplication of all terms by the same factor.

It is usually best to start with the innermost grouping sign when it is desired to remove grouping signs in order to simplify or proceed with the solution of an equation.

The signs of the terms inside a grouping sign do not change when the grouping sign is removed if it is *preceded* by a *plus* sign.

Example:

$$2x + (3b - 4a) = 2$$
$$2x + 3b - 4a = 2$$

The sign of each term inside a grouping sign changes (from plus to minus or from minus to plus) when the grouping sign is removed if the grouping sign is *preceded* by a *minus* sign.

Examples:

1. $2x - (3y + 2a) + 2(a + b) = 6$
$2x - 3y - 2a + 2a + 2b = 6$
$2x - 3y + 2b = 6$

2. $6\{x - [2 + a + 2(x - 1)] - b\} + 2a = 10$
$6\{x - [2 + a + 2x - 2] - b\} + 2a = 10$
$6\{x - 2 - a - 2x + 2 - b\} + 2a = 10$
$6x - 12 - 6a - 12x + 12 - 6b + 2a = 10$
$-6x - 4a - 6b = 10$

Exercise 3:

Remove all grouping signs:

1. $2(a - b) - (2x - z)$
2. $a\{[(2 - c) - b] + 1\}$
3. $-\{a + [-2b - (1 + c)]\} - 5$
4. $x - 2\{[-c + (a - b) + y]\} - 2$
5. $-[2x + c + 4 - (a - b)]$

Solve for unknowns:

6. $6a - 4 = a + 2$
7. $3b - 1 + b = 2(b - 4)$
8. $x - 2(x - 3) = \dfrac{2x + 1}{2}$
9. $I - (2I - 3) + 2 = \dfrac{I - 6}{2}$
10. $R + 4(1 - R) = R - (-1 + 2R)$

33. Exponents. An *exponent* is a number, letter, or symbol, which, when placed at the upper right side of another number, letter, or symbol, indicates how many times the latter is to be taken as a factor. a^3 indicates that a is to be taken as a factor three times.

$$a^3 = a \cdot a \cdot a \tag{5}$$

where a is the base and 3 is the *exponent* or *power*.

34. Law of Exponents for Multiplication. It is self-evident that a^3 multiplied by a will give a^4. From this we may formulate the rule:

(6) *The product of two or more powers with the same base is determined by adding the exponents.* That is,

$$a^x a^y = a^{x+y} \tag{7}$$

$$a^x a^{-y} = a^{x-y} \tag{8}$$

Examples:

1. $a \cdot a^2 \cdot a = a^{1+2+1} = a^4$
2. $a^3 \cdot a^5 = a^{3+5} = a^8$
3. $b^x \cdot b^y \cdot b^c = b^{x+y+c}$
4. $a \cdot a^{-1} \cdot a^2 = a^{1-1+2} = a^2$
5. $a^{-x} \cdot a^{b+1} = a^{b+1-x}$
6. $a^x \cdot a^{2x} = a^{x+2x} = a^{3x}$
7. $a^{x+y} \cdot a^{x+y} = a^{2(x+y)} = a^{2x+2y}$
8. $(a^3b^2c^4)^3 = a^{3\times3}\,b^{2\times3}\,c^{4\times3} = a^9b^6c^{12}$

Exercise 4:

Multiply:

1. $(2a^3)\,(4a^6)$
2. $(-b^2)\,(2ab^3)$
3. $(a^2b)\,(-6ab^2)$
4. $(10ab^3c^2d^{-1})\,(-a^2bcd^4)$
5. $(\frac{1}{3}f^2gh^2)\,(\frac{2}{5}f^3g^4h^3)$
6. $(a^2b^3c^5d)\,(-\frac{17}{18}ab^4cd^3)$
7. $(-\frac{1}{15}x^2y^{b-1}z)\,(\frac{4}{5}x^2y^cz^{2+d})$
8. $(2a^23b^3d^3)^2$

35. Law of Exponents for Division. The law of exponents for division does not vary greatly from that of multiplication. For *multiplication*, the exponents are *added* and with *division*, they are *subtracted.* Thus:

$$\frac{a^x}{a^y} = a^xa^{-y} \text{ or } a^{x-y} \tag{9}$$

which is true for all values of a except zero. Zero raised to *any power* is still *zero.*

It will be noted from equation 9 that a^y was transferred from the denominator to the numerator by changing the sign of its exponent. Likewise, it is possible to transfer a^x from the numerator to the denominator by changing the sign of its exponent.

$$\frac{a^x}{a^y} = \frac{1}{a^{y-x}} \tag{10}$$

Examples:

$$\frac{a^3}{a^2} = a^3a^{-2} = a \text{ or } \frac{a^3}{a^2} = \frac{1}{a^2a^{-3}} = \frac{1}{a^{2-3}} = \frac{1}{a^{-1}} = a$$

$$a^2 = a^{2-3} = a^{-1} \text{ or } \frac{1}{a}$$

3. $\frac{x^{a+b}}{x^{b-a}} = x^{(a+b)-(b-a)} = x^{a+b-b+a} = x^{2a}$

4. $\frac{a^2b^2}{ab} = a^{2-1}b^{2-1} = ab$

It is self-evident that:

$$\frac{a^x}{a^x} = 1, \text{ or that:}$$

$$\frac{a^2}{a^2} = 1, \text{ also that:}$$

$$\frac{3^2}{3^2} = \frac{9}{9} = 1; \text{ however,}$$

$$\frac{a^x}{a^x} = 1 \text{ is also equal to:}$$

$$\frac{a^x}{a^x} = a^{x-x} = a^0 = 1 \tag{11}$$

where a is any base *except zero.*

Then, *any base, except zero, with an exponent of zero is equal to 1.*

Examples:

1. $\frac{x^4y}{xy} = x^{4-1}y^{1-1} = x^3$

2. $\frac{a^2(-b^3)c^4}{a^3b^3c^4} = a^{2-3}(-b^{3-3})c^{4-4} = -a^{-1}$

Exercise 5:

Divide:

1. $\frac{a^4b^3x^c}{a^2bx^d}$

2. $\frac{a^{x-1}}{2a^{x-1}}$

3. $\frac{x^2y^{2b}z^3}{x^b(-y^{2b})}$

4. $\frac{a^3b^2c^4}{a(-b^2)c}$

5. $\frac{x^5y^2d^3}{x^2(-y)^2(d)^2}$

6. $\frac{2f^2g^{-3}h^{-1}k^2}{32f^{-1}(-g)^{-2}h^2k^{-3}}$

7. $\frac{\frac{1}{4}a^{-1}b^{-1}(-c)^{-3}(-d^2)}{\frac{1}{8}a^2(-b^3)c^2d^{-1}}$

36. Radicals. Quite often it is desirable to know what number multiplied by itself (squared) will give a certain known number. When such an operation on a number is performed, it is said that the *square root* of the number is obtained. The radical sign $\sqrt{\ }$ is placed over a number, called the *radicand*, to indicate that the square root of the number is to be obtained. $\sqrt{4} = 2$ and $\sqrt{64} = 8$.

Some mathematical operations require that other roots besides the square root be performed. A small number is placed over the "V" portion of the radical sign to indicate the root. This small number is called the *index of the root.*

Examples:

$$\sqrt[3]{8} = 2$$
$$\sqrt[4]{16} = 2$$
$$\sqrt[5]{32} = 2$$

An index of 2 is not placed over the radical sign to indicate the square root of a number as the radical sign alone is considered sufficient.

Two *negative* numbers or two *positive* numbers multiplied together give a plus result. Then, the square root of a positive number gives two roots, one positive and one negative. Thus $(-2)(-2) = 4$ or $(2)(2) = 4$; therefore, $\sqrt{4} =$ plus 2 or minus 2, written ± 2.

The square root of a *negative* number may be *indicated* (such as $\sqrt{-1}$; $\sqrt{-2}$; $\sqrt{-3}$) but it is not possible to take the square root of a negative number.

37. Addition and Subtraction of Radicals. The addition or subtraction of radicals can only be indicated unless they have like indices and like radicands.

Examples:

1. $2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$
2. $3\sqrt{a} - 2\sqrt{a} = \sqrt{a}$

$2\sqrt[3]{a} - \sqrt[3]{b} + \sqrt[3]{a} = 3\sqrt[3]{a} - \sqrt[3]{b}$

3 Multiplication of Radicals. The multiplication of two radicals with index of 2 and like radicands is accomplished by simply removing e radical sign with the result equal to one radicand. Thus:

$$\begin{aligned} \sqrt{2} \times \sqrt{2} &= 2 \\ \sqrt{5} \times \sqrt{5} &= 5 \\ \sqrt{10} \times \sqrt{10} &= 10 \\ \sqrt{a} \times \sqrt{a} &= a \end{aligned}$$

The multiplication of two radicals with an index of 2 and unlike radicands is accomplished by placing all the radicands under one radical and considering them as a product. Thus:

$$\sqrt{3} \times \sqrt{5} = \sqrt{3 \times 5} = \sqrt{15}$$

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

39. Division of Radicals. The division of one radical by another radical with the same index but with a different radicand is shown by the following equation:

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \tag{12}$$

40. Fractions. There are two types of fractions: *common* and *decimal.* $\frac{1}{2}$ is a common fraction and .5 is a decimal fraction. Common fractions are converted into decimal fractions by dividing the bottom member (*denominator*) into the top member (*numerator*). Common fractions are added or subtracted by first finding a *common denominator,* while decimal fractions require no treatment before adding or subtracting or dividing.

Examples:

1. $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$
2. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12}$ or $1\frac{1}{12}$

In adding or subtracting decimal fractions, the decimal points must be lined up, one with another.

Example:

$$\begin{array}{r} .0206 \\ .41 \\ 1.01 \\ \hline 1.4406 \end{array}$$

Exercise 6:

Perform the operations as indicated:

1. $\frac{3}{5} + \frac{2}{3} =$
2. $\frac{1}{3} + \frac{1}{5} + \frac{1}{2} =$
3. $\frac{5}{6} - \frac{3}{8} =$
4. $\frac{4}{5} \times \frac{1}{2} =$
5. $\frac{6}{11} \times \frac{9}{16} =$
6. $\frac{12}{14} \times \frac{20}{25} =$
7. $\frac{3}{4} \div \frac{1}{4} =$
8. $\frac{12}{20} \div \frac{3}{40} =$
9. $\frac{14}{17} \div \frac{3}{16} =$
10. $\frac{13}{7} \div \frac{4}{5} =$
11. $.25 + .062 + 1.26 =$
12. $4.006 + .010 + .122 =$
13. $.75 - .0003 - 1.07 =$
14. $.0026 + .00032 - .00004 =$
15. $2.67 \div .074 =$
16. $.625 \div .92 =$
17. $.006 \div .03 =$
18. $.107 \div 2.1 =$
19. $.0102 \div .0003 =$
20. $482 \div .00109 =$

Decimal fractions may be converted into common fractions by multiplying the fraction by a multiple of 10 sufficient to make it a whole number which becomes the numerator of a common fraction with the same multiple of 10 as a denominator. Thus:

$$.5 = .5 \times \frac{10}{10} = \frac{5}{10}$$

$$.003 = .003 \times \frac{1000}{1000} = \frac{3}{1000}$$

41. Fractional Exponents. It is possible to consider radicals as fractional exponents. The denominator of a fractional exponent denotes *the root* and the numerator denotes *the power*. Thus:

$$a^{\frac{1}{2}} = \sqrt{a}$$

$$a^{\frac{1}{5}} = \sqrt[5]{a}$$

Other examples of fractional exponents are:

1. $a^{\frac{1}{2}}a^{\frac{1}{2}} = a^{\frac{1}{2}+\frac{1}{2}} = a^1 = a$
2. $a^{\frac{2}{3}} = \sqrt[3]{a^2}$
3. $a^{\frac{1}{2}}a^{\frac{1}{3}} = a^{\frac{1}{2}+\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}$
4. $a^{\frac{3}{2}} = \sqrt{a^3} = \sqrt{a^2a} = a\sqrt{a} = a^{1\frac{1}{2}}$
5. $a^{\frac{6}{2}} = \sqrt{a^6} = \sqrt{a^4a^2} = a\sqrt{a^4} = a\sqrt{a^2a^2} = aaa = a^3$

Also

$$a^{\frac{6}{2}} = a^{\frac{3}{2}+\frac{3}{2}} = \frac{a^{\frac{3}{2}}}{a^{-\frac{3}{2}}} = \frac{\sqrt{a^3}}{\sqrt{\frac{1}{a^3}}} = \frac{\sqrt{a^3}}{\frac{1}{\sqrt{a^3}}} = \sqrt{a^3} \times \sqrt{a^3} = a^3$$

Exercise 7:

Perform indicated operations:

1. $6\sqrt{a} - 2\sqrt{b} - \sqrt{a} + 3\sqrt{b}$
2. $2\sqrt{c} \times 4\sqrt{c}$
3. $2c\sqrt{a} \times 3b\sqrt{a}$
4. $a^{\frac{1}{8}}a^2$
5. $a^{\frac{4}{5}}a^{\frac{2}{3}}$
6. $a^{\frac{1}{4}}a^3a^{\frac{3}{2}}$

Rewrite, expressing each fractional exponent as a radical:

7. $a^{\frac{2}{5}}$
8. $a^{\frac{4}{5}}$
9. $a^{\frac{5}{3}}$
10. $a^{\frac{12}{4}}$

Rewrite, expressing each radical as a fractional exponent:

11. $\sqrt[3]{a^2}$
12. $\sqrt{x^6}$
13. $\sqrt[5]{b^4}$
14. $\sqrt[6]{a^2b^3c}$
15. $\sqrt{x^by^c}$

42. Powers of Ten. Engineers use the slide rule extensively in engineering calculations. Slide-rule rules of operation provide for multiplying or dividing both large and small numbers. However, it is not convenient to work with large numbers or with small fractional numbers.

The powers of 10 provide a system whereby any number, such as a very large one, may be reduced to one sufficiently small to be used easily. Likewise, a very small fractional number may be increased to a whole number that is easier to use.

It is very convenient to multiply any number by 10 or by a multiple of 10: one has only to move the decimal point one digit to the *right* for each time the number is multiplied by 10.

Examples:

1. $168 \times 10 = 1{,}680$
2. $16.8 \times 10 = 168$
3. $1.68 \times 10 \times 10 = 168$
4. $1.68 \times 10 \times 10 \times 10 = 1{,}680$
5. $.5 \times 10 = 5$
6. $.062 \times 10 = .62$
7. $.0017 \times 10 \times 10 \times 10 \times 10 \times 10 = 170$

Any number may be divided by 10 by moving the decimal point one digit to the *left*. If a number is divided by a multiple of 10, the decimal point is moved to the left as many places as multiples of 10 are placed in the denominator.

Examples:

1. $\dfrac{428}{10} = 42.8$

2. $\dfrac{428}{10 \times 10 \times 10} = .428$

3. $\dfrac{.12}{10} = .012$

4. $\dfrac{.006}{10 \times 10} = .00006$

Multiples of 10 may be expressed as 10 with the number of mul ples as an exponent. Such numbers are called *powers of ten.*

$$10 \times 10 \text{ may be written as } 10^2$$

and

$$10 \times 10 \times 10 \text{ may be written as } 10^3,$$

also

$$\frac{1}{10 \times 10} \text{ may be written as } \frac{1}{10^2} \text{ or } 10^{-2}.$$

$$\frac{1}{10 \times 10} \text{ may also be written as } \frac{1}{100}$$

or .01 which may be expressed as

$$.1 \times 10^{-1} \text{ or } \frac{.1}{10} \text{ or } 1 \times 10^{-2} \text{ or } 10^{-2}.$$

Other exponents or powers of 10 are shown in the following tat

TABLE I

One-thousandth $= \dfrac{1}{1000} = \dfrac{1}{10^3} = .001 = 10^{-3}$

One-hundredth $= \dfrac{1}{100} = \dfrac{1}{10^2} = .01 = 10^{-2}$

One-tenth $= \dfrac{1}{10} = \dfrac{1}{10^1} = .1 = 10^{-1}$

One $= 1 = \dfrac{1}{10^0} = 10^0$

Ten $= 10 = \dfrac{1}{10^{-1}} = 10^1$

Hundred $= 100 = \dfrac{1}{10^{-2}} = 10^2$

Thousand $= 1000 = \dfrac{1}{10^{-3}} = 10^3$

Exercise 8:

Express each as a number between 1 and 10, times the proper power of 10:

1. 1,000,000
2. 4820
3. 6742×10^2
4. 1274×10^{-3}
5. 0.071
6. 0.00000042
7. 0.0300
8. 0.041×10^{-6}
9. $\frac{1}{161}$
10. $\frac{1}{648 \times 10^{-2}}$
11. $\frac{1}{.0146 \times 10^3}$
12. $\frac{10^2}{.0041 \times 10^{-1} \times 10^6}$

REVIEW QUESTIONS

1. Define arithmetic.
2. Define algebra.
3. What are mathematical symbols and why used?
4. What are coefficients?
5. What are positive and negative numbers?
6. What is Ohm's law? Give an example.
7. What is an exponent?
8. Give the law of exponents for multiplication.
9. Give the law of exponents for division.
10. Discuss radicals.
11. Give the laws for adding and subtracting radicals.
12. Give the laws for the multiplication and division of radicals.
13. Discuss fractional exponents.
14. What are powers of ten?
15. Why is a thorough knowledge of the powers of ten very important to the radio student?

ANSWERS

Exercise 1, page 21:

1. $E_2 = \frac{E_1R_2}{R_1}$
2. $a = 16c - b$
3. $b = \frac{4a - x - y}{4}$
4. $b = \frac{a + x - 2y}{x - 2y}$
5. $R_1 = \frac{RR_2}{R_2 - R}$

   $R_2 = \frac{RR_1}{R_1 - R}$

Exercise 2, page 23:

1. 450 volts
2. 7.5 ohms
3. .4 ampere
4. 4 amperes
5. 20 ohms

Exercise 3, page 25:

1. $2a - 2b - 2x + z$
2. $3a - ab - ac$
3. $-a + 2b + c - 4$
4. $x + 2c - 2a + 2b - 2y - 2$
5. $-2x - c - 4 + a - b$
6. $a = 1\frac{1}{5}$
7. $b = -3\frac{1}{2}$
8. $x = 2\frac{3}{4}$
9. $I = 5\frac{1}{3}$
10. $R = 1\frac{1}{2}$

Exercise 4, page 26:

1. $8a^9$
2. $-2ab^5$
3. $-6a^3b^3$
4. $-10a^3b^4c^3d^3$
5. $\frac{2}{15}f^5g^5h^5$
6. $-\frac{17}{18}a^3b^7c^6d^4$
7. $-\frac{4}{75}x^4y^{b-1+c}z^{3+d}$
8. $36a^4b^6d^6$

Exercise 5, page 27:

1. $a^2b^2c^{c-d}$
2. $\frac{1}{2}$
3. $-x^{2-b}z^3$
4. $-a^2c^3$
5. x^3d
6. $\dfrac{f^3g^{-1}h^{-3}k^5}{16}$
7. $-2a^{-3}b^{-4}c^{-5}d^3$

Exercise 6, page 30:

1. $1\frac{4}{15}$
2. $1\frac{1}{30}$
3. $\frac{11}{24}$
4. $\frac{2}{5}$
5. $\frac{27}{88}$
6. $\frac{24}{35}$
7. 3
8. 8
9. $4\frac{20}{51}$
10. $2\frac{9}{28}$
11. 1.572
12. 4.138
13. −.3203
14. .00288
15. 36.1
16. .679
17. .2
18. .0509
19. 34
20. 442,000

Exercise 7, page 31:

1. $5\sqrt{a} + \sqrt{b}$
2. $8c$
3. $6abc$
4. $a^{2\frac{1}{3}}$ *or* $a^{\frac{7}{3}}$
5. $a^{1\frac{7}{15}}$ *or* $a^{\frac{22}{15}}$
6. $a^{4\frac{3}{4}}$ *or* $a^{\frac{19}{4}}$
7. $\sqrt[5]{a^2}$
8. $\sqrt[6]{a^4}$ *or* $\sqrt[3]{a^2}$
9. $\sqrt[3]{a^5}$
10. $\sqrt[4]{a^{12}}$ *or* a^3
11. $a^{\frac{2}{3}}$
12. $x^{\frac{6}{2}}$ *or* x^3
13. $b^{\frac{4}{5}}$
14. $a^{\frac{2}{6}}b^{\frac{3}{6}}c^{\frac{1}{6}}$ *or* $a^{\frac{1}{3}}b^{\frac{1}{2}}c^{\frac{1}{6}}$
15. $x^{\frac{b}{a}}y^{\frac{c}{a}}$

Exercise 8, page 33:

1. 1×10^6
2. 4.82×10^3
3. 6.742×10^5
4. 1.274
5. 7.1×10^{-2}
6. 4.2×10^{-7}
7. 3×10^{-2}
8. 4.1×10^{-8}
9. 6.21×10^{-3}
10. 1.543×10^{-1}
11. 6.85×10^{-2}
12. 2.44×10^{-1}

CHAPTER IV *Resistance Circuits*

It is assumed that the student has pursued the mathematical sections of the preceding chapter well enough to delve deeper into the use of Ohm's law and its application to resistance circuits.

43. Division of Current. When a current flowing down a conductor reaches a point where the circuit branches into two paths, the current also divides. If each branch of the circuit has the same resistance, the current will divide evenly. Half of the current will flow through one branch and the other half of the current will flow through the other branch. If the resistances of the two branches are not equal, the one with the *least* amount of *resistance* will allow *more* current to flow through that branch. If several paths are provided, more current will flow through the path of lowest resistance than will flow through any of the other paths. *The current flowing in each path is inversely proportional to the resistance of that path.*

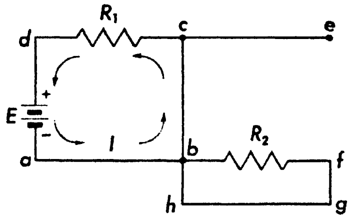


Fig. 11

It must be kept in mind that current does not flow in a circuit of its own accord. It is necessary that the circuit be such that the circuit is continuous and there is a voltage present to push the current through the circuit. Another way of stating the above fact is: There must be a *difference in potential* between two points in order for current to flow. It is not possible to connect a piece of wire such as *ce* to a circuit as in Fig. 11 and get a current to flow in *ce*. The end *e* must be connected to the circuit, through the battery, in such a way that *ce* will be in a continuous circuit.

In Fig. 11, current will flow out of the battery in the path to *a*, then to *b*, to *c*, to *d*, and back to the battery. No current flows through R_2, since the connection at *b* shorts out R_2. In other words, the connection at *b* provides a shorter path, with no resistance, so the current does not flow out to resistor R_2, which does offer resistance to the flow of current. It might seem to the student that current should flow from *a*, to *b*, to *f*, to *g*, to *h*, through *b* again and to *c*, to *d*, and back to the battery. This is not the case, since this portion of the circuit is not in series with the continuous circuit containing the battery.

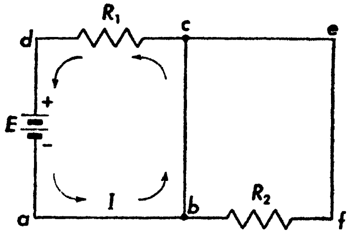


Fig. 12

Figure 12 shows the circuit of Fig. 11 with point *f* of resistor R_2 connected to *e* instead of to *g*, to *h*, and back to *b*.

It may now appear to the student that the current will divide at point *b* and that part of the current will flow through the path *b* to *f* to *e* to *c* and the other part will follow the path *b* to *c*. In Fig. 12, as in Fig. 11, the connecting leads have such a small amount of resistance as compared to the resistance of R_1 and R_2 that they are considered to have no resistance. The connection, *bc*, is considered to have no resistance, just as all the connecting links of the circuit, such as *ab*, *ce*, and *ef*, are considered to have no resistance. The student is cautioned always to consider a circuit from the viewpoint of the resistance offered to the path of current by the various parts of the circuit. In Fig. 12, path *bc* offers no resistance, while path *bfec* does offer resistance, since it contains the resistance of R_2. There is no split of the current at *b*; the entire current flows along the path *bc* and no current flows in the path through R_2.

In Fig. 13, a resistor, R_3, is inserted in the lead *bc* and the current I now divides at *b* into parts I_1 and I_2 as indicated.

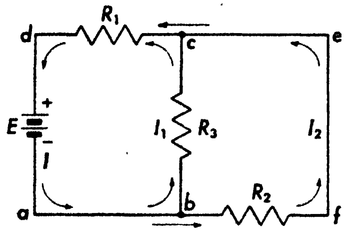


Fig. 13. Division of current.

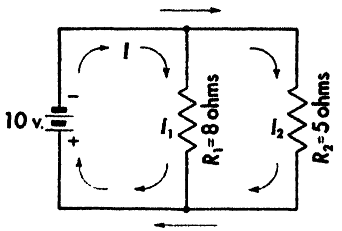


Fig. 14

44. Currents in Resistance Branches. The problem of determining the resulting resistance when resistors are connected in parallel or connected in series parallel is taken up in section 48. The student should refrain from the use of methods employed in section 48 until the principles outlined in this section and sections 45, 46, and 47 are fully mastered.

The current I in Fig. 14 divides into parts I_1 and I_2 as shown. The current through R_1 is determined from Ohm's law.

$$I_1 = \frac{E}{R_1} = \frac{10}{8} = 1.25 \text{ amperes}$$

The current through R_2, is:

$$I_2 = \frac{E}{R_2} = \frac{10}{5} = 2 \text{ amperes}$$

The total current:

$$I = I_1 + I_2 = 1.25 + 2 = 3.25 \text{ amperes}$$

The student should never lose sight of the fact that a decrease in resistance means a *decrease in opposition* to *current* flow which results in an *increase in current* flow. If several resistors are connected across a battery as in Fig. 15, the *greatest* amount of current will flow through the resistor containing the *smallest* amount of resistance. The *smallest* amount of current will flow in the resistor that has the *highest* resistance.

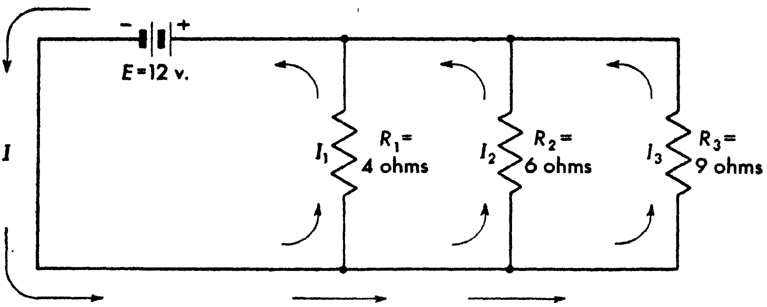


Fig. 15

Figure 15 is similar to Fig. 14 except that the current I now divides into three currents: I_1, I_2, and I_3. These currents can be determined by the use of Ohm's law, as was done in Fig. 14, and are found to be: $I_1 = 3$ amperes; $I_2 = 2$ amperes; $I_3 = 1.33$ amperes.

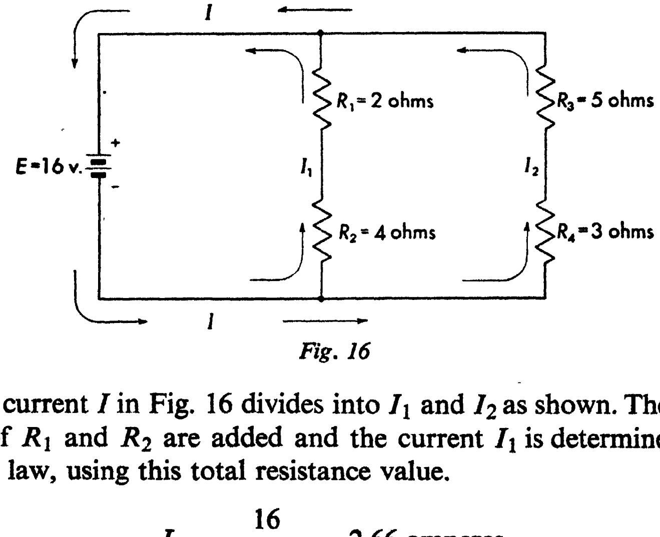


Fig. 16

The current I in Fig. 16 divides into I_1 and I_2 as shown. The resistance of R_1 and R_2 are added and the current I_1 is determined from Ohm's law, using this total resistance value.

$$I_1 = \frac{16}{2 + 4} = 2.66 \text{ amperes}$$

$$I_2 = \frac{16}{5 + 3} = 2 \text{ amperes}$$

45. Voltage Drop. A battery has a positive terminal and a negative terminal. If the battery is connected to a closed circuit, current will flow from the negative terminal around through the circuit to the positive terminal. Current flows in the circuit because there is a difference of potential between the two terminals of the battery. The voltage of the battery in *volts* is a measure of the difference in potential between the two terminals of the battery.

If a battery is connected in a circuit such as Fig. 17, there will be a voltage drop across each resistor as indicated. Each one of these voltages can be measured by means of a voltmeter. If the voltage E_4 is measured between points *f*, and *g*, it would not be expected to be so much as the battery voltage *E*, since other voltage measurements can be made across the remaining resistors. The *sum* of the voltages across all the resistors is *equal* to the battery voltage. The voltage as measured across any resistor is called the *voltage drop* across the resistor. The sum of the voltages across the remaining resistors is that much less than the battery voltage.

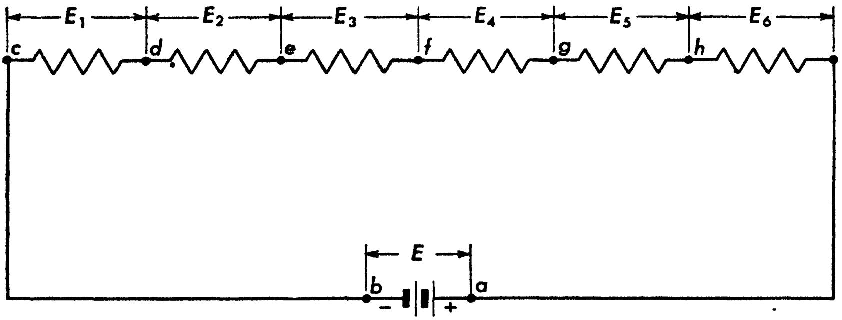


Fig. 17. Voltage drop in a circuit. The sum of the voltage drops across the resistors is equal to and opposite to the battery voltage *E*.

Ohm's law may be used to calculate the voltage drop across any resistor in a circuit if its resistance and the current through it are known.

The full battery voltage is applied across all the resistors in Fig. 17, and is divided up among the resistors in accordance with the resistance of each since the same amount of current passes through all resistors.

The voltage drop in a resistor is *opposite* to the driving voltage, otherwise this voltage would add to the driving voltage and we would have more voltage than we had in the beginning, which is not true.

46. Series Circuits. As has already been pointed out, the current through various resistors connected in series is the same, regardless of the resistance of each resistor. The *total resistance* of all the resistors in the circuit is the determining factor governing how much current flows in the circuit.

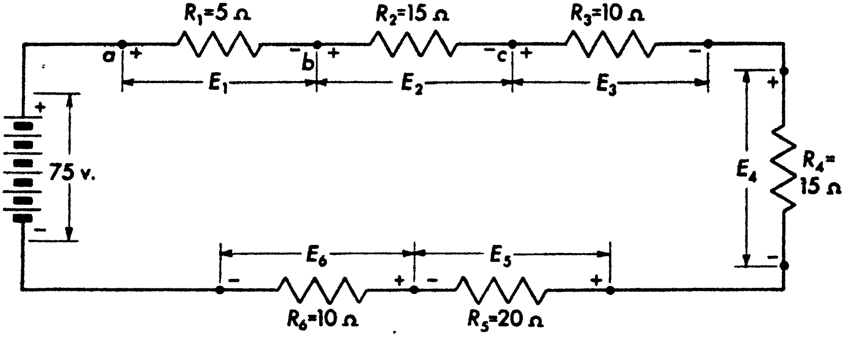


Fig. 18. Series circuit.

The total resistance in the circuit shown in Fig. 18 is 75 ohms and the current from Ohm's law is:

$$I = \frac{E}{R} = \frac{75}{75} = 1 \text{ ampere}$$

The voltage drop across R_1 is:

$$E_1 = IR_1 = 1 \times 5 = 5 \text{ volts}$$

Likewise, the voltage drop across each of the remaining resistors is determined the same way. The terminal of resistor R_1 connected to the battery is plus, since the terminal of the battery is plus. The other end of the resistor must be minus, since there is a difference of potential between the ends of the resistor and this difference in potential is in opposition to the battery voltage.

One terminal of resistor R_2 is joined to the *negative* end of R_1 at point *b*, but the R_2 terminal is *plus* since point *b* is *minus* with respect to point *a* but *plus* with respect to point *c*.

A measure of a voltage drop across a resistor is a measure of the potential difference of one end of the resistor with respect to the other end.

A measure of voltage across a resistor does not mean that the resistor is acting as a battery furnishing an electromotive force in the circuit. On the contrary, resistors in a closed circuit such as Fig. 18 oppose the action of the electromotive force of the battery in the circuit. On the other hand, the voltage drop as measured across a resistor has a definite polarity that is due to the *electromotive force* of the *battery* in the circuit.

The + and – symbols at the ends of resistors in the diagrams and sketches of this text are placed there for the convenience of the student as an indication of the polarity of the voltage drops across the resistors.

D–c voltmeter connections are labeled + and –. The pointer moves from left to right across the scale when the + terminal of the instrument is connected to the + terminal of a battery or + point of a resistor and the – terminal of the instrument is connected to the – terminal of the battery or – point of a resistor. If these two connections are reversed, the pointer tries to swing in the opposite direction, striking the left-hand stop pin. Therefore, a voltmeter is quite useful in determining the proper polarity of the voltage drop across resistors as well as that of batteries.

Example:

Determine the current flowing, the voltage drop across R_2 and R_3, and the resistance of R_2 in Fig. 19.

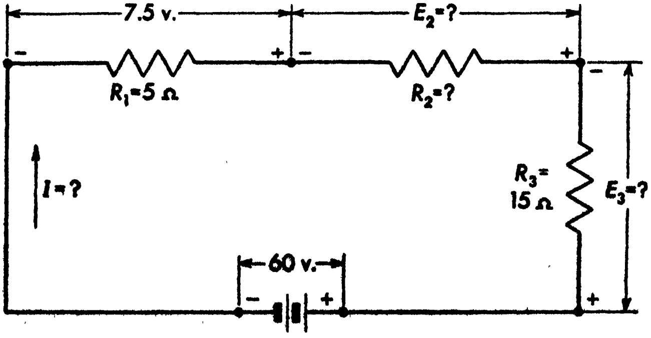


Fig. 19. To determine the quantities as indicated. (I, R_2, E_2 and E_3.)

$$I = \frac{7.5}{5} = 1.5 \text{ amperes}$$

$$E_3 = 1.5 \times 15 = 22.5 \text{ volts}$$

$$E_2 = 60 - 7.5 - 22.5 = 30 \text{ volts}$$

$$R_2 = \frac{30}{1.5} = 20 \text{ ohms}$$

Exercise 9:

1. Three resistors, having resistances respectively of 4 ohms, 7 ohms, and 9 ohms, are connected in parallel across the terminals of a 6-volt battery. Draw the circuit and determine the current in each resistor.

2. Two resistors, R_1 and R_2, are connected in parallel to a 1½-volt dry cell. The current drawn from the cell is 200 milliamperes. What is the resistance of R_2 if R_1 has a resistance of 10 ohms?

3. Four resistors of 8 ohms, 18 ohms, 28 ohms, and 36 ohms are connected in series with a battery. What is the battery voltage if the current is 260 milliamperes?

4. Determine the battery voltage necessary to furnish 300 milliamperes of current to three resistors connected in parallel across the battery if each resistor has a resistance of 1,000 ohms.

5. A tube requires a filament voltage of 6.3 volts at 300 milliamperes. What series resistance will be required to operate the tube from a 12-volt battery? What is the resistance of the tube filament?

6. A 4-tube radio receiver uses the d-c 1.4-volt filament-type tubes with filament currents of the tubes as follows: 1R5 tube—50 milliamperes; 1T4 tube—50 milliamperes; 1S4 tube—100 milliamperes; 1S5 tube—50 milliamperes. What is the filament resistance of each tube?

47. Series Parallel Circuits. Radio circuits are not restricted to simple series circuits alone. In most instances they are more complex.

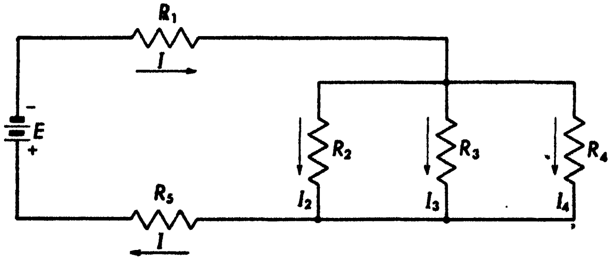


Fig. 20

Quite often one is confronted with circuits which contain elements that contain both *series* and *parallel* parts. A simple series circuit may branch off into several resistors providing additional paths for the current as in Fig. 20. The same amount of current flows in R_5 that flows in R_1. The current flowing in each of the resistors R_2, R_3, and R_4 is *inversely* proportional to the resistance of that resistor.

The voltage across R_2, R_3, and R_4 is the same; therefore, the calculated voltage drops across these resistors should be the same.

Let $E = 6$ volts, $R_1 = 4$ ohms, $R_2 = 8$ ohms, $R_3 = 12$ ohms, $R_4 = 6$ ohms, and $R_5 = 8$ ohms in Fig. 20. Find the current through each of the resistors R_2, R_3, and R_4 when the current through the battery is .41 ampere.

The voltage drop across R_1 is:

$$E_1 = IR_1 = .41 \times 4 = 1.64 \text{ volts}$$

The voltage drop across R_5 is:

$$E_5 = IR_5 = .41 \times 8 = 3.28 \text{ volts}$$

The voltage drop across R_2, R_3, R_4 is:

$$E_2 = 6 - (1.64 + 3.28) = 1.1 \text{ volts}$$

$$I_2 = \frac{E_2}{R_2} = \frac{1.1}{8} = .137 \text{ ampere}$$

$$I_3 = \frac{1.1}{12} = .091 \text{ ampere}$$

$$I_4 = \frac{1.1}{6} = .183 \text{ ampere}$$

$$\begin{aligned} I &= .137 + .091 + .183 \\ &= .411 \text{ ampere, which checks with the given current of .41 ampere} \end{aligned}$$

Exercise 10:

1. Let $E = 20$ volts, $R_1 = 10$ ohms, $R_2 = 6$ ohms, $R_3 = 10$ ohms, $R_4 = 15$ ohms, and $R_5 = 11$ ohms in Fig. 20. The current through R_1 is 833 milliamperes. Find the current in each of the resistors R_2, R_3, and R_4.

2. Let $E = 15$ volts, voltage drop across R_1 be 6 volts, $R_2 = 5$ ohms, $R_3 = 10$ ohms, $R_4 = 15$ ohms, and voltage drop across R_5 be 4 volts in Fig. 20. Find: I_2, I_3, I_4, R_1, and R_5.

48. Parallel Connected Resistors. By now the student has doubtless become aware of the fact that two or more resistors connected in parallel and placed in a circuit offer an *equivalent resistance* which is *less* than the *resistance of either resistor* taken singly. He can probably see a need for a method whereby the resultant resistance of several resistors connected in parallel may be calculated. Such a method is available and is quite simple. It adds considerable flexibility to the solution of circuit problems as discussed in sections 44, 45, 46, and 47.

The resultant resistance R of several resistors connected in parallel may be determined from the following expression:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \tag{13}$$

where R_1, R_2, and R_3 represent the parallel resistors. If more than three resistances are connected in parallel, additional terms should be added to equation 13, such as $\frac{1}{R_4}$, $\frac{1}{R_5}$, and so forth.

Equation 13 simply states that the reciprocal is equal to the sum of the reciprocals.

If only two resistors are connected in parallel, the general equation may be reduced to a special form in the following manner:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$\frac{1}{R} = \frac{R_2 + R_1}{R_1R_2}$$

$$R = \frac{R_1R_2}{R_1 + R_2} \tag{14}$$

A special equation may be worked out in the same manner for three resistors connected in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{R_2R_3}{R_1R_2R_3} + \frac{R_1R_3}{R_1R_2R_3} + \frac{R_1R_2}{R_1R_2R_3}$$

$$\frac{1}{R} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1R_2R_3}$$

$$R = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} \tag{15}$$

Additional equations may be worked out in the same manner for 4, 5, or any number of resistors. Such additional equations are not recommended. Equations 13 and 14 should prove to be satisfactory for the determination of the resultant resistance of all parallel connected resistors. Too many special-purpose equations are such a burden on the memory as to offset any advantages otherwise.

Example:

Determine the resistance of two resistors connected in parallel if one has a resistance of 2 ohms and the other has a resistance of 6 ohms.

$$R = \frac{R_1R_2}{R_1 + R_2} = \frac{2 \times 6}{2 + 6} = \frac{12}{8} = 1.5 \text{ ohms}$$

An inspection of the result of the above example reveals an important fact in regard to resistors connected in parallel.

The equivalent resistance of any number of resistors connected in parallel is always less than that of the resistor with the lowest resistance.

Further observation of equation 14 shows that the resultant resistance of two resistors of equal resistance connected in parallel is equal to *half the resistance of one resistor.* If three resistors of like resistance are connected in parallel, the resultant resistance is *one-third* the resistance of one resistor. Likewise, it is one-fourth for four, one-fifth for five, and so on.

Example:

Determine the total current flowing through the circuit of Fig. 21.

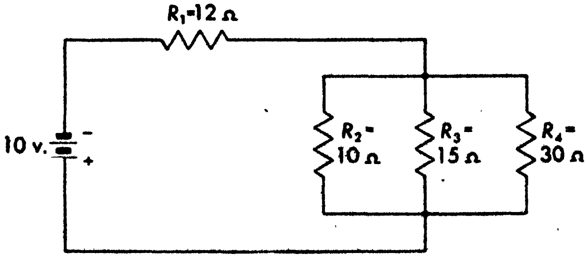


Fig. 21

From equation 13:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{15} + \frac{1}{30}$$

Obtaining 60 as a common denominator, we have:

$$\frac{1}{R} = \frac{6}{60} + \frac{4}{60} + \frac{2}{60}$$

$$\frac{1}{R} = \frac{12}{60}$$

$$R = \frac{60}{12} = 5 \text{ ohms resultant resistance of resistors } R_2, R_3, \text{ and } R_4.$$

$$12 + 5 = 17 \text{ ohms total resistance of circuit.}$$

$$I = \frac{E}{R} = \frac{10}{17} = .588 \text{ ampere}$$

Exercise 11:

Solve for resultant resistance for the resistors connected in parallel as indicated.

1. 10 ohms, 15 ohms
2. 10 ohms, 20 ohms
3. 10 ohms, 15 ohms, 20 ohms
4. 8 ohms, 6 ohms, 12 ohms
5. 40 ohms, 100 ohms
6. 30 ohms, 150 ohms, 300 ohms
7. 2,000 ohms, 1,500 ohms, 500 ohms
8. 10,000 ohms, 1,500 ohms, 6,000 ohms
9. 5,000 ohms, 50,000 ohms, 100,000 ohms

Example:

Find the resultant resistance in Fig. 22.

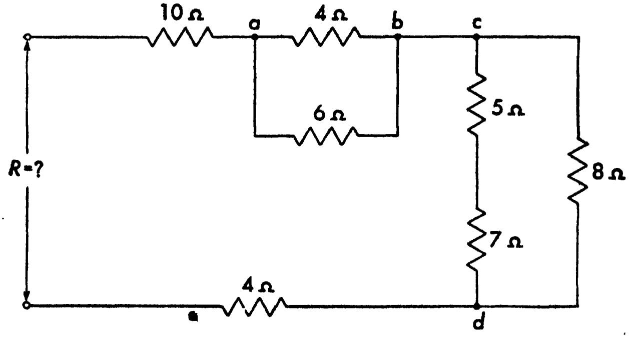


Fig. 22

$$R_{a-b} = \frac{4 \times 6}{4 + 6} = \frac{24}{10} = 2.4 \text{ ohms equivalent resistance between points } a \text{ and } b.$$

$$R_{c-d} = \frac{8(5 + 7)}{8 + (5 + 7)} = \frac{8 \times 12}{8 + 12} = \frac{96}{20} = 4.8 \text{ ohms equivalent resistance between points } c \text{ and } d.$$

$$\begin{aligned} R &= R_1 + R_2 + R_3 + R_4 \\ &= 10 + 2.4 + 4.8 + 4 \\ &= 21.2 \text{ ohms resistance} \end{aligned}$$

Exercise 12:

1. Determine the equivalent resistance in Fig. 23.

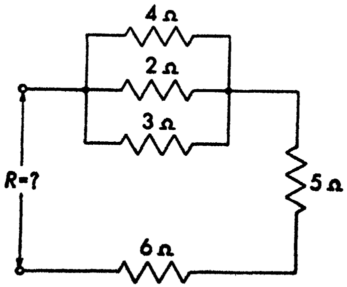


Fig. 23

2. Determine the equivalent resistance in Fig. 24.

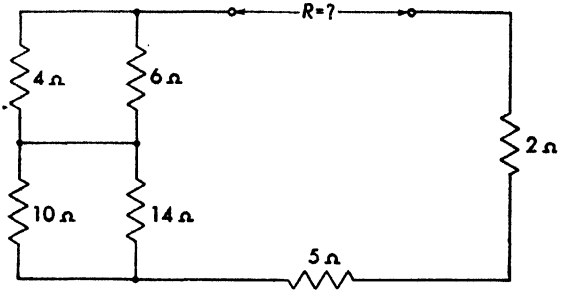


Fig. 24

3. Determine the equivalent resistance in Fig. 25.

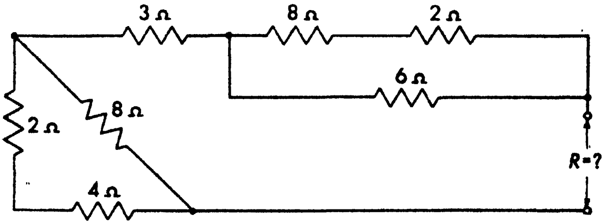


Fig. 25

4. Determine the equivalent resistance in Fig. 26.

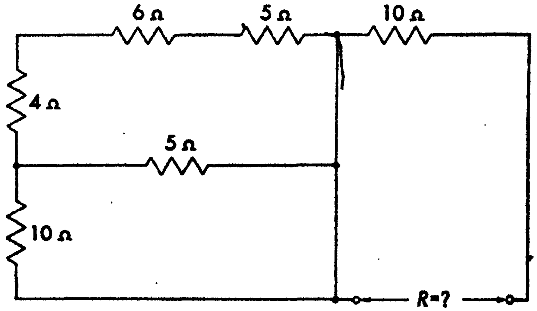


Fig. 26

5. Determine the equivalent resistance in Fig. 27.

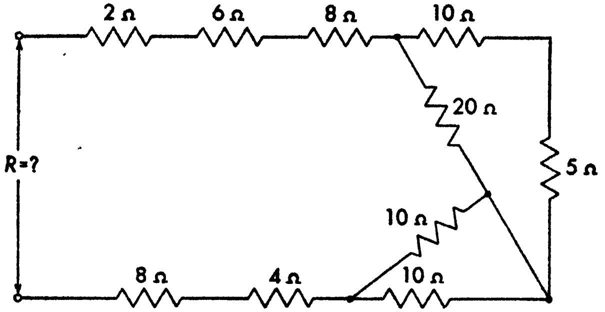


Fig. 27

6. The voltage-divider resistor on a receiver power supply becomes damaged and an inspection shows that a 5,000-ohm section of the resistor is open. A quick search in the spare resistor box nets four resistors that have sufficient current-carrying capacity to be used provided a proper series-parallel combination approaching 5,000 ohms can be found. The resistors have a resistance of: R_1 = 1,000 ohms; R_2 = 7,000 ohms; R_3 = 8,000 ohms; R_4 = 9,500 ohms. Determine the resistance combination whose resultant resistance is nearest 5,000 ohms.

49. Conductance. Resistance was defined (section 12) as that property of a material which opposes the flow of electric current through the material. It may appear that resistance in a circuit is an objectionable feature at all times. In the next chapter it will be shown that work cannot be produced in an electrical circuit without *opposition* to the flow of current. The electric light becomes incandescent because its filament is constructed of resistance wire which becomes white hot when the proper amount of current flows through it.

Electrical current is transmitted from one point to another by means of conductors. It is important that such conductors have a minimum amount of resistance. In fact, engineers often consider conductors in the light of their *ability to conduct* electric current.

The property of a material to conduct electrical current is called *conductance.*

The conductance properties of a material are, naturally, opposite to its resistance properties. Conductance, therefore, is the *reciprocal* of resistance:

$$G = \frac{1}{R} \tag{16}$$

where the conductance G is measured in *mhos*. It may be observed that the word *mho* is the reverse of *ohm*.

Conductance values may be used in the solution of circuit elements since:

$$I = \frac{E}{R} \text{ from Ohm's law and}$$

$$I = E\frac{1}{R} = EG$$

The conductance of several resistors connected in parallel may be determined from the expression:

$$G = G_1 + G_2 + G_3 + \ldots \tag{17}$$

This expression is derived from equation 13 which states that:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

G_1, G_2, and G_3 represent the conductance of each resistor.

Example:

Determine the total conductance of the circuit in Fig. 28.

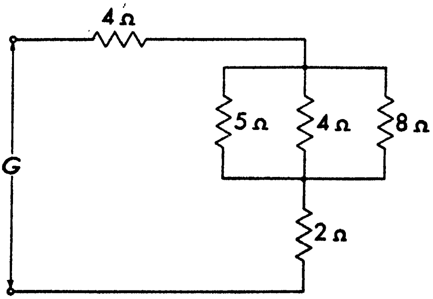


Fig. 28

$$\begin{aligned} G &= G_1 + G_2 + G_3 \\ &= \frac{1}{5} + \frac{1}{4} + \frac{1}{8} \\ &= .2 + .25 + .125 \\ &= .575 \text{ mho} \end{aligned}$$

$$R = \frac{1}{G} = \frac{1}{.575} = 1.74 \text{ ohms}$$

The total circuit resistance is:

$$4 + 1.74 + 2 = 7.74 \text{ ohms}$$

$$G = \frac{1}{7.74} = .129 \text{ mho}$$

Exercise 13:

1. Determine the total conductance in Fig. 29.

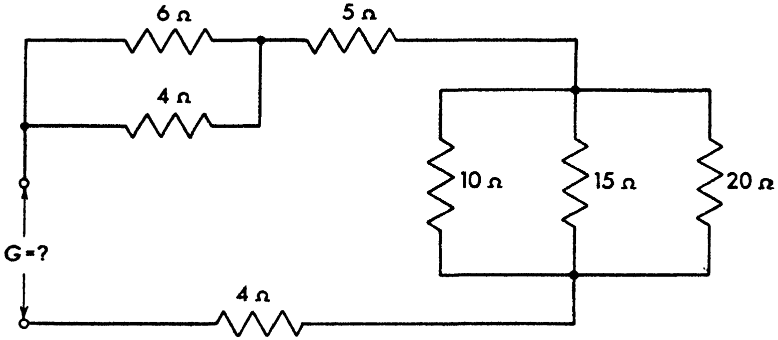


Fig. 29

2. In Fig. 30, determine: (1) R_2; (2) R_1; and (3) G.

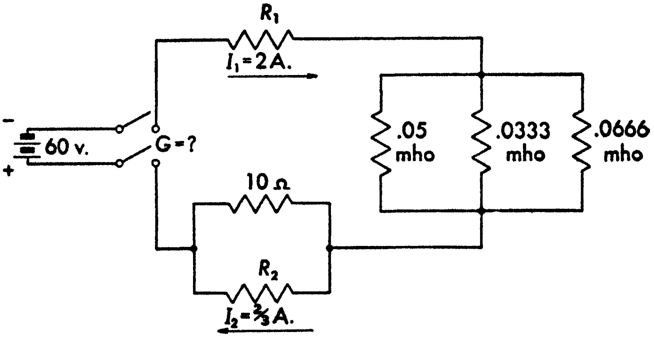


Fig. 30

REVIEW QUESTIONS

1. What is meant by a division of current?
2. Explain why current will not flow into the bfgh section of the circuit of Fig. 11.

3. Two resistors are connected in parallel. How does the current through each resistor compare with the resistance of that resistor?

4. Explain "voltage drop."

5. How does the sum of the voltage drops of Fig. 17 compare in value and algebraic sign to the battery voltage E?

6. What is meant by a "series circuit"?

7. A voltmeter connected across any resistor in Fig. 18 such as R_1 will indicate a voltage and a polarity of voltage as indicated. Why is this true since resistors offer opposition to the flow of current and certainly are not a source of voltage?

8. Why is the current through all the resistors of a series circuit the same even though the resistance of the resistors may each be different?

9. What is meant by a series parallel circuit?

10. Give the equation for determining the equivalent resistance of several resistors, unlike in resistance values, connected in parallel.

11. Define conductance.

12. In what unit is conductance measured?

ANSWERS

Exercise 9, page 42:

1. $I_1 = 1.5$ amperes; $I_2 = .86$ ampere
$I_3 = .67$ ampere; $I_t = 3.03$ amperes
2. $R_2 = 30$ ohms
3. $E = 23.4$ volts
4. $E = 100$ volts
5. $R = 19$ ohms
Fil. Res. = 21 ohms
6. 1R5 tube filament = 28 ohms
1S4 tube filament = 14 ohms
1T4 tube filament = 28 ohms
1S5 tube filament = 28 ohms

Exercise 10, page 43:

1. $I_2 = .417$ ampere
$I_3 = .251$ ampere
$I_4 = .167$ ampere
2. $I_2 = 1$ ampere
$I_3 = .5$ ampere
$I_4 = .33$ ampere
$R_1 = 3.27$ ohms
$R_5 = 2.18$ ohms

Exercise 11, page 46:

1. 6 ohms
2. 6.66 ohms
3. 4.61 ohms
4. 2.66 ohms
5. 28.6 ohms
6. 23.1 ohms
7. 316 ohms
8. 1,071 ohms
9. 4,348 ohms

Exercise 12, page 47:

1. 11.922 ohms
2. 15.23 ohms
3. 10.18 ohms
4. 10 ohms
5. 41.57 ohms
6. R_1 in series with R_2 and R_4 in parallel.

Exercise 13, page 51:

1. .0623 mho
2. (1) $R_2 = 20$ ohms
   (2) $R_1 = 16.66$ ohms
   (3) $G = .0333$ mho

CHAPTER V *Direct-Current Power*

Power must always be expended in forcing current through any circuit containing resistance.

Power companies deal in large values of power since the power industry's primary function is the distribution and sale of electrical power.

The radio industry has many divisions, but the industry, as a whole, is principally engaged in the *transmission* and *reception* of *intelligence.* The power involved is only a means to an end. In many instances, the actual power involved is insignificant. The feeble voltage generated by a person speaking into a microphone has to be amplified many times. The feeble voltages induced in a receiving antenna by a distant transmitting station has to be amplified many thousand times before it is of sufficient strength to give up the intelligence that it carries. In both instances, the power involved was infinitesimally small and unimportant from a power engineer's viewpoint, but of supreme importance to a radio engineer.

50. Work. We say that it is hard work moving a heavy piece of machinery because considerable force had to be exerted on it in order to move it. The heavy piece of machinery offered an opposition or opposing force toward being moved.

A force *in motion,* acting against an opposing force, constitutes *work.* A heavy pack on one's back may be very fatiguing but no work is done unless we are in motion. The foundation of a building may be holding up a tremendous weight but no work is being done because no motion is involved.

51. Power. From the natural law of the *conservation* of *energy,* we have a fundamental fact:

Energy can neither be created nor destroyed but can be changed from one form to another.

Electricity provides a most flexible means of changing energy from one form to another. The power of a huge steam turbine is converted to electrical power by coupling an electric generator to the shaft of the turbine. The electrical energy is transmitted to the consumer by means of electrical conductors where it is converted into some other form of energy, such as the sound energy from the radio, the light energy from the electric light, or the mechanical energy from the electric motor.

Power may be considered as the *ability* of an electrical circuit *to do work* and may be defined as the *time rate at which work can be done.*

Before work can be realized in an electrical circuit, there must exist some opposing or resisting force. It was pointed out in Chap. II that no conductor is perfect; therefore, some power must be used in *forcing the current through the conductors* carrying the current from the battery or generator to its destination. Since any conductor contains resistance, which opposes the flow of current, it follows that a movement of electrons constituting an electric current is not accomplished without the use of power.

It will be remembered that the definition of the unit of current (ampere) is associated to a time rate. The ampere is defined as a certain number of electrons passing a given point in a *given length of time*, which is 6.3×10^{18} electrons per second.

Power is determined from the expression:

$$P = EI \tag{18}$$

where P is in watts, E in volts, and I is measured in amperes. Large amounts of power are usually measured in *kilowatts.* Watts are converted to kilowatts by dividing watts by 1,000. Small amounts of power are measured in *milliwatts.* Watts are multiplied by 1,000 to obtain milliwatts.

52. Heating Effect of Current. From the law of the conservation of energy, some of the electrical energy is used to force the current through the conductor. Therefore, this energy is transformed from electrical to some other form of energy. Actually, it is transformed into *heat.* As the current increases, the heating effect increases in proportion to the square of the current.

Conductors are rated in current-carrying capacity in amperes. The current-carrying capacity increases with the size of the conductor.

53. Other Expressions of Power. Equation 18 stated that;

$$P = EI$$

Other expressions of power may be derived from this equation by substituting in the equation for E and I. From Ohm's law:

$$I = \frac{E}{R}$$

$$P = EI = E\left(\frac{E}{R}\right)$$

(19) $$P = \frac{E^2}{R}$$

From Ohm's law:

$$E = IR$$

$$P = EI = (IR)I$$

(20) $$P = I^2R$$

All three equations for the determination of power are useful in radio work and should be fully understood. Equation 20 is used extensively in computing the radio-frequency power in the antenna of a transmitting station.

Example 1:

Determine the power taken from the battery in Fig. 31.

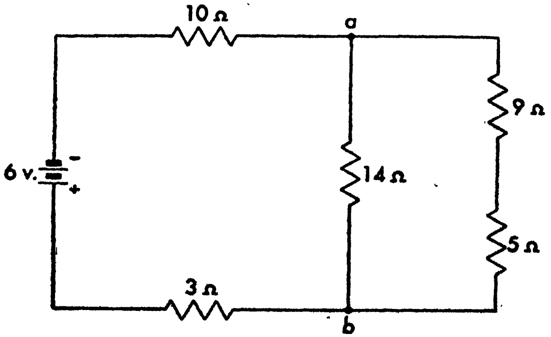


Fig. 31

$R_{ab} = 7$ ohms

$10 + 7 + 3 = 20$ ohms total resistance

$$P = \frac{E^2}{R} = \frac{(6)^2}{20} = \frac{36}{20} = 1.8 \text{ watts}$$

Example 2:

The filaments of two tubes, each rated at 2 volts and 60 milliamperes are connected in parallel across a 3-volt battery with a proper voltage dropping resistor to ensure proper voltage to the tubes. Draw diagram and determine the required resistor resistance and its power rating.

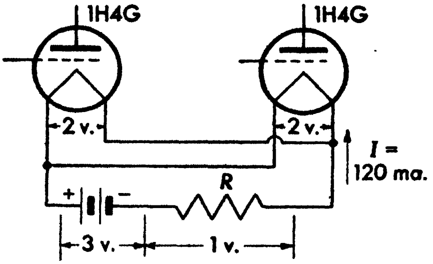


Fig. 32. Voltage dropping resistor *R* required to drop the 3v. battery voltage to the required 2v. at the tube filament terminals.

$$I = .06 + .06 = .12 \text{ ampere}$$

$3 - 2 = 1$ volt, the required voltage drop in the series resistor R.

$$R = \frac{E}{I} = \frac{1}{.12} = 8.33 \text{ ohms}$$

$$P = EI = 1 \times .12 = .12 \text{ watt}$$

Exercise 14:

1. Determine the power in an antenna with a radiating resistance of 48.6 ohms and a current of 3.25 amperes.
2. A tube with a 2-volt filament must be operated from a 6-volt storage battery. What value of resistance must be connected in series with the tube for proper operation, if the filament draws 240 milliamperes?
3. A power of 7.8 watts is supplied to the plate circuit of a tube. What is the plate resistance if the plate current is 35 milliamperes?
4. What resistance will be required to be placed in series with two parallel connected 60-watt 110-volt light bulbs across a 500-volt d-c source of power? How much power will be used by the resistor?

54. Efficiency. No discussion of power is complete without the mention of efficiency. Any device or machine, no matter how carefully designed, must have losses that keep one from fully realizing all the power from it that was put into it. An electric motor has resistance losses, iron losses, friction losses, and others that prevent it from producing mechanical energy equal to the electrical energy that was put into it.

Efficiency may be defined as the output power divided by the input power.

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} \tag{21}$$

55. Power Used by Resistors. The heating effects of current flowing through a conductor were discussed in section 52. Conductors are usually operated with currents flowing through them that are far below the safe allowable current. If the current is continually increased, the temperature keeps increasing. Conductors operated at currents above the safe allowable current may get hot enough to burn the insulation from the conductors and even ignite other combustible material near by. When conductors are operated within the standards prescribed by law and good engineering practice, the heat to be dissipated per unit length of conductor is well taken care of by radiation into the surrounding air.

Radio circuits contain many resistance devices called *resistors.* In such devices, varying amounts of circuit resistance are confined to a relatively small space since in many instances many feet of resistance wire are wound on a small spool. Some of these devices in use in receivers, transmitters, amplifiers, and other radio equipment are designed to operate at a temperature that will burn the fingers if contact is maintained for longer than a split second. Resistors required for other circuit positions carry such low currents as to show no temperature effects of current flow.

Resistors are rated in *wattage sizes* as well as resistance values. The more common sizes for use in receivers, amplifiers, low-power stages of transmitters, and testing equipment will be found in the size range of $\frac{1}{2}$ watt to 50 watts and a majority of those will be below the 10-watt size. Other applications involving more power require resistors that have still higher wattage ratings.

Resistors that dissipate very much power must be mounted so that they will obtain a *maximum amount of ventilation.* As the wattage ratings are increased above 100 watts, they depend more and more on the surrounding conditions of ventilation. The wattage rating of the resistor depends to a large degree upon its ability to radiate the heat that is being continually generated within it by the current flowing through it.

56. Variable Resistors. Many circuits require resistance values that have to be determined under operating conditions or even under operating conditions that change from time to time. Such circuits are often supplied with *variable resistors.* If the resistance value does not have to be changed after the initial setting, or if its value needs to be changed at infrequent intervals, a resistor is usually supplied that has a *variable slider.* Connections are made as in Fig. 33 *a* and *b.* Method *b* is more desirable as it minimizes the possibility of burning out the resistor at point of contact.

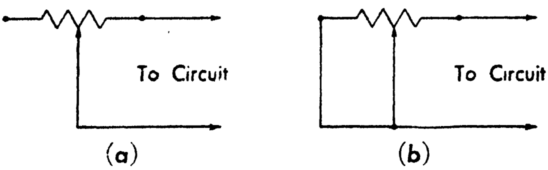


Fig. 33. Showing connections to a resistor made variable by use of a sliding clamp contact. Method *b* is more desirable.

If a 10-watt resistor is connected in series with another 10-watt resistor, the wattage of the two resistors in the circuit gives a total of 20 watts that can be dissipated by the two resistors. Obviously, if one of the resistors is removed, the wattage is only one-half the original, the remaining wattage being 10 watts. Suppose the 10-watt resistor still remaining in the circuit is equipped with a variable slider. If the slider is moved to the center of the resistor so that only half the resistor is now in the circuit, the wattage is cut in half also. The resistance *remaining in the circuit* will dissipate 5 watts. The other half of the resistor is also capable of dissipating 5 watts, but it is now out of the circuit. This fact should be well remembered. When a variable resistor is required, every attempt should be made to determine as closely as possible the exact resistance required and obtain a variable resistor such that nearly all the resistance will be in use. If a 50-watt variable resistor has only *one-tenth* of its resistance in the circuit, then that part of its resistance has a *capacity of 5 watts.*

57. Sources of D-c Power. The chief sources of direct-current power are: (1) *batteries,* (2) *motor generators,* (3) *vibrators,* and (4) *a-c rectifiers.* All these play an important role in the radio industry.

Batteries have many uses. One of the chief uses of batteries is in the operation of portable receivers, transmitters, amplifiers, and other portable equipment.

At one time, d-c generators were used extensively to furnish voltages up to 3,000 or 4,000 volts for vacuum-tube plate power in radio transmitters. Now the use of d-c generators in the radio industry is confined chiefly to use in connection with portable equipment where the power required is sufficiently large that the use of high-voltage batteries is not practical.

The vibrator power pack has practically replaced the generator and special batteries for the operation of automobile receivers. It is also used extensively for the operation of low-power police mobile transmitters and other portable equipment which is operated intermittently.

Modern equipment, except portable equipment, is designed to operate from alternating-current power which is available in most communities. The alternating current is converted into direct-current power by means of a rectifier.

58. Batteries. Early experimenters discovered that electrical power could be obtained from certain chemical reactions. Two dissimilar electrodes were placed in a container containing the proper chemical solution. If the two terminals were connected to an external circuit, current would flow from one terminal out through the circuit and back into the battery through the other terminal, allowing a chemical action to take place between the solution and the electrodes. Such a unit is called a *cell.*

Present-day batteries operate on the same principle and are divided into two general classifications: (1) *wet cells;* and (2) *dry cells.*

The lead cell containing a diluted solution of sulphuric acid is the most common type of wet cell and is usually called a *storage battery.* It can be *recharged* by forcing current through it in the *opposite direction* to the normal flow of current. The voltage of a lead cell depends upon its *condition of charge.* It is normally considered to be *2 volts* under average operating conditions. Under full charge, it may increase to 2.1 volts. A lead cell should be recharged when its voltage drops to approximately 1.8 volts.

Fig. 34. Various types of dry cell batteries for radio use. (Courtesy, National Carbon Company.)

A wet-cell type battery is not always convenient to move about for fear of spilling the solution. The *dry cell* was developed to eliminate this difficulty. In reality, *the dry cell is a wet cell that is not nearly so* *wet*. Less solution is used, and it is absorbed by sawdust or other binder and ground carbon which fills the space between the electrodes. The chemical solution is not sulphuric acid, but is usually manganese dioxide and ammonium chloride. The dry cell is convenient to use because of its portability, but it is not rechargeable and must be thrown away and new batteries obtained when the charge is used. A dry cell has a voltage of approximately 1.5 volts.

Batteries containing many cells are made up for commercial use. The most common size of lead storage battery is the 6-volt or three-cell size. Dry-cell batteries are made in sizes ranging from one cell up to thirty cells, as found in the standard 45-volt B battery.

Batteries deteriorate with age even though not in use. A lead-cell storage battery will deteriorate very little with age if kept fully charged at all times. A lead-cell storage battery should never be allowed to sit for a very long period of time in a discharged condition as it will deteriorate very rapidly under such conditions.

Dry-cell batteries have a definite shelf life. One should look for the *date* that is usually stamped on a battery to determine if it is fresh. The battery should be placed in service before this date to ensure satisfactory service from it.

59. Series Connected Batteries. If four dry-cell batteries are connected in series, as in Fig. 35, the total voltage E is the sum of the voltages of all four batteries, or 6 volts. The current capacity is the same as for one battery. The power available from the four batteries is four times that of one battery, if all batteries have like current and voltage ratings.

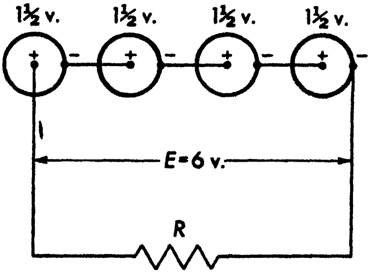


Fig. 35. Dry cells connected in series.

60. Parallel Connected Batteries. Four batteries are connected in parallel as in Fig. 36. The total circuit voltage is the same as for one cell, but the *current capacity* has been increased to *four times* that of one battery, since the total current is divided evenly between the four batteries if they are identical. Here, as in the series connected batteries in Fig. 35, the available power from the four batteries is four times that of one battery.

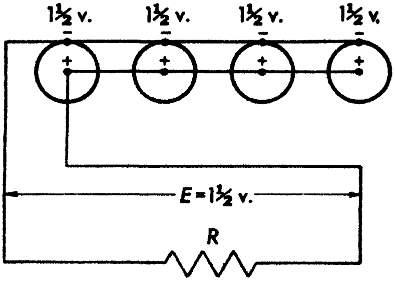


Fig. 36. Parallel connected dry cells.

61. Motor Generators and Vibrators. As stated in section 57, generators and vibrators are used to supply high-voltage power to portable equipment where the current drain is too high for dry-cell batteries or else the voltage drop in the batteries as they begin to lose their charge cannot be tolerated.

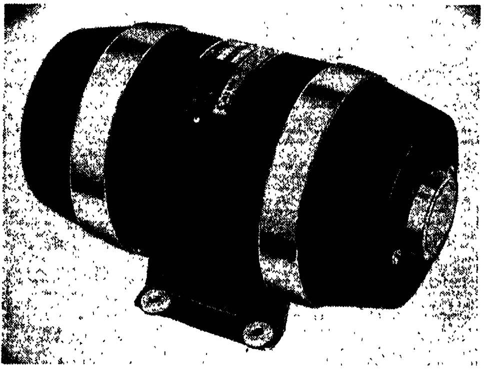


Fig. 37. Portable Direct-Current Generator. (Courtesy, Carter Motor Company, Chicago, Illinois.)

These units are operated from wet storage cells and when operated in automobiles, they are operated directly from the automobile battery. In police service, this places an extra strain on the battery. Police cars that are in continuous service not only use a larger storage battery, but also use a larger battery charging generator.

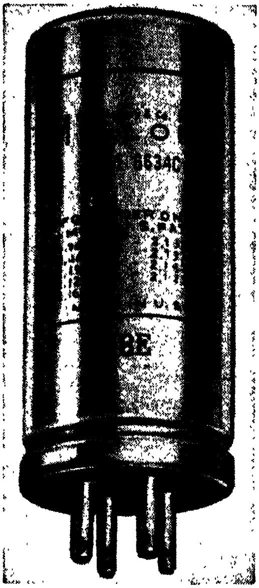


Fig. 38 a. Vibrator. (Courtesy, P. R. Mallory & Company, Inc.)

A vibrator contains principally a vibrating reed which interrupts the direct-current voltage from the battery. This interrupted voltage is increased by means of a transformer to the required voltage and then converted back to direct current by means of a rectifier and filter. The working parts of a typical vibrator are shown in the cut-away view of the vibrator in Fig. 38 *b*.

62. Alternating-current Rectifiers. Alternating current is converted into pulsating direct current by means of a rectifier tube. The pulsating current passes through a filter circuit consisting of inductances and condensers which converts it into pure direct current. The actual operation of the rectifier-type power supply is not taken up at this time since all of the necessary prerequisites have not been covered.

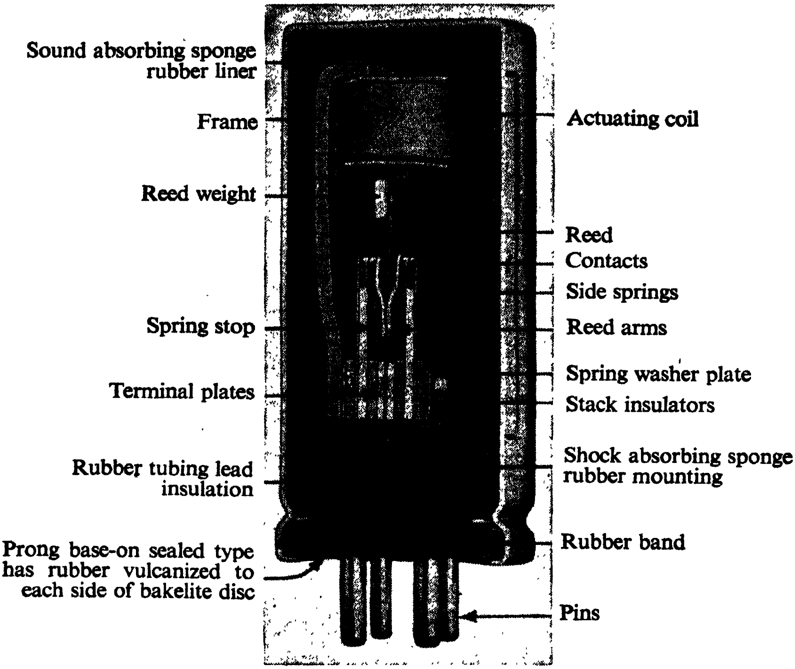

Sound absorbing sponge rubber liner
Frame
Reed weight
Spring stop
Terminal plates
Rubber tubing lead insulation
Prong base-on sealed type has rubber vulcanized to each side of bakelite disc
Actuating coil
Reed
Contacts
Side springs
Reed arms
Spring washer plate
Stack insulators
Shock absorbing sponge rubber mounting
Rubber band
Pins

Fig. 38 b. Cut-away section of vibrator, showing construction. (Courtesy, P. R. Mallory & Company, Inc.)

Exercise 15:

1. What current, neglecting losses, will a 6-volt storage battery be required to furnish to a motor-generator set that has an output of 350 volts at 50 milliamperes?

2. A police mobile transmitter draws 30 watts from its power pack. What current is drawn from the 6-volt storage battery if the power pack has 80 percent efficiency?

3. A 6-volt storage battery furnishes 10 watts power to a vibrator power supply. What is the reduction in power to the power supply if the cell voltages of the battery drop to 1.85 volts each?

4. The bulb of a three-cell flash light draws 60 milliamperes with a fresh set of batteries. What current will the bulb draw when the voltage of each cell drops to 1.2 volts?

5. A receiver designed to operate from a 6-volt battery is operated from a 12-volt battery by using a series resistor. What will be the required resistance and wattage of the resistor if the receiver draws 5 amperes?

63. Internal Resistance of Power Source. All sources of direct-current power have *internal resistance.* The internal resistance of a good lead-cell storage battery at full charge is relatively low as is the case with a fresh dry cell with a full charge.

The internal resistance of batteries increases with the age of the batteries and also as the charge of the batteries decreases. The internal resistance of a worn-out dry cell B battery is many times what it was when fresh and with a full charge.

The internal resistance of a power source becomes more important as the current drain from it increases.

64. Voltage Regulation. The internal resistance of the power source and the resistance of the conductors to a device being supplied with power cause a voltage drop in the supply voltage to the device. If the current used by the device is not constant and it requires different amounts of current from time to time, the voltage drop will change. This means that the voltage supplied to the terminals of the device is not constant.

If the voltage variations are relatively small, it is said that the *voltage regulation* is good.

$$\text{Voltage regulation in percentage} = \frac{\textit{Increase}\text{ in voltage from full load to no load}}{\text{full-load voltage}} \times 100$$

Many radio circuits require that the supply voltage be maintained at a value that must not vary beyond allowable specified limits. When these limits are sufficiently small, it is necessary to incorporate into the power supply some means of manual or automatic *voltage regulation control* to hold the voltage within the specified allowable limits.

REVIEW QUESTIONS

1. Define work.

2. Define electrical power.

3. Compare the importance of power to a radio engineer to its importance to an electrical power engineer.

4. What property of an electrical circuit causes it to use electrical power?

5. Discuss the heating effect of current.

6. Give three expressions for determining the power of a circuit.

7. Explain what is meant by the efficiency of a piece of radio apparatus.

8. Explain why the wattage rating of a resistor is for the whole resistor and the wattage dissipation of a portion of a resistor is in proportion to its resistance to that of the whole resistor.

9. Give four sources of d-c power.

10. What reaction inside batteries makes them a source of voltage, current and power.

11. What is a wet cell? A dry cell?

12. What is a storage battery?

13. Give: The usual voltage of a fully charged lead cell. The voltage of a fresh dry cell.

14. What is meant by the shelf life of dry cell batteries and the date that is usually stamped on such batteries?

15. Compare series and parallel connected cells.

16. What is meant by voltage regulation?

ANSWERS

Exercise 14, page 57:

1. 513 watts
2. 16.67 ohms
3. 6,375 ohms
4. R = 357 ohms
P = 425 watts

Exercise 15, page 65:

1. 2.92 amperes
2. 6.25 amperes
3. .75 watt
4. 48 milliamperes
5. 1.2 ohms; 30 watts

CHAPTER VI *Equipment Wiring*

Circuit diagrams found in radio literature are usually drawn with vertical and horizontal lines with right-angle bends. Equipment symbols are so located on the diagram that there will be a minimum of crossed lines. Such a practice makes it easier to follow the wiring plan.

At an early stage of the radio art, equipment placement and the actual wiring closely followed the diagram practice outlined above. Rigid and bare tinned copper bus bar was used for wiring the parts. An 18-inch piece of bus bar with six or more right-angle bends might be used to connect two points only 7 or 8 inches apart. Such wiring practices have long since been discarded for the more simple and efficient *point-to-point* and form systems of wiring. Either system of wiring naturally makes the actual wiring harder to trace. In fact, one with little experience in working with circuit diagrams or with the actual equipment may fail to find any semblance at all at first glance. Circuit diagrams of receivers or other equipment are of little value unless their application to the equipment is fully understood.

This chapter is designed to give the student a more thorough knowledge of circuits and to train him to follow circuits more readily, as he finds them, under normal operating conditions.

65. Point-to-point Wiring. Point-to-point wiring is in wide use by broadcast receiver manufacturers. There are many reasons why this system is used. In the old method, a maximum of undesirable magnetic and electric coupling from one part of the circuit to the next resulted from many wires being parallel to each other. This undesirable feature is minimized by the crisscrossing of wires which are run directly from one point to another.

Elimination of fancy and complicated bus-bar connections, with many square bends, has simplified the wiring and shortened the actual work of wiring equipment. Bare wire is still used in some

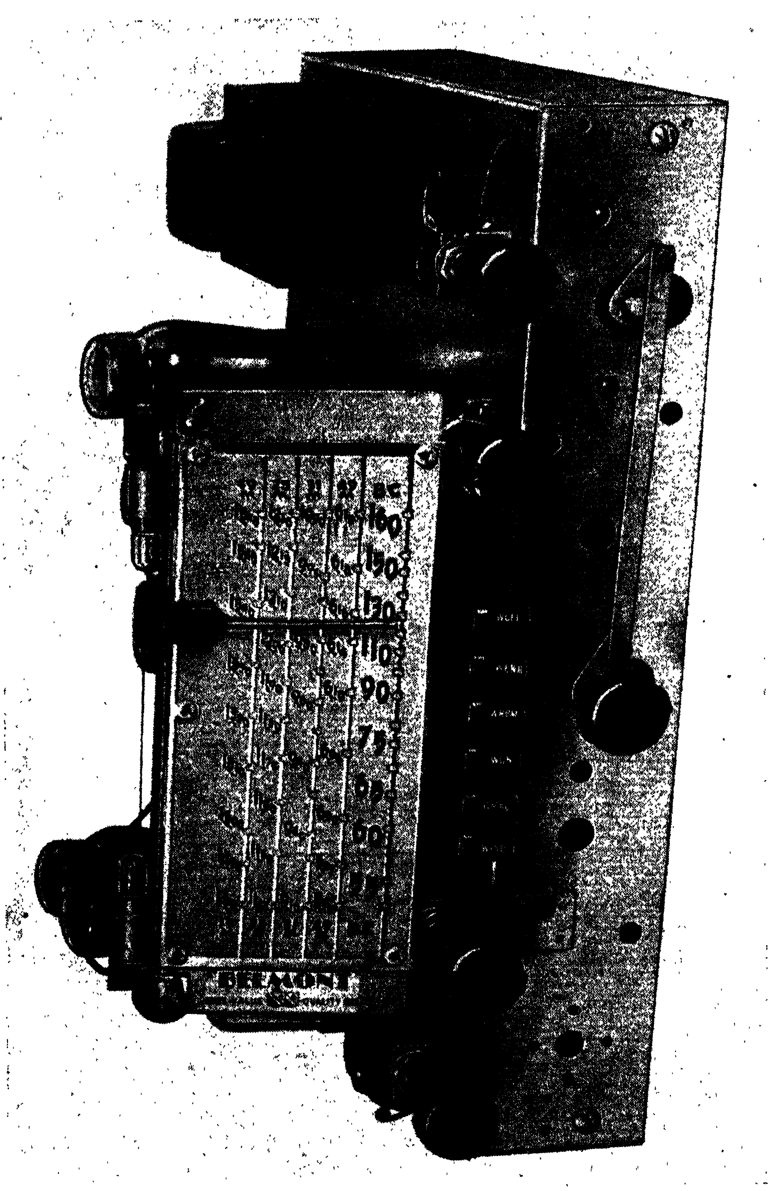


Fig. 39 a. Chassis of a modern type pushbutton broadcast and short wave receiver. (Courtesy, Belmont Radio Corporation.)

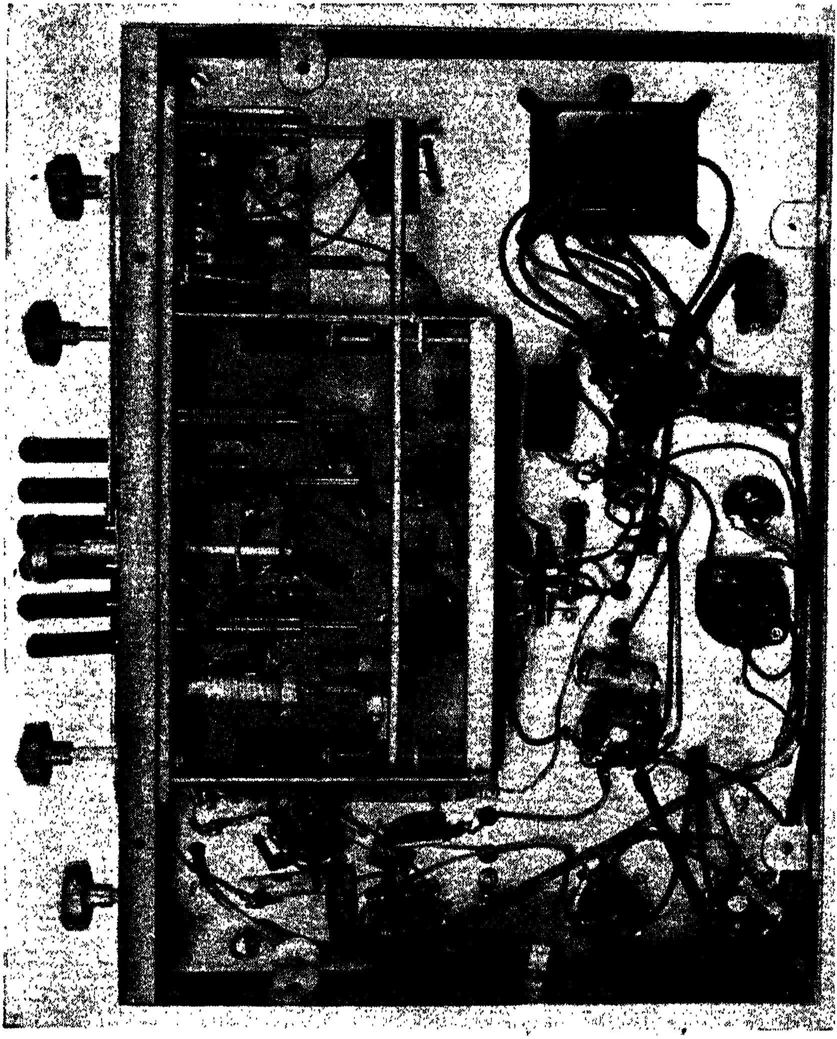


Fig. 39 b. Point-to-point wiring of receiver shown in Fig. 39 *a.* (Courtesy, Belmont Radio Corporation.)

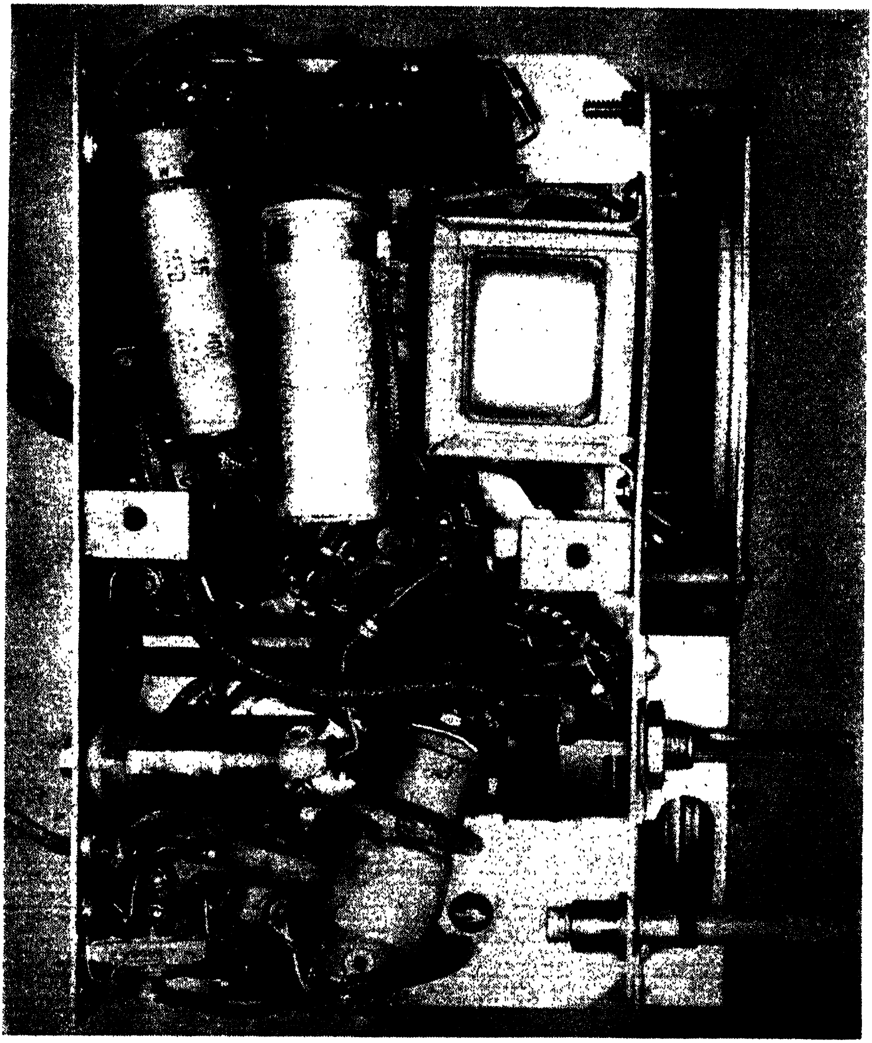


Fig. 39 c. **Point-to-point wiring of midget type broadcast receiver.**

instances where there is little danger of adjacent wires getting disarranged and touching each other and where the circuit is relatively simple. The use of insulated wire eliminates the hazard of short circuits. The use of flexible or semiflexible insulated *hook-up* wire simplifies wiring problems still further. The chassis of a modern type push-button broadcast and short wave receiver is shown in Fig. 39 *a* and an under view of its point-to-point wiring is shown in Fig. 39 *b*. Figure 39 *c* shows the wiring of one of the smaller type broadcast receivers.

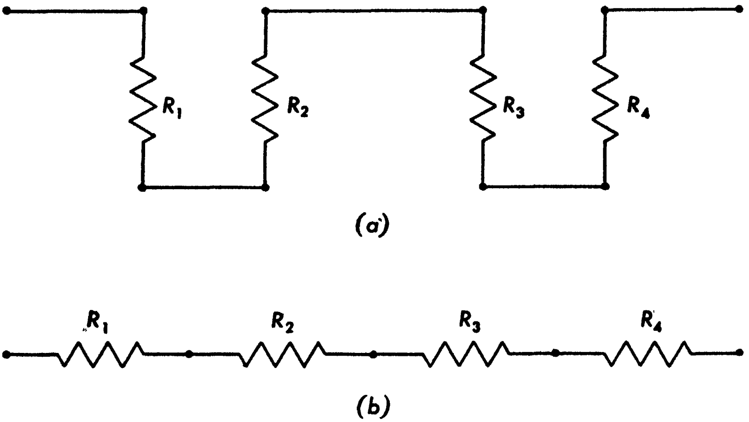


Fig. 40. a. Simple series circuit. *b.* Same circuit as *a* but redrawn with identical connections as *a.*

66. Form Wiring. Form wiring is a system whereby numerous wires are grouped together and usually laced together with cord. This type of wiring gives a neat appearance and is used in many instances where a great number of wires are involved and where the placement of the wires adjacent to each other causes no serious effects because of electrical or magnetic coupling.

Failure of resistors and fixed condensers is a common cause of receiver and transmitter failures. Some radio manufacturers mount such parts together at a point convenient for servicing and use form wiring from this point to other parts of the receiver or transmitter.

Insulated circuit hook-up wire is manufactured in various insulation colors and color stripes. It is the usual practice to use such wire in form wiring to assist in tracing the wires at some later date whenever servicing is necessary.

67. Simple Circuits. The uninitiated have many surprises awaiting them when they turn a receiver bottom side up for the first time. The first impression may be that the circuit diagram is wrong. An inspection of Figs. 39 *b* and 39 *c* shows that it is not always an easy matter to trace the wiring of a commercial receiver. Especially is this true when the equipment is crowded into such a small space as shown in Fig. 39 *c*. In all probability, the physical placement of the parts in the receiver will be quite different from the placement of their symbols on the electrical circuit diagram. The student must learn to readily follow electrical circuit diagrams and circuit wiring regardless of the relative placement of the symbols in the electrical diagram or the actual parts in the receiver or other piece of apparatus. The clarity of electrical circuit diagrams can often be improved by rearranging the position of some of the symbols and connecting wires without changing the actual electrical significance of the diagram. This is demonstrated in Fig. 40 and succeeding figures in this chapter. A simple set of wiring as shown in Fig. 40 *a* can easily be resolved into its simplest form, shown in Fig. 40 *b*. *a* is redrawn in *b* without actually changing the connections. This circuit is easy to see but who could say, at first glance, that Fig. 41 *a* resolves into the simple circuit of

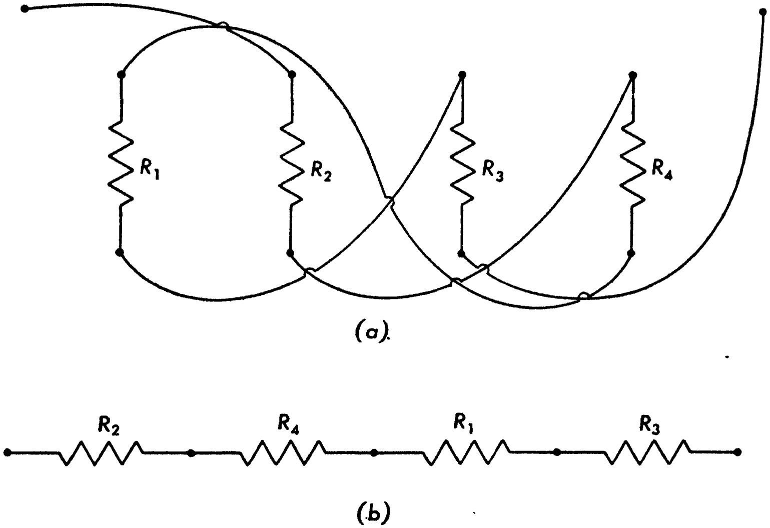


Fig. 41. a. Simple series ciruit. *b.* Same circuit as *a* but with resistor positions reversed.

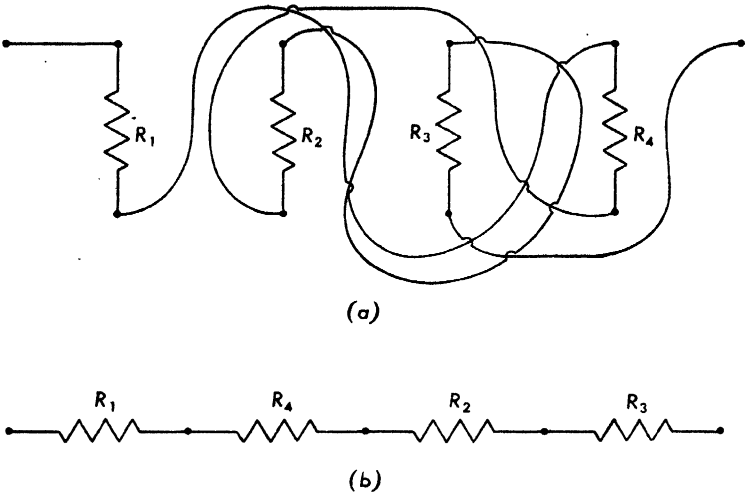


Fig. 42. a. Series circuit. *b.* Same circuit as *a* resolved into its simplest form.

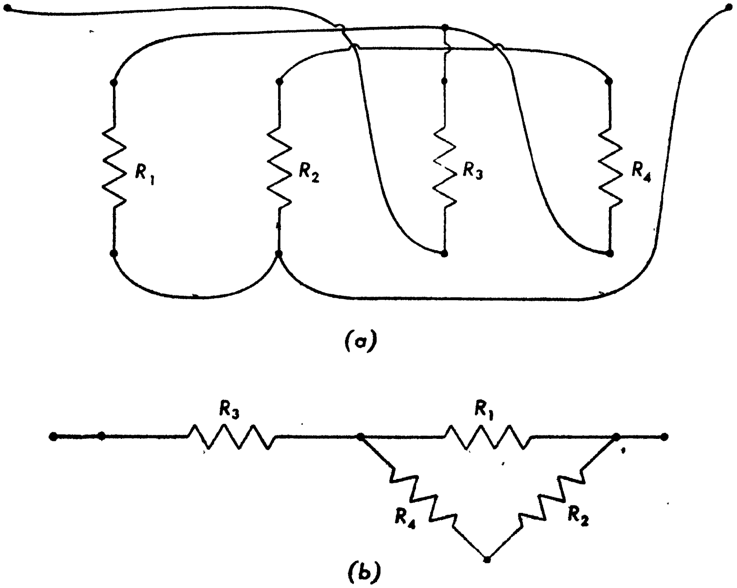


Fig. 43. a. Series parallel circuit. *b.* Part *a* simplified.

Fig. 41 *b*? Circuit *b* is exactly the same as *a*. No connections have been added or omitted. The student should verify this by tracing out the circuit of *a*.

Part *a* of Fig. 42 shows another series circuit which resolves into the simple circuit of part *b*.

Figure 43 shows a slightly more difficult circuit since it is a series-parallel circuit. Here again part *b* is a simplified version of part *a*.

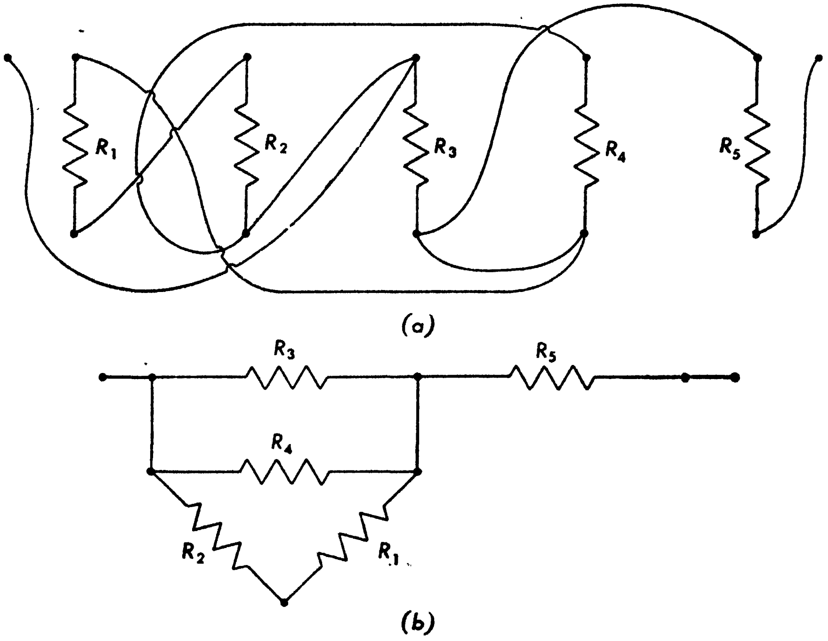


Fig. 44. a. Series parallel circuit. *b.* Part *a* simplified.

68. Advanced Circuits. The circuits as covered in section 67 were designed to introduce the student to *scrambled circuits*. In practice, most circuits are found to be more complex and require more effort on the part of the individual in following them.

Figures 44 and 45 are similar to Figs. 40, 41, 42, and 43. In each instance, as in the previous figures, the student should verify part *b* by tracing out the circuit of *a*.

Up to this point the discussion has been restricted to the tracing of existing circuits. Many times the procedure is reversed, and one must wire an amplifier or some other radio unit using an electrical circuit diagram. Lack of ability to follow scrambled circuits successfully can easily result in circuit units being short-circuited or *by-passed* by means of a connecting link of hook-up wire. A thorough check of the circuit in Fig. 46 shows that all resistors are short-circuited or by-passed and the net resistance from *f* to *g* is zero. Parts *b* and *c* show how the circuit of *a* may be rearranged but still remain the same circuit. The student must realize that shifting the resistors around in position does not change the electrical circuit. Furthermore, the end of R_2 connected to R_3 may be shifted to the end of R_4 or R_5 without changing the electrical circuit.

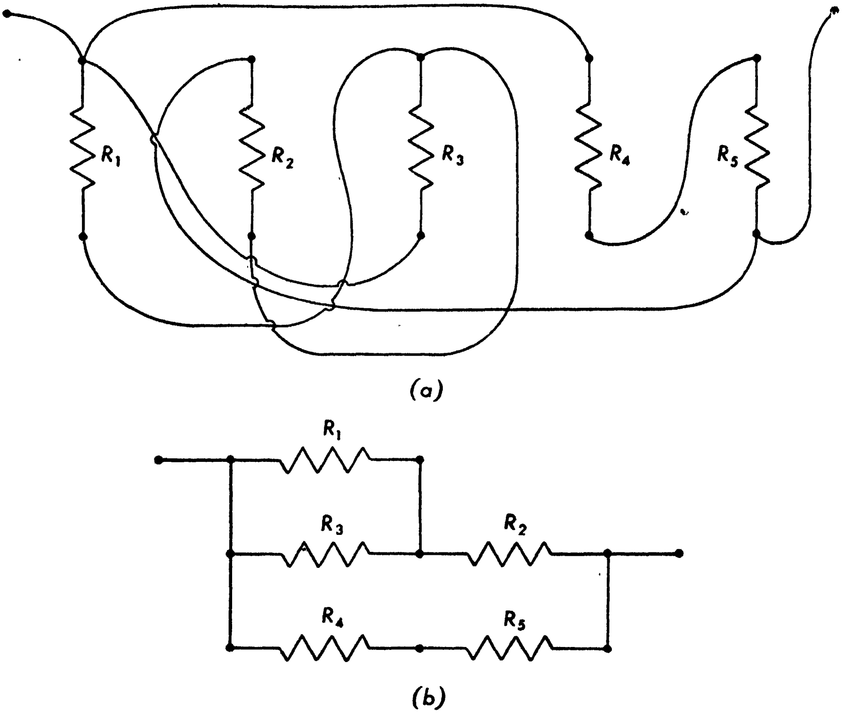


Fig. 45. a. Series parallel circuit. *b.* Part *a* simplified.

Figure 47 is designed to give the student an idea of the various ways that a single circuit may be drawn and still maintain all of its original electrical aspects. The electrical characteristics are essentially the same in all the various forms of the circuit shown. The only thing that was changed in each case was the physical appearance of the circuit.

The circuit in Fig. 48 *a* is a simple one-tube regenerative receiver.

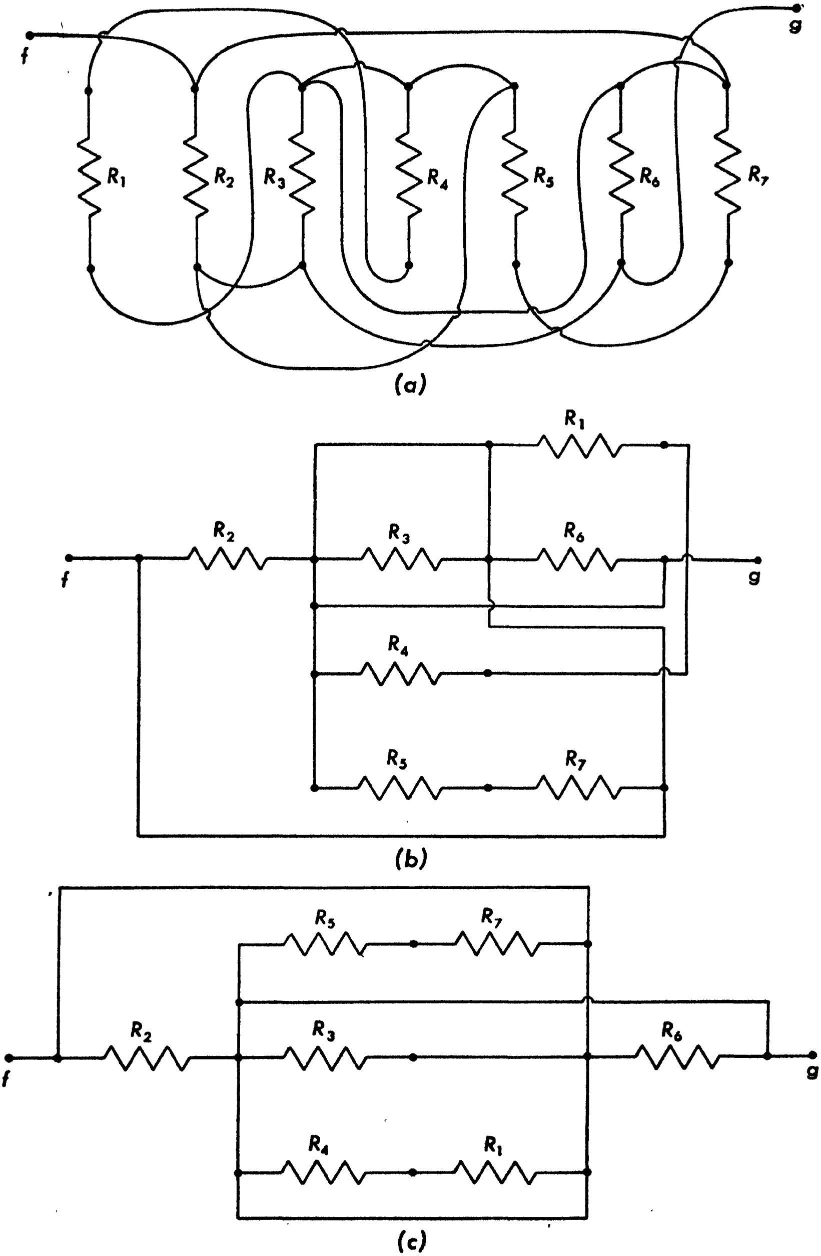


Fig. 46. a. Scrambled circuit. *b* and *c.* Part *a* simplified.

Anyone tracing the circuit from the actual wiring of the receiver itself might obtain a diagram such as Fig. 48 *b*.

Exercise 16:

1. Draw the equivalent circuit of Fig. 41 *a* if the top end of resistors R_3 and R_4 are connected together.

2. Draw the equivalent circuit of Fig. 41 *a* if the bottom end of resistor R_2 is connected to the top end of R_3.

3. Draw the equivalent circuit of Fig. 42 *a* if a resistor R_5 is connected between the bottom ends of resistors R_2 and R_3.

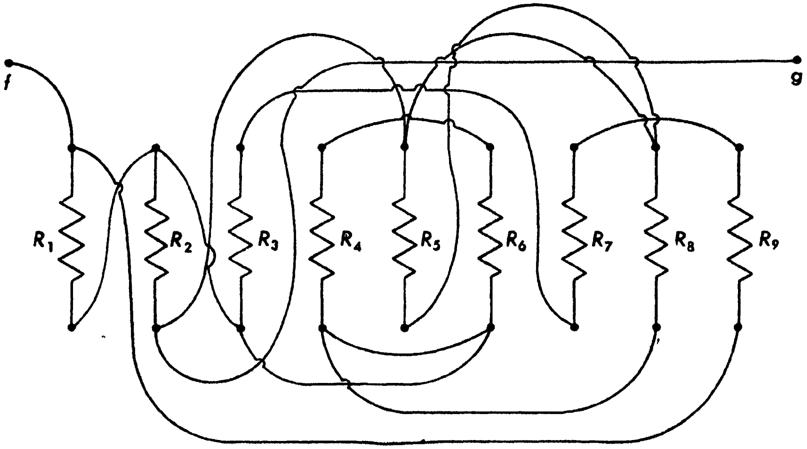


Fig. 47 a. Scrambled circuits. Figs. 47 *b*, *c*, *d* and *e* are other forms of part *a*.

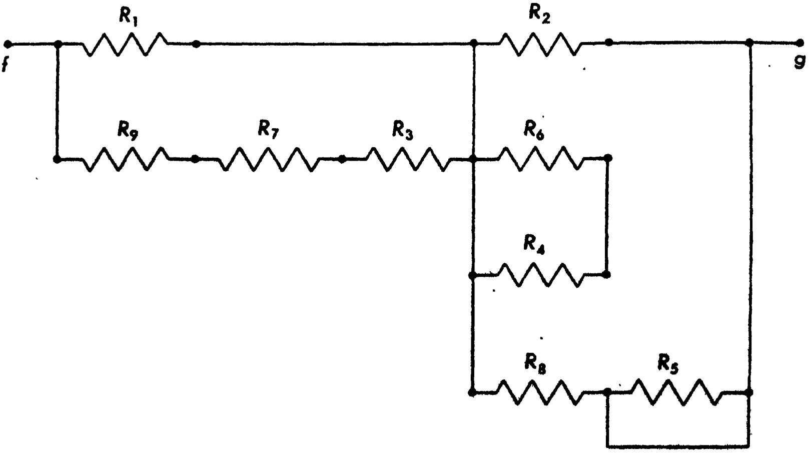


Fig. 47 b

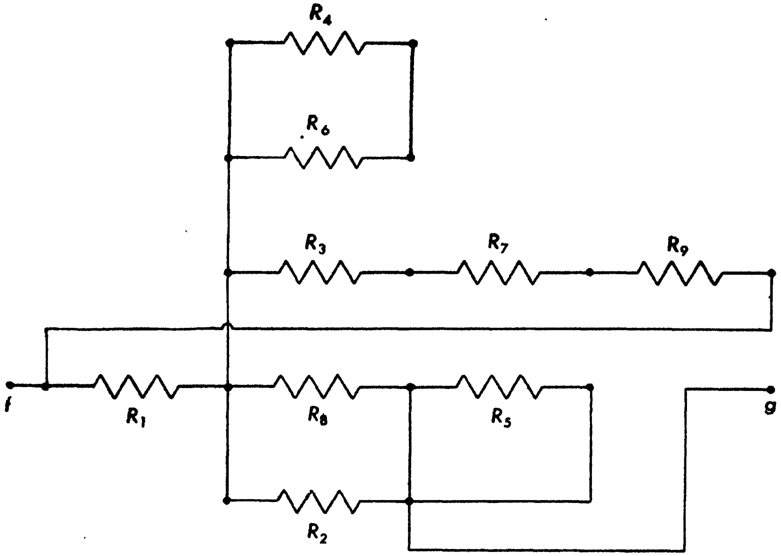


Fig. 47 c

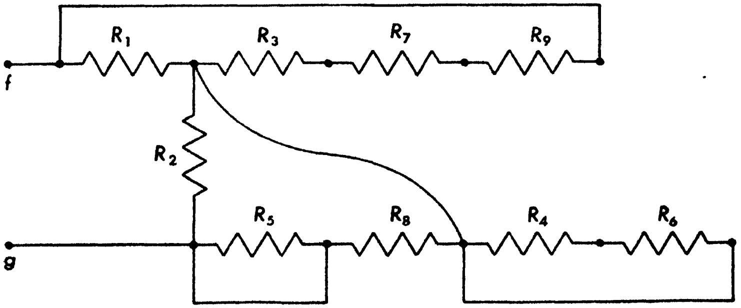


Fig. 47 d

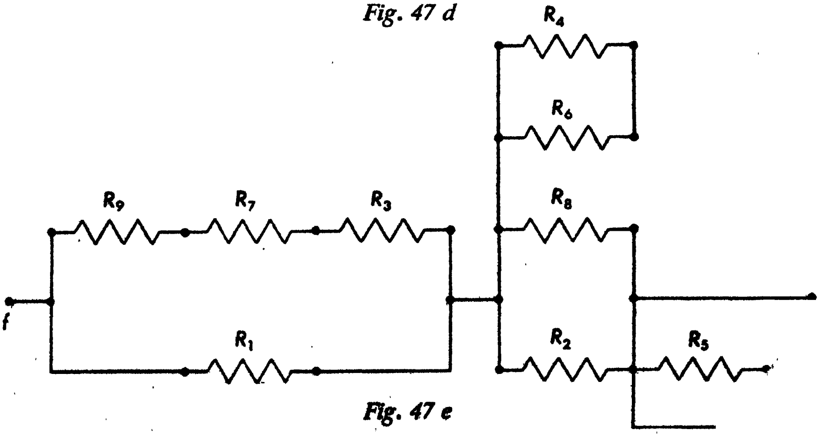


Fig. 47 e

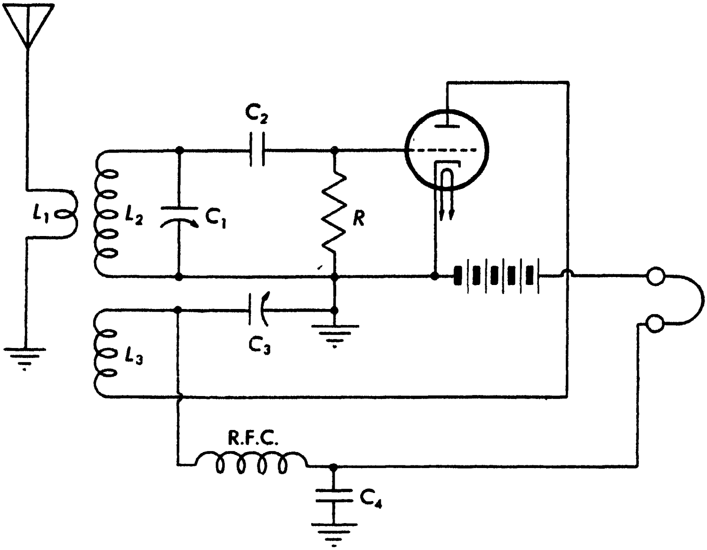
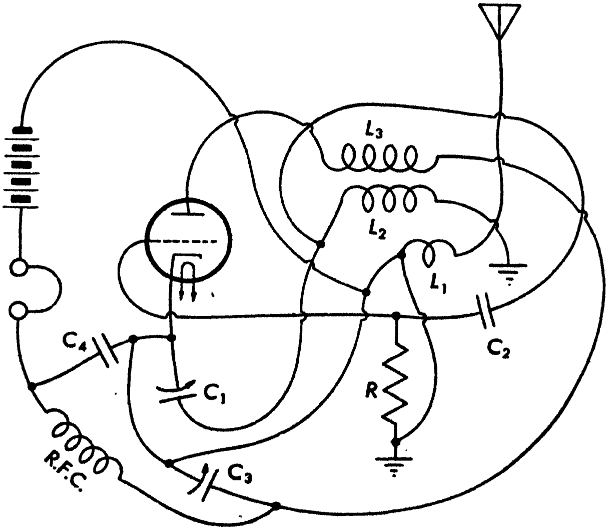


Fig. 48 a. Circuit of a simple single tube regenerative receiver.



b. The same circuit as *a* in the probable form that would result from drawing the circuit directly from the receiver wiring.

4. Draw the equivalent circuit of Fig. 43 *a* if the connecting link between the bottom ends of resistors R_1 and R_2 is removed and a resistor R_5 is connected in this position.

5. Determine the equivalent circuit of Fig. 44 *a* if the connections at the top ends of resistors R_4 and R_5 are switched.

6. Determine the equivalent circuit of Fig. 44 *a* if the connections on the top end of resistor R_3 are switched with the connections on the bottom end of R_3.

7. Determine the equivalent circuit of Fig. 45 *a* if a strap is placed from the top end of R_5 to the bottom end of R_1.

8. Determine the equivalent circuit of Fig. 47 *a* if the connection from the bottom of R_3 to the bottom of R_6 is removed.

9. Determine the equivalent resistance of the circuit in problem 8 if the values of the resistors are as follows:

$R_1 = 20$ ohms
$R_2 = 4$ ohms
$R_3 = 10$ ohms
$R_4 = 8$ ohms
$R_5 = 5$ ohms
$R_6 = 4$ ohms
$R_7 = 4$ ohms
$R_8 = 5$ ohms
$R_9 = 6$ ohms

69. Summary. The student can obtain an amazing amount of knowledge and good experience that will be highly valuable in working with actual receiver and transmitter circuits by building up and working with a simple resistor board.

Several resistors should be mounted on any small board that is available. Each resistor should be provided with clips, binding posts, or other convenient connecting terminals. A number of connecting wires should be so cut that their length is about four times as long as the actual length required for each connection.

Such a device, although crude and simple in appearance, is bound to gain the respect of the student after he has laid out and solved a few circuits of his own design.

REVIEW QUESTIONS

1. What is meant by the point-to-point system of wiring radio circuits?

2. What is meant by the form system of wiring radio circuits?

3. Why is it necessary that the student familiarize himself with the scrambled type of circuits?

4. Is there any essential electrical difference between the circuit of Fig. 46 *b* and the circuit of Fig. 46 *c* (neglecting the resistance of the wiring)?

ANSWERS

Exercise 16, page 78:

9, 14 ohms

CHAPTER VII *Kirchhoff's Laws*

One of the early experimenters in electricity was Gustav Robert Kirchhoff. He investigated many other fields of science, contributing much information that is indispensable in man's ever-increasing search for knowledge and the truths of science.

Kirchhoff studied the division of currents in circuits where various paths are provided for the flow of current. He made two fundamental discoveries in regard to their behavior.

These laws are so simple and fundamental that any present-day student of electricity should discover them for himself. One must remember that practically nothing was known about electricity when Kirchhoff discovered them. Even today one often hears the expression: "Little is known about electricity." In reality, there is a wealth of knowledge of electricity, far more advanced than in the age when Kirchhoff and a few other experimenters began to piece together the few fragmentary bits then available.

70. First Law. Kirchhoff's first law is almost as elementary and as obvious as the statement: "A ball tossed off a building will fall downward." Kirchhoff's first law states:

(22) *All currents flowing into a junction point flow out of the junction point.*

In applying this law to Fig. 49, it can be said that the current flowing into the junction point A through path 1 must flow out of

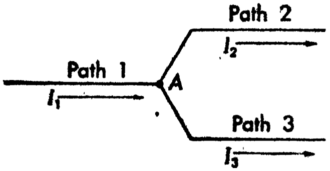


Fig. 49. **From Kirchhoff's law of currents flowing in and out of a junction point:** $I_1 = I_2 + I_3$.

the junction point. Obviously, the only paths of exit for the current are paths 2 and 3. It follows that since I_1 represents the current flowing into the junction point A, and I_2 and I_3 represent the current flowing out of point A that:

$$I_1 = I_2 + I_3$$

Then, another way of stating the law is: The algebraic sum of the currents in all the conductors connecting at a point is zero.

71. Second Law. In Fig. 17, Chap. IV, it was considered that the sum of all the voltage drops around the closed circuit was equal to the battery voltage. Quite often circuits are not simple series circuits as were considered at that time. It is entirely possible to have such a maze of series-parallel sections in a circuit that one particular section may have several resistors, and the current in each resistor may be from a different voltage source. This leads to a more general statement of the relation of the voltages and voltage drops around a closed circuit.

(23) *The algebraic sum of all the voltages and voltage drops around a closed circuit is equal to zero.*

This law may be applied to the circuit of Fig. 50. The algebraic sum of the battery voltages and the voltage drops across the resistors may be equated to zero. One may start at any point in the circuit such as E_1. If this voltage is considered to have a plus sign, the sign of all the

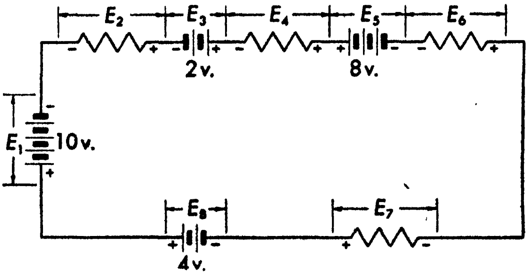


Fig. 50. Circuit illustrating Kirchhoff's second law.

other voltages and voltage drops will be determined by whether they aid or oppose the current flow from E_1. The first voltage encountered is E_2. It is a voltage drop across a resistor so its *algebraic sign* is minus (because this voltage opposes E_1). The next voltage, E_3, is a battery voltage but it opposes the current flow from E_1 so its algebraic sign is also minus. The remaining voltages around the circuit are considered in the same light and the equation becomes:

$$E_1 - E_2 - E_3 - E_4 + E_5 - E_6 - E_7 - E_8 = 0$$

In the consideration of any circuit, the student is cautioned to take into account the *polarity* of the voltage drop across *each resistor,* as well as that of *all batteries* or other source of potential. It is quite possible to have resistors in the particular circuit under observation whose voltage drops oppose the voltage drops of other resistors in the circuit. This is a common possibility when two or more sources of voltage are involved, but it is also possible with a single voltage source as illustrated in the *a-b-c-d* portion of the circuit of Fig. 51. There are two voltage drops around the circuit *a* to *b* to *c* to *d* to *a.* These voltages are E_1 and E_2.

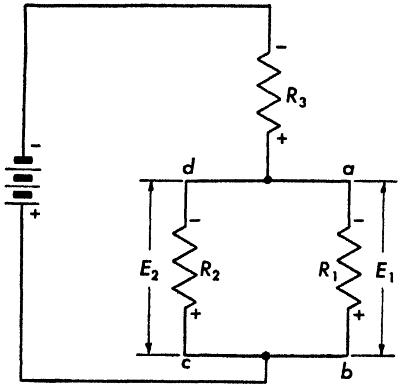


Fig. 51. Illustration of Kirchhoff's second law.

$$E_1 = E_2$$

since resistors R_1 and R_2 are connected in parallel and any measure of voltage drop across one resistor is a measure of the voltage drop across the other resistor as well. Then:

$$E_1 - E_2 = 0$$

72. Further Studies of Mathematics. The successful application of Kirchhoff's laws to the proper solution of circuit problems requires a knowledge of *algebraic addition, subtraction, multiplication, division,* *factoring*, and *simultaneous equations.* Only portions of the above-mentioned topics have been covered up to this point as that is all that has been needed. Additional material on those topics already partially considered and material on those topics not as yet covered will be found in the next few sections.

73. Addition and Subtraction. In most instances, equations contain several terms. Unless the terms are expressed in the same unit, their addition or subtraction can only be indicated by a plus or minus sign. In the equation

$$2x - 4y + 3z = 0$$

the term $4y$ is preceded by a minus sign indicating that it is to be subtracted from $2x$. The term $3z$ is preceded by a plus sign, indicating that it is to be added to $2x$. In the equation

$$6R + 8R - 2R = 15$$

the $8R$ term is added to $6R$ and the $2R$ term is subtracted from $6R$; the equation now becomes:

$$12R = 15$$

One term or *polynomial* (an algebraic expression consisting of two or more terms), called the *subtrahend*, may be subtracted from another term or polynomial, called the *minuend*, by changing all signs of the subtrahend and adding, *algebraically*, to the minuend.

Example:

Subtract $(4a - 2b + c)$ from $(7a + 6b - 3c)$

minuend $(7a + 6b - 3c)$

subtrahend $(4a - 2b + c)$

Change signs of all terms of subtrahend and add to the minuend.

$$\begin{array}{lr} & 7a + 6b - 3c \\ & -\ 4a + 2b - c \\ \hline \text{result} & 3a + 8b - 4c \end{array}$$

Powers with like exponents can be added or subtracted.

Examples:

1. $a^2 + 2a^2 + 6b + 3a^2 + 2b$ simplifies to $6a^2 + 8b$
2. $a^2b + a^2b^2 + 3a^2b + b^2 + 2a^2b^2$ simplifies to $4a^2b + 3a^2b^2 + b^2$

This may be further simplified by factoring as shown in section 76.

The addition or subtraction of powers with unlike exponents can only be indicated unless the operations indicated by the exponents can be performed. The addition of a^2 plus a^3 cannot be performed, but can be indicated by being written thus: $a^2 + a^3$. $(2)^2$ plus $(2)^3$ can be performed only after the first term is squared and the second term cubed. Thus: $(2)^2 + (2)^3 = 4 + 8 = 12$.

Exercise 17:

Add:

1. $2R, 6I, 3E, 7R$
2. $6I^2R, I^2R_1, -4I^2R, 2IR$
3. $x - y + 4z, 2x + 3y$
4. $3a - y, 2b + y, a - 3y$
5. $4a - 3x, x + 2a, 3x - 3a$

Subtract:

Subtrahend Minuend

6. $61b - 4a - 3c$ from $6c - 16b + 2a$
7. $20x + 4y - 2x$ from $10y + 5y - 4x$
8. $2y - 14a - a^2$ from $6b - 2a + a^3$
9. $2c - y^2 + x^3$ from $2cy^2 + 4y^2 - cx^3$
10. $ab - 4ac - 3bc$ from $6c + 4ac + 3bc$

74. Multiplication. Multiplication is accomplished by applying some of the principles already learned. Suppose that it is desired to obtain the product of $(x - y)(x + y)$. This operation is performed as follows:

Step 1. Set down all terms. (Lines (1) and (2).)
Step 2. Multiply each term of line (1) by each term of line (2). (Starting at left side.)
Step 3. Add results.

Solution:

$$\begin{array}{lll}
\textit{Step 1.} & (1) & x - y \\
 & (2) & \underline{x + y} \\
\textit{Step 2.} & & x^2 - xy \\
 & & \underline{\qquad\; xy - y^2} \\
\textit{Step 3.} & & x^2 \qquad\; - y^2
\end{array}$$

Examples:

Multiply:

1. $3x^2y - ab^3 + 2ax^3$ by $2bx$

Solution:

$$\begin{array}{r} 3x^2y - ab^3 + 2ax^3 \\ 2bx \\ \hline 6bx^3y - 2ab^4x + 4abx^4 \end{array}$$

2. $2a^2 - 6x + ab^3$ by $(x - a)$

Solution:

$$\begin{array}{l} 2a^2 - 6x + ab^3 \\ \qquad\quad x - a \\ \hline 2a^2x - 6x^2 + ab^3x \\ \qquad\qquad\qquad\qquad\quad - 2a^3 + 6ax - a^2b^3 \\ \hline 2a^2x + ab^3x - a^2b^3 + 6ax - 2a^3 - 6x^2 \end{array}$$

3. $(2a + 4b)(a^2 - ab + b^2)$

Solution:

$$\begin{array}{l} a^2 - ab + b^2 \\ \qquad 2a + 4b \\ \hline 2a^3 - 2a^2b + 2ab^2 \\ \qquad 4a^2b - 4ab^2 + 4b^3 \\ \hline 2a^3 + 2a^2b - 2ab^2 + 4b^3 \end{array}$$

Exercise 18:

Multiply:

1. $(x^2 - x + 2)(3x^2)$
2. $(x^3 + x^2 - 2x + 4)(x^2 - 2)$
3. $(2a^2 + 3a - 2ab - 3)(a^2 - b^2)$
4. $(2x - 3)(2x + 3)$
5. $(4b - 8bx + 4x)(b - x)$

75. Division. The student may have less occasion to use division than he will multiplication except for the operations of division as covered in Chap. III, section 35 on exponents. However, division is sufficiently important to justify mastering the required steps.

To divide one polynomial by another:

Step 1. Arrange the dividend and divisor in either ascending or descending powers of some *common base.*

Step 2. Divide the *first term* of the *dividend* by the *first term* of the *divisor* and write the result as the *first term of the quotient.*

Step 3. Multiply *all terms* of the *divisor* by the *first term* of the *quotient* and place these new terms in a line under the dividend and subtract from the dividend (by *changing the sign* of each new term).

Step 4. Consider the remainder as obtained in step 3 as a *new dividend* and repeat steps 1, 2, and 3 until division is completed or is completed as far as is possible.

Example:

Divide:

$$(x^4 - 10 + x^3 - x^2) \text{ by } (3 + x)$$

$$\begin{array}{llll}
\text{Quotient} & & x^3 - 2x^2 + 5x - 15 & \\
\text{Divisor} & \underline{x + 3}\big/ & \overline{x^4 + x^3 - x^2 - 10} & \text{dividend} \\
 & & \underline{x^4 + 3x^3} & \\
 & & \quad - 2x^3 - x^2 - 10 & \\
 & & \quad \underline{- 2x^3 - 6x^2} & \\
 & & \qquad\qquad 5x^2 - 10 & \\
 & & \qquad\qquad \underline{5x^2 + 15x} & \\
 & & \qquad\qquad\quad - 15x - 10 & \\
 & & \qquad\qquad\quad \underline{- 15x - 45} & \\
 & & \qquad\qquad\qquad\quad + 35 & \text{remainder}
\end{array}$$

The full quotient is:

$$x^3 - 2x^2 + 5x - 15 + \frac{35}{x + 3}$$

The quotient may be checked by multiplying the quotient by the divisor to obtain the dividend.

Exercise 19:

Divide:

1. $(x^2 + 2x + 3)$ by $(x - 1)$
2. $(a^3 - 2 + 2a - 3a^2)$ by $(2 + a)$

3. $(a^2b + 2ab - b^2)$ by $(a - b)$
4. $(d - 4x + x^4 + 3x^2 - 3x^3)$ by $(x^3 - 2x^2 + x - 3)$
5. $(2x^5 - 5x^4 - 10 + 13x + 11x^3 - 15x^2)$ by $(x^2 - x + 2)$

76. Factoring. In the expression:

$$2a^2b - 4ab + 6ab^2$$

the factors $2ab$ are *common to all terms* and the expression may be rewritten as:

$$2ab(a - 2 + 3b)$$

2, a, b, and $(a - 2 + 3b)$ are *factors* since they may *be multiplied to obtain the original expression.*

Factoring of the above was accomplished *by inspection.* It is not always possible to factor an expression as readily from inspection as in the above instance because the factors are not always so obvious.

The use of certain expressions and their factors has become so common that they are recognized as *standard forms.* An expression to be factored should first be inspected to determine whether any of these standard forms apply, or if the expression can be reduced to one or more of the standard forms. The following are the more common types:

	Factors		*Product*
1.	$x(a + b)$	$=$	$ax + bx$
2.	$(x - y)(x - y)$	$=$	$x^2 - 2xy + y^2$
3.	$(x + y)(x + y)$	$=$	$x^2 + 2xy + y^2$
4.	$(x - y)(x + y)$	$=$	$x^2 - y^2$

It should be noted that not only can the proper factors be obtained from the use of the above standard-type forms, but the process may be reversed and used to determine the product by inspection.

Example:

Determine the product from inspection:

$(a + 2b)^2$ This is type 3.

Step 1. Square the first term: a^2

Step 2. Take the product of twice the first term times the second: $4ab$

Step 3. Square the second term: $4b^2$

Result: $(a + 2b)^2 = a^2 + 4ab + 4b^2$

This process may be reversed and the factors obtained from the product by extracting the square roots of the two outside terms and connecting them with the sign of the middle term.

Exercise 20:

Use the standard types to determine the following products by inspection:

1. $2b(a + c - d)$
2. $3ab(a^2 + b + c)$
3. $(a + 6b)^2$
4. $(a - 3b)^2$
5. $(a^2 + b^2)^2$
6. $(a - 1)(a + 1)$
7. $(2x - 3b)^2$

Factor:

8. $a^2bc^3 + 3abc^2 - 2a^3b^2c$
9. $4a^2 + 4a + 1$
10. $4a^2 - 8ab + 4b^2$

77. Simultaneous Equations. In many instances it is possible to write two or more equations involving the same set of unknowns for a specific set of circuit conditions. A good example is the use of the principles set forth in Kirchhoff's laws in setting up equations for the solution of resistance network problems.

Any solution of two linear equations with a set of values that satisfy both linear (straight line) equations is called a *simultaneous solution.*

Various methods or systems are used for the simultaneous solution of equations. Principal of these are: (1) graphical, (2) addition, (3) subtraction, (4) substitution, (5) comparison, and (6) determinants.

78. Graphical Solution. The graph of each equation may be plotted on proper rectangular graph paper. If the two graphs intersect at any point, the two equations are said to be linear simultaneous equations. The values of the coordinates at the point of intersection are common to both graphs and form the solution.

79. Addition and Subtraction Solution. Two linear equations with two common unknowns can be solved by adding or subtracting the two equations if one of the unknowns can be eliminated by either operation. It may be necessary to multiply both sides of one or both equations by some quantity before either form of simultaneous solution can be used.

Example 1:

Solve for x and y: (1) $3x + 2y = 4$
(2) $x - 2y = 8$

Solution by *addition:*

Add (1) and (2)

$$\begin{array}{r} 3x + 2y = 4 \\ x - 2y = 8 \\ \hline 4x = 12 \\ x = 3 \end{array}$$

Substitute this value of x in equations 1 and 2 and solve for y. The same value should be obtained in each case which provides a check for correctness of solution.

Substituting the value of x in (1)

$$\begin{aligned} 9 + 2y &= 4 \\ 2y &= 4 - 9 \\ y &= -2.5 \end{aligned}$$

Substituting the value of x in (2)

$$\begin{aligned} 3 - 2y &= 8 \\ -2y &= 8 - 3 \\ y &= -2.5 \end{aligned}$$

The set of values: $x = 3$ and $y = -2.5$ satisfy both equations 1 and 2.

The method of simultaneous solution of linear equations by subtraction is the same as for addition except for the first step.

Example 2:

Solve for x and y: (1) $x + 2y = 2$
(2) $x + 6y = 4$

Solution by *subtraction:*

Subtract (2) from (1)

$$\begin{array}{r} x + 2y = 2 \\ x + 6y = 4 \\ \hline -\ 4y = -2 \\ y = \frac{1}{2} \end{array}$$

Substitute this value of y in (1)

$$\begin{aligned} x + 1 &= 2 \\ x &= 1 \end{aligned}$$

Substitution of the value of y in (2)

$$\begin{aligned} x + 3 &= 4 \\ x &= 4 - 3 \\ x &= 1 \end{aligned}$$

Example 3:

Solve for x and y

$$\begin{aligned} (1)\ 3x + 2y &= 2 \\ (2)\ 6x + y &= 3 \end{aligned}$$

Solution by subtraction:

Multiply (1) by 4 and (2) by 2:

Subtract (2) from (1)

$$\begin{aligned} 12x + 8y &= 8 \\ \underline{12x + 2y} &\underline{= 6} \\ 6y &= 2 \\ y &= \tfrac{1}{3} \end{aligned}$$

Substitute this value of y in (1)

$$\begin{aligned} 3x + \tfrac{2}{3} &= 2 \\ 3x &= \tfrac{4}{3} \\ x &= \tfrac{4}{9} \end{aligned}$$

Substitution of the value of y in (2)

$$\begin{aligned} 6x + \tfrac{1}{3} &= 3 \\ 6x &= \tfrac{8}{3} \\ x &= \tfrac{4}{9} \end{aligned}$$

Exercise 21:

Solve graphically and substitute values in the original equations for check.

1. $2x + y = 4$
$x - 3y = 3$

2. $4x - y = 8$
$5x + 2y = 10$

Solve by addition or subtraction:

3. $y - 2x = 3$
$3y + 2x = 13$

4. $x + 4y = 16$
$x - 3y = 2$

5. $8x + 6y = 30$
$3x - 6y = 14$

6. $7x - 13y = 36$
$2x - 13y = -14$

7. $4x + 6y = 7$
$5x + 4y = 10$

8. $3x - 7y = 20$
$7x + 6y = 9$

80. Substitution, Comparison, Determinants.

This section covers the simultaneous solution of equations by substitution, comparison, and determinants.

1. *Substitution:*

Example:

Solve for x and y

$$\begin{aligned} (1)\ 2x - y &= 1 \\ (2)\ 3x + y &= 4 \end{aligned}$$

Solution:

Solve equation (2) for x

$$\begin{aligned} 3x &= 4 - y \\ x &= \frac{4 - y}{3} \end{aligned}$$

Substitute this value of x in (1)

$$\begin{aligned} \frac{2(4 - y)}{3} - y &= 1 \\ \frac{8 - 2y}{3} - y &= 1 \\ 8 - 2y - 3y &= 3 \\ -5y &= 3 - 8 \\ y &= 1 \end{aligned}$$

Substitute this value of y in (2)

$$\begin{aligned} 3x + 1 &= 4 \\ 3x &= 4 - 1 \\ x &= 1 \end{aligned}$$

The value of y may be substituted in equation 1 and the equation solved for x as a check for correctness of solution.

Thus:

$$\begin{aligned} 2x - 1 &= 1 \\ 2x &= 1 + 1 \\ x &= 1 \end{aligned}$$

2. *Comparison:*

This method closely parallels that of substitution. Each equation is either solved for x or for y and the two expressions equated to each other. This eliminates one unknown, and the new expression can be solved for the other unknown. The value of the unknown thus found can be substituted in either equation in order to solve for the other unknown.

3. *Determinants:*

This system provides still other methods for the simultaneous solution of equations but is beyond the scope of this book.

Exercise 22:

Solve by Substitution:

1. $3x + 2y = 16$
$8x + 11y = 13$

2. $7x - 15y = 17$
$2x + y = 4$

3. $x - y = 1$
$2x - y = 2$

Solve by any method:

4. $3x - 3y = 11$
$3y - 2x = -4$

5. $2y - 3x = 2$
$17x + 2y = 13$

6. $8x + 9y = 7$
$11x - 3y = 5$

7. $5x + 11y = 10$
$3x - 7y = 3$

8. $15x + 23y = 57$
$13x - 2y = 14$

9. $16x - 27y = -21$
$24x + 15y = 38$

10. $14x + 7y = 63$
$10x + 21y = 3$

81. Simple Applications of Kirchhoff's Laws. Many circuits are impossible of solution or cannot readily be solved by the methods given in Chap. IV. Often such solutions, when possible, are complicated and quite involved.

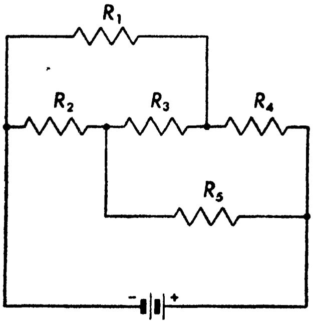


Fig. 52 a. Cross connected resistance circuit that is more readily solved by the use of Kirchhoff's laws.

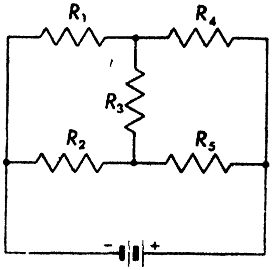


Fig. 52 b. Same circuit as *a* redrawn to show that the resistors form the well-known "H" network.

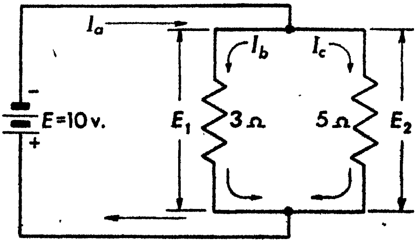


Fig. 53. Application of Kirchhoff's laws.

Figure 52 is a typical example. Such circuits are more readily analyzed by means of Kirchhoff's laws.

The total current flowing in the circuit of Fig. 53 can be readily determined by the application of Kirchhoff's laws and also by methods introduced in Chap. IV. The latter methods may be used to check the following results as obtained from the use of Kirchhoff's laws.

Solution:

$$\begin{aligned} E_1 &= E_2 = E = 10 \text{ volts} \\ I_a &= I_b + I_c \\ &= \frac{10}{3} + \frac{10}{5} \\ &= \frac{50}{15} + \frac{30}{15} \\ &= \frac{80}{15} = 5\tfrac{1}{3} \text{ amperes} \end{aligned}$$

Exercise 23:

1. Solve for total current in Fig. 53 if a 3-volt battery is placed in series with the 5-ohm resistor and top end of battery is positive.
2. Find the total current in Fig. 53 if the 3-volt battery of problem 1 remains in place and the 10-volt battery terminals are reversed.
3. Solve for total current in Fig. 53 if a 3-volt battery is placed in each resistor branch and the top terminal of each battery is negative.
4. Determine I_b in Fig. 54.

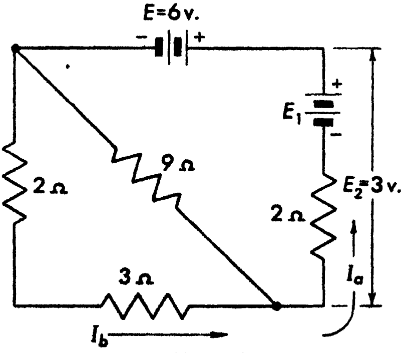


Fig. 54

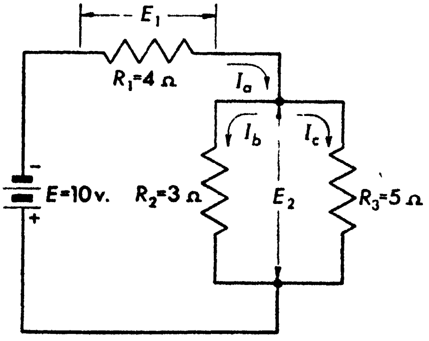


Fig. 55

Example:

Determine E_1, E_2, I_a, I_b, and I_c in Fig. 55.

Solution:

From Kirchhoff's second law (23):

(1) $$E_1 + E_2 = E = 10$$

From Kirchhoff's first law (22):

(2) $$I_b + I_c = I_a$$

From Ohm's law:

$$I_a = \frac{E_1}{4}; \quad I_b = \frac{E_2}{3}; \quad I_c = \frac{E_2}{5}$$

Substituting in equation 2:

$$\frac{E_2}{3} + \frac{E_2}{5} = \frac{E_1}{4}$$

$$\frac{5E_2}{15} + \frac{3E_2}{15} = \frac{E_1}{4}$$

$$\frac{8E_2}{15} = \frac{E_1}{4}$$

(3) $$\frac{8E_2}{15} - \frac{E_1}{4} = 0$$

Equations 1 and 3 may be solved by simultaneous equations by multiplying equation 1 by 8/15 and subtracting equation 3 from equation 1.

$$\begin{array}{l} \dfrac{8E_2}{15} + \dfrac{8E_1}{15} = \dfrac{80}{15} \\ \dfrac{8E_2}{15} - \dfrac{E_1}{4} = 0 \\ \hline \dfrac{8E_1}{15} + \dfrac{E_1}{4} = \dfrac{80}{15} \end{array}$$

$$\frac{32}{60}E_1 + \frac{15}{60}E_1 = \frac{80}{15}$$

$$\frac{47}{60}E_1 = \frac{80}{15}$$

$$E_1 = \frac{60 \times 80}{47 \times 15} = 6.8 \text{ volts}$$

$$I_a = \frac{6.8}{4} = 1.7 \text{ amperes}$$

$$E_2 = 10 - 6.8 = 3.2 \text{ volts}$$

$$I_b = \frac{3.2}{3} = 1.07 \text{ amperes}$$

$$I_c = \frac{3.2}{5} = .64 \text{ ampere}$$

Exercise 24:

1. Determine E_1, E_2, I_a, I_b, and I_c in the circuit of Fig. 55 if $E = 15$ volts, $R_1 = 6$ ohms, $R_2 = 10$ ohms, and $R_3 = 8$ ohms.
2. Determine E_1, E_2, I_a, I_b, and I_c in the circuit of Fig. 55 if a 4-ohm resistor is connected in series with the 3-ohm resistor and $R_1 = 10$ ohms and $R_3 = 10$ ohms.
3. Determine E_1, E_3, R_4, I_a, I_b, and I_c in the circuit of Fig. 56.
4. Determine E_1, E_2, I_a, I_b, and I_c in the circuit of Fig. 57.

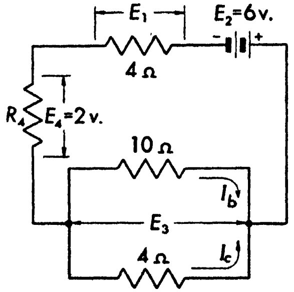


Fig. 56

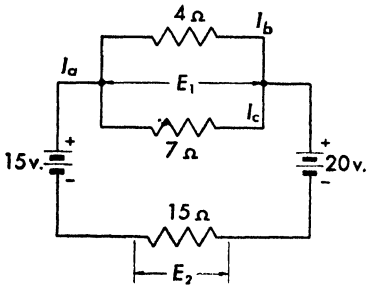


Fig. 57

82. Determination of Direction of Current Flow. The student knows that the current flowing out of a battery flows away from the negative terminal. The direction of the current flowing through a resistor acted upon by several batteries is not so easy to determine by inspection. Often, when it is hard to determine the direction of current flow by inspection, it is more convenient *to assume* a direction of current flow. If the assumption is right, the sign of the current found from the solution will be plus. If the assumption is wrong, the sign of the current found from the solution will be minus showing that the current is actually flowing in the opposite direction.

Example:

Determine E_1, E_2, E_3, I_a, I_b, I_c, and the direction of currents in the circuit of Fig. 58.

Solution:

Assume the currents I_a and I_c to flow into the junction point O and the current I_b to flow out of the junction point O as shown by arrows.

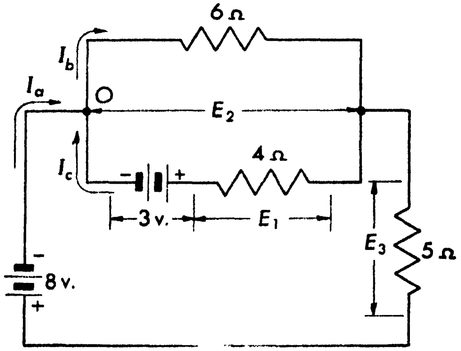


Fig. 58

Then: (1) $I_b = I_a + I_c$

Also: (2) $3 = E_1 + E_2$

(3) $8 = E_2 + E_3$

Substituting in equation 1 (4) $\frac{E_2}{6} = \frac{E_3}{5} + \frac{E_1}{4}$

From equation 2 (5) $E_1 = 3 - E_2$

From equation 3 (6) $E_3 = 8 - E_2$

Substituting in equation 4 (7) $\frac{E_2}{6} = \frac{(8 - E_2)}{5} + \frac{(3 - E_2)}{4}$

Collecting terms:

$$\frac{10E_2 - 12(8 - E_2) - 15(3 - E_2)}{60} = 0$$

$$\frac{10E_2 - 96 + 12E_2 - 45 + 15E_2}{60} = 0$$

$$\frac{37E_2}{60} = \frac{141}{60}$$

$$\begin{aligned} E_2 &= \frac{141 \times \cancel{60}}{37 \times \cancel{60}} \\ &= 3.81 \text{ volts} \end{aligned}$$

Solve for I_b:
$$I_b = \frac{E_2}{6} = \frac{3.81}{6} = .636 \text{ ampere}$$

From equation 6
$$\begin{aligned} E_3 &= 8 - E_2 \\ &= 8 - 3.81 \\ &= 4.19 \text{ volts} \end{aligned}$$

Solving for I_a:
$$I_a = \frac{E_3}{5} = \frac{4.19}{5} = .838 \text{ ampere}$$

From equation 5:
$$\begin{aligned} (8) \quad E_1 &= 3 - E_2 \\ &= 3 - 3.81 \\ &= -.81 \text{ volts} \end{aligned}$$

Solve for I_c:
$$I_c = \frac{E_1}{4} = -\frac{.81}{4} = -.202 \text{ ampere}$$

The minus sign in front of the current I_c indicates that the *assumed direction of current flow is wrong* and the current actually flows in the *opposite direction* to that shown by the arrow I_c. The minus sign in front of the voltage E_1 (answer of equation 8) indicates that the assumed direction of voltage drop as indicated by the sign preceding E_1 in equation 2 is also wrong. The sign in front of the voltage E_1 in equation 2 should be minus. It appeared logical to make the assumptions that were made because the current would normally flow from the negative end of the 3-volt battery around through the 6-ohm and 4-ohm resistors and back to the battery were it not for the presence of the 8-volt battery. The solution proves that the current I_c is not the result of the voltage from the 3-volt battery. The 8-volt battery offsets the voltage of the 3-volt battery, forcing current through it in the opposite direction, through the 4-ohm and 5-ohm resistors and back to the positive terminal of the 8-volt battery.

Since the current through the 3-volt battery is being forced through it in the opposite direction, its action is that of another resistor in the circuit with a voltage drop of 3 volts across its terminals. Such a resistance would have a resistance of: $R = \frac{E}{I} = \frac{3}{.202} = 14.85$ ohms.

From equation 1: $I_b = I_a + I_c$

Substituting in equation 1 $.636 = .838 + (-.202)$

$= .636$ amperes

Exercise 25:

1. Assume the currents flowing in the right directions in the above example, as proved by the solution, set up new equations accordingly and solve for all required values of voltages and currents.

2. Solve for E_1, E_2, E_3, I_a, I_b, I_c and the directions of current flow in the circuit of Fig. 59.

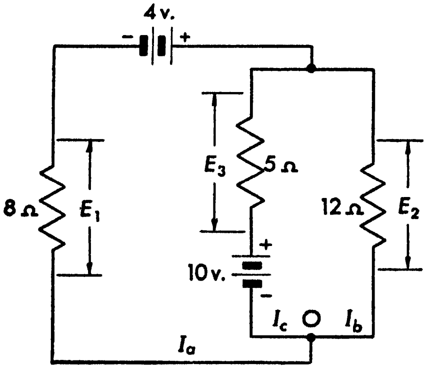
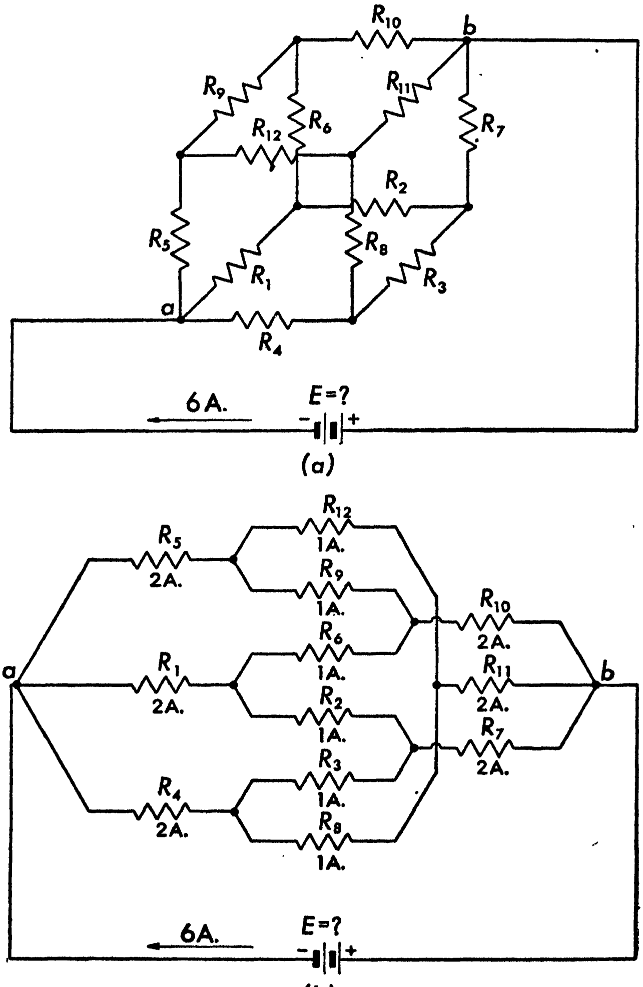


Fig. 59

83. Extended Applications of Kirchhoff's Laws. Numerous examples and problems have been presented in order to familiarize the student with the solution of network problems by the use of Kirchhoff's laws. The complexity of these circuits was kept to a minimum in order that the steps of solution might be few and easy to follow. The use of these laws may be extended to the solution of networks much more complex and difficult to solve.

The circuit of Fig. 60 is one that has attracted widespread interest among laymen during the past twenty years because of its apparent simplicity but stubborn opposition to solution by means of the conventional Ohm's law method of solution.

The twelve resistors of the circuit represent the twelve edges of a box or cube. Consider each resistor to have a resistance of 1 ohm. Determine the resistance from one of the bottom corners *a* to the most distant upper corner *b* of the circuit as shown in Fig. 60 *a*. The symmetrical nature of this circuit due to all resistors having the same resistance makes the problem comparatively simple. This symmetry



(*a*)

(*b*)

Fig. 60. a. Twelve one ohm resistors represent 12 edges of a box. *b.* The same circuit as *a*, redrawn with the connections the same but with the resistors in different positions.

of the circuit is more evident from Fig. 60 *b* which is the same as Fig. 60 *a* but has the positions of the resistors rearranged.

Solution:

$$\begin{aligned} E &= I_5R_5 + I_{12}R_{12} + I_{11}R_{11} \\ &= (2 \times 1) + (1 \times 1) + (2 \times 1) \\ &= 2 + 1 + 2 \\ &= 5 \text{ volts} \end{aligned}$$

$$R_{a-b} = \frac{E}{I} = \frac{5}{6} \text{ ohms}$$

Example 1:

Determine I_a, I_b, and I_c in the circuit of Fig. 61.

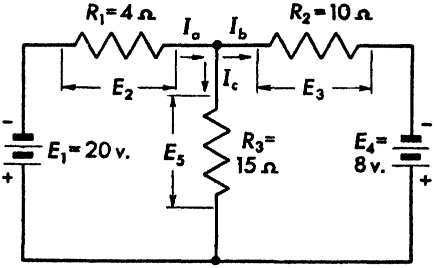


Fig. 61

Solution:

The assumed directions of current flow are indicated by the arrows shown in the circuit diagram.

From this assumption	(1)	$I_a = I_b + I_c$
and	(2)	$E_1 - E_2 - E_3 - E_4 = 0$
Substituting in equation 2	(3)	$20 - 4I_a - 10I_b - 8 = 0$
Collecting terms	(4)	$12 - 4I_a - 10I_b = 0$
For the left portion of the circuit	(5)	$E_1 - E_2 - E_5 = 0$
Substituting in equation 5	(6)	$20 - 4I_a - 15I_c = 0$
From equation 1	(7)	$I_c = I_a - I_b$

Substituting for the value of I_c in equation 6 (8) $20 - 4I_a - 15I_a + 15I_b = 0$

Collecting terms (9) $20 - 19I_a + 15I_b = 0$

Equations 4 and 9 may be solved simultaneously by:

Multiplying equation 4 by 3 (10) $36 - 12I_a - 30I_b = 0$

and multiplying equation 9 by 2 (11) $40 - 38I_a + 30I_b = 0$

$$\overline{76 - 50I_a \qquad = 0}$$

$$I_a = \frac{76}{50} = 1.52 \text{ amperes}$$

Substituting in equation 4 $12 - 4(1.52) - 10I_b = 0$

$$12 - 6.08 - 10I_b = 0$$

$$I_b = \frac{5.92}{10} = .592 \text{ ampere}$$

From equation 7 $I_c = I_a - I_b$

$$= 1.52 - .592$$

$$= .928 \text{ ampere}$$

Substitute in equation 6 for check:

$$20 - 4(1.52) - 15(.928) = 0$$

$$20 - 6.08 - 13.92 = 0$$

$$20 - 20 = 0$$

Example 2:

Determine I_1, I_2, I_3, I_4, I_5 and I_t in the circuit of Fig. 62.

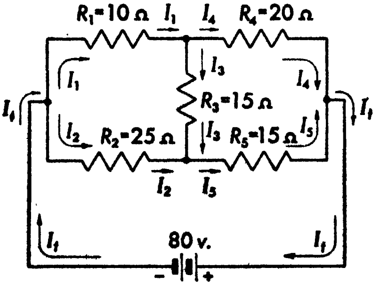


Fig. 62

Solution:

Equations 1–7 represent various paths of current flow.

(1) $I_2R_2 - I_3R_3 + I_4R_4 = 80$

(2) $I_1R_1 + I_3R_3 + I_5R_5 = 80$

(3) $I_1R_1 + I_4R_4 = 80$

(4) $I_2R_2 + I_5R_5 = 80$

(5) $I_1 = I_3 + I_4$

(6) $I_5 = I_2 + I_3$

(7) $I_2 = I_5 - I_3$

Equations from the above list are paired together and properly treated for simultaneous solution in order to solve for the various unknowns.

(3) $10I_1 + 20I_4 = 80$

Substitute value of I_1 from (5) $10I_3 + 10I_4 + 20I_4 = 80$

(8) $10I_3 + 30I_4 = 80$

(4) $25I_2 + 15I_5 = 80$

Substitute value of I_2 from (7) $25I_5 - 25I_3 + 15I_5 = 80$

(9) $40I_5 - 25I_3 = 80$

Multiply (8) by 5 $50I_3 + 150I_4 = 400$

Multiply (9) by 2 $-50I_3 + 80I_5 = 160$

Adding (10) $150I_4 + 80I_5 = 560$

(1) $25I_2 - 15I_3 + 20I_4 = 80$

Substitute the value of I_2 from (7) $25I_5 - 25I_3 - 15I_3 + 20I_4 = 80$

(11) $-40I_3 + 20I_4 + 25I_5 = 80$

(2) $10I_1 + 15I_3 + 15I_5 = 80$

Substitute the value of I_1 from (5) $10I_3 + 10I_4 + 15I_3 + 15I_5 = 80$

(12) $25I_3 + 10I_4 + 15I_5 = 80$

$$\begin{array}{ll}
\text{Multiply (11) by 5} & -200I_3 + 100I_4 + 125I_5 = 400 \\
\text{Multiply (12) by 8} & \underline{200I_3 + 80I_4 + 120I_5 = 640} \\
\text{Add} & (13) \qquad 180I_4 + 245I_5 = 1{,}040 \\
\text{Multiply (13) by 5} & 900I_4 + 1{,}225I_5 = 5{,}200 \\
\text{Multiply (10) by 6} & \underline{900I_4 + 480I_5 = 3{,}360} \\
\text{Subtract} & 745I_5 = 1{,}840 \\
 & I_5 = 2.47 \text{ amperes} \\
\text{Multiply (10) by 245} & 36{,}750I_4 + 19{,}600I_5 = 137{,}200 \\
\text{Multiply (13) by 80} & \underline{14{,}400I_4 + 19{,}600I_5 = 83{,}200} \\
\text{Subtract} & 22{,}350I_4 = 54{,}000 \\
 & I_4 = 2.42 \text{ amperes}
\end{array}$$

Substitute the computed values of I_4 and I_5 in (4) and (6) and (5):

$$\begin{aligned}
(4)\ 25I_2 + 15(2.47) &= 80 \\
25I_2 + 37.05 &= 80 \\
25I_2 &= 42.95 \\
I_2 &= 1.72 \text{ amperes} \\
(6)\ 2.47 &= 1.72 + I_3 \\
I_3 &= 2.47 - 1.72 \\
I_3 &= .75 \text{ ampere} \\
(5)\ I_1 &= .75 + 2.42 \\
I_1 &= 3.17 \text{ amperes} \\
I_t &= I_1 + I_2 \\
&= 3.17 + 1.72 \\
&= 4.89 \text{ amperes}
\end{aligned}$$

Check:

$$\begin{aligned}
I_t &= I_4 + I_5 \\
&= 2.42 + 2.47 \\
&= 4.89 \text{ amperes}
\end{aligned}$$

Exercise 26:

1. What is the equivalent resistance between points *a* and *b* in the circuit of Fig. 60 if each resistor has a resistance of 10 ohms? If each resistor is 6 ohms?

2. What will be the total resistance in the circuit of Fig. 60 if resistors R_1, R_4, and R_5 are shorted?

3. What is the total resistance of the circuit of Fig. 60 if a shorting strap is placed from the junction of R_8 and R_{12} to the junction of R_3 and R_7?

4. Solve for I_a, I_b, and I_c in the circuit of Fig. 61 if E_4 is changed to 15 volts and R_1 to 5 ohms.

5. Determine I_1, I_2, and I_3 in the circuit of Fig. 63.

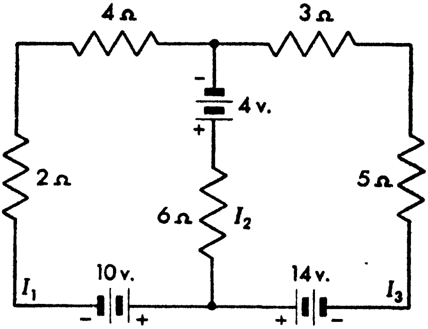


Fig. 63

84. Superposition Solution of Circuits. Circuit solution by means of Kirchhoff's laws may be long and tedious in some instances as in the examples of Figs. 61 and 62. Numerous theorems have been developed that aid in the solution of circuit-network problems.

One of these theorems is the *superposition theorem.* This is a system whereby *the effect of each circuit voltage is considered separately.*

Example:

Determine the currents I_a, I_b, and I_c in the circuit of Fig. 61 by means of the superposition theorem.

Solution:

Consider first the effect of the 20-volt battery in the circuit, neglecting altogether, for the moment, the *voltage effect* of the 8-volt battery. If the 8-volt battery had been considered to have internal resistance, then this resistance would have to be considered.

Figure 64 shows Fig. 61 with only the 20-volt battery in the circuit. This becomes a simple Ohm's law problem for the solution of the currents through the resistors, if only the 20-volt battery were acting upon the circuit.

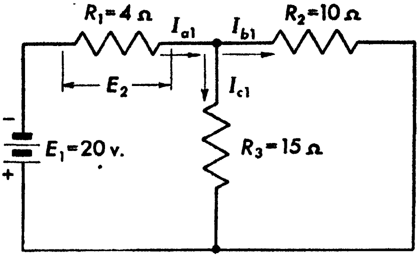


Fig. 64. Circuit of Fig. 61 showing effect of the 20-volt battery alone in the circuit. (Superposition method of solution.)

The combined resistance of R_2 and R_3 is:

$$R_{2-3} = \frac{R_2 R_3}{R_2 + R_3} = \frac{10 \times 15}{10 + 15} = \frac{150}{25} = 6 \text{ ohms}$$

$$I_{a1} = \frac{E_1}{R} = \frac{20}{4+6} = 2 \text{ amperes}$$

$$E_2 = I_{a1} R_1 = 2 \times 4 = 8 \text{ volts}$$

$$I_{b1} = \frac{E_1 - E_2}{R_2} = \frac{20 - 8}{10} = 1.2 \text{ amperes}$$

$$I_{c1} = \frac{12}{15} = .8 \text{ ampere}$$

Consider the circuit of Fig. 61 with only the voltage of the 8-volt battery acting upon the circuit. This is shown in Fig. 65.

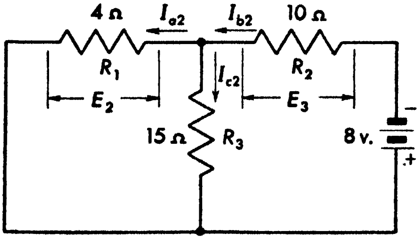


Fig. 65. Circuit of Fig. 61 showing the effect of the 8-volt battery in the circuit. (Further application of the superposition theorem in the solution of circuits.)

$$R_{1-3} = \frac{R_1 R_3}{R_1 + R_3} = \frac{4 \times 15}{4 + 15} = \frac{60}{19} = 3.16 \text{ ohms}$$

$$I_{b2} = \frac{E}{R} = \frac{8}{3.16 + 10} = \frac{8}{13.16} = .608 \text{ ampere}$$

$$I_{a2} = \frac{8 - E_3}{4} = \frac{8 - 6.08}{4} = .48 \text{ ampere}$$

$$I_{c2} = \frac{1.92}{15} = .128 \text{ ampere}$$

The actual current flowing in each branch of the circuit of Fig. 61 is determined by taking the algebraic sum of the currents in each branch. I_{a1} from Fig. 64 is 2 amperes and I_{a2} from Fig. 65 is .48 ampere and flowing in the opposite direction. Therefore:

$$I_a = I_{a1} - I_{a2} = 2 - .48 = 1.52 \text{ amperes}$$

Likewise:

$$I_b = I_{b1} - I_{b2} = 1.2 - .608 = .592 \text{ ampere}$$

and:

$$I_c = I_{c1} + I_{c2} = .8 + .128 = .928 \text{ ampere}$$

Exercise 27:

Solve by superposition method:

1. Problem No. 2 of Exercise 23.
2. Problem No. 4 of Exercise 24.
3. In Fig. 66, $R_1 = 2$ ohms, $R_2 = 6$ ohms, $R_3 = 4$ ohms, $R_4 = 5$ ohms, $E_1 = 6$ volts, $E_2 = 9$ volts, and $E_3 = 24$ volts. Solve for the current through each resistor and voltage drop across each resistor.

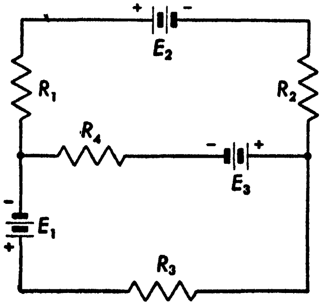


Fig. 66

REVIEW QUESTIONS

1. Give Kirchhoff's two laws of the electric circuit.
2. What is a linear equation?
3. What are simultaneous equations?

4. Give some of the methods used for simultaneous solutions.

5. Why is a knowledge of simultaneous equations necessary for a more thorough understanding of the solution of circuits by means of Kirchhoff's laws?

6. What is meant by the superposition solution of circuits?

ANSWERS

Exercise 17, page 86:

1. $3E + 6I + 9R$
2. $I^2R_1 + 2I^2R + 2IR$
3. $3x + 2y + 4z$
4. $4a + 2b - 3y$
5. $3a + x$
6. $6a - 77b + 9c$
7. $11y - 22x$
8. $a^3 + a^2 + 12a + 6b - 2y$
9. $5y^2 + 2cy^2 - cx^3 - x^3 - 2c$
10. $8ac + 6bc + 6c - ab$

Exercise 18, page 87:

1. $3x^4 - 3x^3 - 6x^2$
2. $x^5 + x^4 - 4x^3 + 2x^2 + 4x - 8$
3. $2a^4 + 3a^3 - 2a^3b - 3a^2 - 2a^2b^2 - 3ab^2 + 2ab^3 + 3b^2$
4. $4x^2 - 9$
5. $4b^2 - 8b^2x + 8bx^2 - 4x^2$

Exercise 19, page 88:

1. $x + 3 + \left(\frac{6}{x - 1}\right)$
2. $a^2 - 5a + 12 - \left(\frac{22}{a + 2}\right)$
3. $ab + 2b + b^2 + \left(\frac{b^2 + b^3}{a - b}\right)$
4. $x - 1 + \left(\frac{d - 3}{x^3 - 2x^2 + x - 3}\right)$
5. $2x^3 - 3x^2 + 4x - 5$

Exercise 20, page 90:

1. $2ab + 2bc - 2bd$
2. $3a^3b + 3ab^2 + 3abc$
3. $a^2 + 12ab + 36b^2$
4. $a^2 - 6ab + 9b^2$
5. $a^4 + 2a^2b^2 + b^4$
6. $a^2 - 1$
7. $4x^2 - 12bx + 9b^2$
8. $abc(ac^2 + 3c - 2a^2b)$
9. $(2a + 1)^2$
10. $(2a - 2b)^2$

Exercise 21, page 92:

3. $y = 4; x = .5$
4. $y = 2; x = 8$
5. $x = 4; y = -.333$
6. $x = 10; y = 2.618$
7. $x = 2.28; y = -.357$
8. $x = 2.728; y = -1.686$

Exercise 22, page 94:

1. $x = 8.82; y = -5.23$
2. $x = 2.081; y = -.162$
3. $x = 1; y = 0$
4. $x = 7; y = 3.333$
5. $x = .55; y = 1.825$
6. $x = .537; y = .3$
7. $x = 1.515; y = .22$
8. $x = 1.324; y = 1.613$
9. $x = .8; y = 1.252$
10. $x = 5.82; y = -2.63$

Exercise 23, page 95:

1. 5.93 amperes
2. 4.73 amperes
3. 3.73 amperes
4. .6 ampere

Exercise 24, page 97:

1. $E_1 = 8.62$ volts
   $E_2 = 6.38$ volts
   $I_a = 1.44$ amperes
   $I_b = .64$ ampere
   $I_c = .8$ ampere

2. $E_1 = 7.08$ volts
   $E_2 = 2.92$ volts
   $I_a = .708$ ampere
   $I_b = .416$ ampere
   $I_c = .292$ ampere

3. $E_1 = 2\frac{1}{3}$ volts
   $E_3 = 1\frac{2}{3}$ volts
   $R_4 = 3.43$ ohms
   $I_a = .583$ ampere
   $I_b = .1665$ ampere
   $I_c = .4165$ ampere

4. $E_1 = .725$ volt
   $E_2 = 4.275$ volts
   $I_a = .285$ ampere
   $I_b = .1813$ ampere
   $I_c = .1037$ ampere

Exercise 25, page 100:

1. $E_1 = .81$ volt
   $E_2 = 3.81$ volts
   $E_3 = 4.19$ volts
   $I_a = .838$ ampere
   $I_b = .636$ ampere
   $I_c = .202$ ampere

2. $E_1 = 2.12$ volts
   $E_2 = 6.12$ volts
   $E_3 = 3.88$ volts
   $I_a = .265$ ampere away from Junction point O.
   $I_b = .511$ ampere away from Junction point O.
   $I_c = .776$ ampere into Junction point O.

Exercise 26, page 105:

1. *a.* 8.34 ohms
   b. 5 ohms
2. .5 ohm
3. $\frac{5}{6}$ ohm
4. $I_a = 1$ ampere
   $I_b = 0$ ampere
   $I_c = 1$ ampere
5. $I_1 = .1812$ ampere
   $I_2 = .8128$ ampere
   $I_3 = .637$ ampere

Exercise 27, page 108:

1. 4.73 amperes
2. $E_1 = .725$ volt
   $E_2 = 4.275$ volts
   $I_a = .285$ ampere
   $I_b = .1813$ ampere
   $I_c = .1037$ ampere

3.

	I through resistor	*E* across resistor
R_1	2.28 amperes	4.56 volts
R_2	2.28 amperes	13.68 volts
R_3	.68 ampere	2.72 volts
R_4	2.97 amperes	14.85 volts

CHAPTER VIII *Magnetism*

The introduction to a chapter on *magnetism*, with subsequent discussions on *magnetic fields* and *electromagnetic-energy* transfers from point to point through space, needs a word of explanation as did Chap. II on elementary electricity. It is difficult to talk of magnetism without also talking about *electromagnetism* (covered in the next chapter) as the two are closely associated.

Magnetism is the property of particles of certain substances to attract others. Such an attraction is accomplished by means of an associated *magnetic field* surrounding the attracting material.

The existence of magnetic properties in certain materials is well known. At some time, a majority of the students probably have been fascinated by the magnetic properties of a magnet and have observed those properties with keen interest. The ability of a magnet to attract iron or steel objects placed near the magnet is truly phenomenal. It is self-evident that the magnet exerts a force of some nature in the space surrounding it. We say that a magnetic field exists around the magnet, and the force is a *magnetic force*.

The student knows that to name something does not necessarily explain what it is. To say that the magnet attracted certain objects because of its magnetic force does not explain the absolute nature of the force. If this baffles the student, he may find consolation in the fact that it is a problem which baffled the great scientists down through the ages and still stands as a challenge to the minds of men. Perhaps, someday, we may be able to say more than: Magnetic lines of force are magnetic lines of force.

85. Magnetic Field. It is a well-known fact that a current flowing in a conductor produces a magnetic force in the region around the conductor. This force creates a *magnetic field.* It can be said that *an electric current gives rise to a magnetic field.* It was learned from Chap. II that an electric current is a movement of electrons. Thus, it appears that an electron in motion creates a magnetic field in its vicinity. The strength of this field *decreases as the square of the distance* from the moving electron.

As one studies electricity and, one by one, the great truths of nature unfold he cannot help being filled with profound respect and admiration: admiration that, when the whole is known, science obeys well-laid plans. The whole electrical industry is based upon the above principle: An electrical current gives rise to a magnetic field.

86. Magnetic Properties of Materials. There is considerable logic to one contemporary thought on magnetism which considers that the *motion of the electrons* in a material must have some bearing upon the magnetic properties of that material. From this assumption, the motion[1] of the electrons in nonmagnetic materials is such that the magnetic fields associated with them tend to cancel each other. The motion of the electrons in a material having magnetic properties is such that there is not a complete cancellation of the magnetic fields associated with the electrons.

87. Magnetic Materials. Few materials other than iron or iron alloys possess magnetic properties. *Cobalt* and *nickel* show strong magnetic properties but they are much less than those for iron. Materials possessing magnetic properties are called *ferromagnetic.*

Radio-equipment applications of magnetic materials require a wide range of magnetic characteristics. The magnetic-core material of a high-quality audio transformer would be unsatisfactory for use in a large power-line transformer. Likewise, the magnetic-core material used in the power transformer would give poor performance if used in the high-quality audio transformer. Neither of these materials would be suitable for use in the construction of a permanent-magnet loud-speaker. The magnetic material that is actually used in the construction of permanent-magnet loud-speakers is not suitable for use in any type of transformer.

Numerous alloys have been developed to satisfy the various needs of the electrical industry. One of the best known of these is called *Permalloy*. It is composed of approximately 20 per cent iron and

---

[1] Not the motion due to a difference of potential considered in section 85, but the general motion found in any material unless the material is at a temperature of −273° C. At such an extremely cold temperature, all molecular action is purported to stop.

80 per cent nickel. Other materials known by their trade names are *Perminvar*, *Nicoloi*, and *Hipernick*.

88. Theory of Magnetism. A piece of ferromagnetic material placed in a magnetic field will attract other pieces of ferromagnetic material. It is thought that a piece of ferromagnetic material is made up of many very small individual magnets. When a piece of iron is acted upon by a magnetic field, the forces of the many tiny magnets become additive. A soft iron bar maintains its magnetic properties only so long as it is in a magnetic field, and is called a *temporary magnet*. A piece of hard steel still has magnetic properties after it is removed from the magnetizing field and is called a *permanent magnet*.

The haphazard positions of the small magnetized particles that make up a bar of hard steel are represented by sketch *a* of Fig. 67.

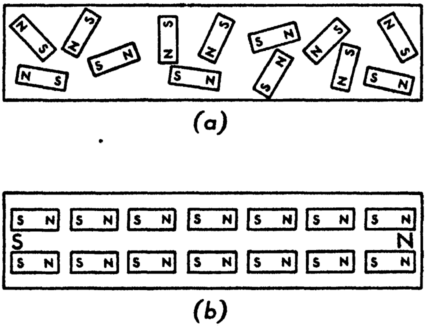


Fig. 67. a. Arrangement of individually magnetized particles of a steel bar which is not magnetized because of the haphazard positions of the particles. *b.* The additive arrangement of the particles after the steel bar has been magnetized.

The positions of these same particles after the bar has been magnetized are shown in *b* of Fig. 67.

89. Magnetic Poles. The attracting force of a magnet is concentrated at the ends of the magnet and these points are called the *poles* of the magnet.

If a bar magnet is suspended so that its ends are free to rotate freely, it will assume a north-south position. The same end will always point toward the *north magnetic pole of the earth*. This pole is the *north-seeking pole* of the magnet and is actually its *south pole*. The other pole of the magnet is its *north pole*. This principle is utilized in the construction of the *magnetic compass* which has been in use as a navigation aid for centuries. Sometimes the north-seeking pole of a compass is erroneously called its north pole.

If *like poles* of two magnets are placed close together, they will *repel* each other. If *unlike poles* are placed close together, they will *attract* each other. Obviously, unmagnetized ferromagnetic material has no polarity and will be attracted by either pole of a magnet.

If a magnet is divided into several pieces, each piece will form a magnet having a north and south pole as shown in Fig. 68.

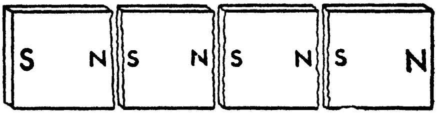


Fig. 68. Showing how a magnet divided into several parts forms several smaller magnets.

90. Magnetic Lines of Force. Several elementary facts about the magnet can be determined by means of relatively simple experiments. For instance, it is easy to determine that the magnetic field is concentrated at the poles and diminishes as a square of the distance from either pole or that the field has *magnetic polarity.*

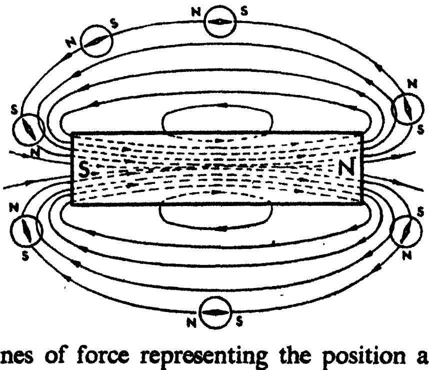


Fig. 69. Showing lines of force representing the position and direction of the magnetic field about a magnet.

If the field about a magnet is explored with a very small compass as in Fig. 69, it will be found that the field also has *direction.* The lines of Fig. 69 were obtained by moving the compass from one pole to the other in such a way that a line through the pointer position for each succeeding position of the compass was continuous as shown.

From an inspection of Fig. 69, it can be seen that, from a practical standpoint, it is most convenient to visualize the magnetic field as consisting of *lines of magnetic force*. These magnetic lines of force are called *Maxwells* and their total number is represented by the Greek letter phi, whose symbol is ϕ. Magnetic lines of force are often referred to as *magnetic flux*.

The magnetic lines of force are continuous from the north pole through the surrounding space to the south pole and through the magnet from the south pole to the north pole.

91. Magnetic Fields of Magnets. If two magnets are placed near each other, their fields will be either aiding or opposing. If a pole of one is placed near the like pole of the other, the two fields oppose each other as illustrated in Fig. 70. The lines of force around the north pole of each magnet are distorted due to the repelling action of the two fields. The position of the lines of force should be compared to the lines of force in Fig. 69. Any object having magnetic properties and placed in a magnetic field will distort the field. If the unlike poles of two magnets are placed near each other, the lines of force from one magnet will extend to the other as shown in Fig. 71. The attraction or repulsion between two magnetic poles is inversely proportional to the square of the distance between the poles.

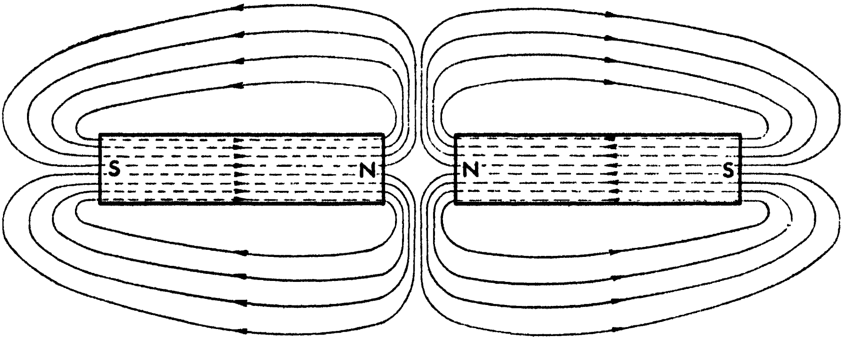


Fig. 70. When like poles of two magnets are placed together, they repel each other, distorting the normal positions of the lines of force from each.

92. Practical Uses of Magnets. The permanent magnet has many applications. Headphones, speakers, voltmeters, ammeters, microphones, motors, and generators are a few of the devices that make use of the permanent magnet.

Devices that contain permanent magnets are rugged enough for normal handling but one should avoid jarring them severely as this will cause the magnets to lose some of their magnetism. For instance, headphones will lose their sensitivity if carelessly handled and allowed to be severely jarred when placed on a table or other hard surface.

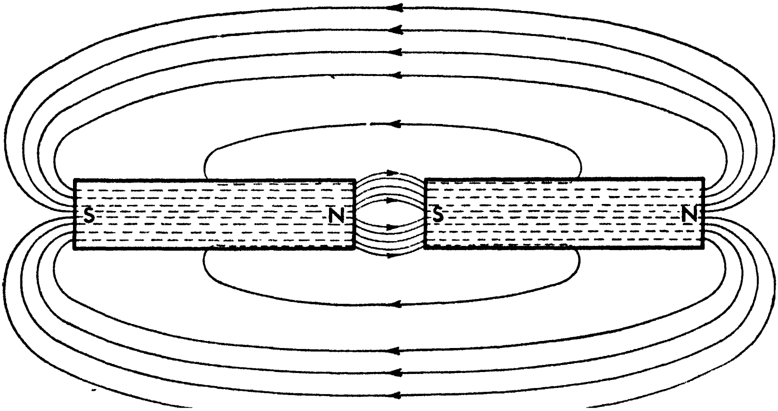


Fig. 71. Like magnetic poles attract and the lines of force extend from the poles of one magnet to those of the other magnet.

Most voltmeters and ammeters and several types of microphones use permanent magnets. Dropping them not only injures the permanent magnets that they contain but will almost certainly damage one or more of the delicate parts.

Extreme heat increases the molecular action of a magnet and will cause it to lose part or all of its magnetism, depending upon the temperature.

93. Logarithms. Logarithm, from the Greek meaning, is: "the number of the ratio." John Napier was credited with the invention of logarithms about the beginning of the seventeenth century.

Logarithms are particularly useful in working with fractional exponents such as $(4.73)^{1.64}$. Such an operation reduces to one simple multiplication when logarithms are used. Logarithms not only can be used for raising to a power or extracting a root but they may also be applied to multiplication and division.

The logarithm of a number is the exponent to which a base must be raised to obtain the number.

(24) If $$a^x = N$$

then

(25) $$x = \log_a N$$

which is read as: x equals the logarithm of N to the base a. Equation 25 holds true only when the base is positive and its value is neither 1 nor 0.

94. Different Bases. From equation 24, a base of 2 with an exponent of 1 gives a value of N equal to 2. Also, $(2)^3 = 8$. As the exponent of 2 is increased in small fractional steps (1.1; 1.2) from 1 to 3, the value of N increases from 2 to 8. The exponent values may be formed into a table with their corresponding values of N. Such a table is called a *table of logarithms* and its range of numbers may be extended to embrace any desired values of N.

Logarithm tables, like units of measure, should be standard if designed for general use. Two systems in general use are the *common system* and the *Napierian* or *natural system.* The common system uses the base 10 and the Napierian system uses the base $e = 2.71828\ldots$.

It is logical that common logarithms should be in more general use since 10 is also the base of our number systems. Equation 25 may be written:

(26) $$x = \log_{10} N$$

where the base is 10. In writing common logarithms of numbers, the base is usually omitted. For example, $\log_{10} 25$ is written log 25. Therefore, when any base other than 10 is used, it must be indicated.

95. Logarithm Tables. The student should note that the value of the logarithm increases from 0 to 1 as the value of N increases from 1 to 10 as shown in Table II. Table II is very brief as it is evident that the value of N may be increased by tenths (6.1; 6.2; 6.3) or even by hundredths (6.10; 6.11; 6.12).

Table V on page 273 is a more complete set of logarithms.

96. Characteristic and Mantissa. The logarithm of a number is divided into two parts, the characteristic and the mantissa. The *whole number part* of a logarithm is called the *characteristic* and the *decimal fraction part* is called the *mantissa.*

TABLE II

$10^{0} = 1$ or $\log 1 = 0.0000$
$10^{.3010} = 2$ “ $\log 2 = 0.3010$
$10^{.4771} = 3$ “ $\log 3 = 0.4771$
$10^{.6021} = 4$ “ $\log 4 = 0.6021$
$10^{.6990} = 5$ “ $\log 5 = 0.6990$
$10^{.7782} = 6$ “ $\log 6 = 0.7782$
$10^{.8451} = 7$ “ $\log 7 = 0.8451$
$10^{.9031} = 8$ “ $\log 8 = 0.9031$
$10^{.9542} = 9$ “ $\log 9 = 0.9542$
$10^{1} = 10$ “ $\log 10 = 1.0000$
$10^{2} = 100$ “ $\log 100 = 2.0000$
$10^{3} = 1000$ “ $\log 1000 = 3.0000$
$10^{3.0414} = 1100$ “ $\log 1100 = 3.0414$

From Table II:

$$\log 1100 = 3.0414$$

3 is the characteristic, and .0414 is the mantissa.

The *numerical value* of the characteristic of a number equal to or greater than one is positive and *one less* than the total number of digits in the *whole-number* portion of the number.

From Table II:

The characteristic of log 1 is 0
The characteristic of log 10 is 1
The characteristic of log 100 is 2
The characteristic of log 1000 is 3

The mantissa in each of the above examples is 0 since 10, 100, and 1,000 are even multiples of 10 and 1 is an even submultiple of 10.

TABLE III

$10^{0} = 1$ or $\log 1 = 0.0000$
$10^{-.0458} = .9$ “ $\log .9 = -1 + .9542$ written as $\log .9 = \bar{1}.9542$
$10^{-.2218} = .6$ “ $\log .6 = -1 + .7782$ written as $\log .6 = \bar{1}.7782$
$10^{-.6990} = .2$ “ $\log .2 = -1 + .3010$ written as $\log .2 = \bar{1}.3010$
$10^{-1} = .1$ “ $\log .1 = -1$ written as $\log .1 = \bar{1}$
$10^{-2} = .01$ “ $\log .01 = -2$ written as $\log .01 = \bar{2}$
$10^{-3} = .001$ “ $\log .001 = -3$ written as $\log .001 = \bar{3}$
$10^{-4} = .0001$ “ $\log .0001 = -4$ written as $\log .0001 = \bar{4}$

The minus sign is written *above* a negative characteristic of a logarithm to indicate that *only the characteristic and not the mantissa is negative* as shown in Table III. It must be kept in mind that

$$\log .2 = \bar{1}.3010 \text{ means } \log .2 = -1 + .3010$$

or
$$.2 = 10^{-1+.3010} = 10^{-.6990} = 10^{-1} \times 10^{.3010} = 10^{-1} \times 2$$
$$= \frac{10^{.3010}}{10} = \frac{1}{10^{1} \times 10^{-.3010}} = \frac{1}{10^{.6990}}$$

When the value of N is less than 1, the characteristic of its logarithm is *minus* and its numerical value is one more than the number of zeros immediately to the right of the decimal point. A logarithm with a negative characteristic may be written in another form by adding $+10$ to the minus characteristic and indicating the subtraction of -10 to the left of the mantissa. For example, $\log .046 = \bar{2}.66276$ may be written $\log .046 = 8.66276 - 10$.

97. To Find the Logarithm of a Given Number. Table V on page 273 can be used. Only the mantissas are given in Table V (except condensed portion for numbers 1–99 on the first page) as the characteristics may be determined from the rules given in section 96. The first column headed by N gives the first three digits of the number while the numbers *heading* the next ten (0 through 9) columns represent the fourth digit of the number. The additional columns, headed "Prop. Pts.", are used to determine the fifth digit of the number.

The numbers in the first ten columns immediately to the right of the N column represent the mantissa of the corresponding numbers, while the numbers in the last section of columns represent proportionate parts of the mantissa used for *interpolation*. This is known as a five-place table since it gives the correct values of the mantissa or number to five decimal places.

Examples:

1. Find the log of 132.

Solution:

The characteristic is 2 since 132 has three whole numbers and the numerical value of the characteristic is one less than the number of whole numbers in the number.

The mantissa is found in the 0 column (since 132 has no fourth significant figure), in line with 132 in the N column, and is .12057.

Thus the whole logarithm is 2.12057.

2. Find the logarithm of 6236.

Solution:

The characteristic is 3 and the mantissa is found, from Table V, in line with 623 of column *N* and in the 6 column next adjacent. The mantissa is .79491 and the whole logarithm is 3.79491.

3. Find the logarithm of 51026.

Solution:

The characteristic is 4. The mantissa of 51020 is found in Table V in the 2 column and in line with 510 in the *N* column and is .70774. The mantissa of 51030 is found in the 3 column and also in line with 510 in the *N* column and is .70783. Obviously, the mantissa we seek lies between .70774 and .70783 since our number, 51026, lies between 51020 and 51030. We must use the Proportional Parts to interpolate for the mantissa of 51026.

$$\begin{aligned} \log 51030 &= 4.70783 \\ \log 51020 &= \underline{4.70774} \\ \text{tabular difference} &= .00009 \end{aligned}$$

Hence, a change of 10 in the number causes a change of .00009 in the mantissa. This is called the *tabular difference.* The right-hand column (Prop. Pts.) of the table shows columns of figures for tabular differences of 7, 8, and 9 (shown as whole numbers with decimal point and zeros dropped). Under a tabular difference of 9, we find a proportional part of 5.4, corresponding to 6 which is the difference of 51026 − 51020.

$$\begin{aligned} \log 51020 &= 4.70774 \\ \text{Prop. Pt.} &= \underline{.000054} \\ \log 51026 &= 4.707794 \end{aligned}$$

Exercise 28:

Find the logarithms of the following:

1. 11
2. 15
3. 65
4. 88
5. 115
6. 108
7. 14.2
8. 19.6
9. 259
10. 8
11. 4
12. .6
13. .36
14. .126
15. .023
16. .096
17. 794,000
18. .0028
19. 1,000,000
20. .00001
21. 6,458
22. 8,462
23. .004763
24. .000006241
25. 8.564
26. 714.62
27. 164.73
28. 8,475.6
29. 31,476.4
30. 185,685

98. Antilogarithm. If a logarithm is given, Table V, page 273, may be used to determine the corresponding number called *antilogarithm* and written *antilog*. The antilog of a number may also be written $\log^{-1}$. Thus N = antilog 2.61745 may be written $N = \log^{-1} 2.61745$, meaning "N is a number whose log is 2.61745." For example, it is desired to find the antilog of 3.46613. The mantissa is .46613; from Table V this set of values is found in the line opposite to 292 (which is in the N column) and in the 5 column. The digits of the number are 2-9-2-5. The characteristic of the logarithm is 3; therefore, the number has 4 significant figures and the number is 2925.

Exercise 29:

Find the antilogarithm of the following logarithms:

1. 1.47712
2. 3.08991
3. 2.88024
4. $\bar{2}.93258$
5. 6.92947 − 10
6. 3.74044 − 10
7. 2.39400
8. 1.3165
9. $\bar{2}.66889$
10. $\bar{6}.00432$
11. 3.08429 − 10
12. 8.13258 − 10
13. 3.61745
14. 1.11721
15. $\bar{3}.47124$

99. Multiplication by Logarithms.

$$\log_a MN = \log_a M + \log_a N \tag{27}$$

Example:

Find the product of 628 × 49.

Solution:

$$\log (628 \times 49) = \log 628 + \log 49$$

From Table V:

$$\begin{aligned} \log 628 &= 2.79796 \\ \log 49 &= \underline{1.69020} \\ \text{adding logs} &= 4.48816 \end{aligned}$$

The product of 628 × 49 is a number N whose logarithm is 4.48816.

$$\begin{aligned} \log 30{,}780 &= 4.48827 \\ \log 30{,}770 &= \underline{4.48813} \\ \text{tabular difference} &= 14 \end{aligned}$$

$$\begin{aligned} \log N &= 4.48816 \\ \log 30{,}770 &= \underline{4.48813} \\ \text{Proportional part of tabular difference} &= 3 \end{aligned}$$

From the right-hand column of proportional parts in the tabular difference column 14, a proportional part of 3 (2.8 is the closest approach to 3) gives a corresponding value of 2 as the fifth significant figure of N. Therefore, our required number whose log is 4.48816 is 30,772.

Exercise 30:

Compute by logarithms:

1. 3×6
2. 8×12
3. $3 \times .06$
4. 2.2×3.6
5. 614×1.2
6. 127×389
7. $.0041 \times 721$
8. $.00012 \times .12$
9. 784×3941
10. $3 \times 16 \times 14$

100. Division by Logarithms.

$$\log_a \frac{M}{N} = \log_a M - \log_a N \tag{28}$$

Example:

Use logarithms to find the value of $\frac{126}{275}$

Solution:

Let $\frac{126}{275}$ = Number desired

$$\begin{aligned} \log 126 &= 2.10037 \textit{ or } && 12.10037 - 10 \\ \log 275 &= && \underline{2.43933} \\ \text{subtracting logs} & && 9.66104 - 10 \textit{ or} \\ & && \bar{1}.66104 \end{aligned}$$

Number desired = $\log^{-1}$ $\bar{1}.66104$

$$\begin{aligned} \log .4582 &= \bar{1}.66106 \\ \log .4581 &= \underline{\bar{1}.66096} \\ \text{tabular difference} &= \phantom{\bar{1}.660}10 \end{aligned}$$

$$\begin{aligned} \log \text{Number desired} &= \bar{1}.66104 \\ \log .4581 &= \underline{\bar{1}.66096} \\ \text{Proportional part of tabular difference} &= \phantom{\bar{1}.6610}8 \end{aligned}$$

The fifth significant figure of the number desired is obtained as shown in the example of section 99 and is 8. Therefore, the number desired is .45818.

101. Cologarithms. The logarithm of the reciprocal of a number is called the *cologarithm* or *colog* of the number.

$$\log_a \frac{1}{N} = \log_a 1 - \log_a N = -\log_a N = \operatorname{colog}_a N \tag{29}$$

102. Logarithm of Powers and Roots. Equations 30 and 31 may be used to perform the operations indicated in the absence of a suitable slide rule.

$$\log_a N^x = x \log_a N \tag{30}$$

$$\log_a \sqrt[x]{N} = \frac{1}{x} \log_a N \tag{31}$$

where N is a real positive root.

Examples:

1. Find the value of $(6.28)^{1.41}$ using logarithms.

Solution:

$\log (6.28)^{1.41} = 1.41 \log 6.28 = 1.41 \times .79796 = 1.12512$ which is the logarithm of a number equal to $(6.28)^{1.41}$. From Table V, the number is found to be 13.339.

2. Find the value of $\sqrt{100}$ by using logarithms.

Solution

$$\log \sqrt{100} = \frac{1}{2} \log 100 = \frac{2}{2} = 1$$

$$\operatorname{antilog} 1 = 10$$

Exercise 31:

Find the values of the following by the use of equations (30) and (31)

1. $(16)^2$
2. $(716)^3$
3. $(4)^{1.6}$
4. $(707)^{1.6}$
5. $(10{,}000)^{1.6}$
6. $(100)^{2.5}$
7. $(10)^{3.47}$
8. $(1000)^{1.293}$
9. $\sqrt{10000}$
10. $\sqrt{625}$
11. $\sqrt[3]{8}$
12. $\sqrt[1.61]{4}$

103. Change of Logarithm Base. It was mentioned in section 94 that there are two principal systems of logarithms in use: the common system used in this text and the Napierian or natural system.

The Napierian system of logarithms is used in higher mathematics. The student need not be concerned with it at this point. When he finishes this text and takes up the succeeding and more advanced text that follows, he will need to concern himself more and more with the Napierian system of logarithms.

One may change from one system to the other by means of equations 32 and 33. Equation 32 is used to change the logarithm of a number with base 10 to a logarithm of the same number to base *e*. Equation 33 is used to change the logarithm of a number with base *e* to a logarithm of the same number to base 10.

$$\log_{10} N = .4343 \log_e N \tag{32}$$

$$\log_e N = 2.3026 \log_{10} N \tag{33}$$

REVIEW QUESTIONS

1. What is a magnetic field?

2. Discuss the magnetic properties of materials.

3. Materials possessing magnetic properties are usually found in what general class or group?

4. Explain the theory of magnetism.

5. What are magnetic poles?

6. What is the polarity of the north magnetic pole of the earth? What is the polarity of the north-seeking pole of a magnetic compass? Why?

7. Discuss magnetic lines of force.

8. What practical proof do we have to support our consideration of the existence of magnetic lines of force?

9. Do magnetic lines of force have direction? If so, what is their direction through: (a) A magnet; (b) The space surrounding the magnet?

10. What are logarithms?

11. Explain the use and meaning of the following logarithm terms: (a) base; (b) characteristic; and (c) mantissa.

12. Give several advantages for having as few standard logarithm bases as possible.

13. What are the purpose and advantages of logarithm tables?

14. What is meant by a "five place logarithm table"?

15. What is an antilogarithm?

ANSWERS

Exercise 28, page 120:

1. 1.04139
2. 1.17609
3. 1.81291
4. 1.94448
5. 2.06070
6. 2.03342
7. 1.15229
8. 1.29226
9. 2.41330
10. 0.90309
11. 0.60206
12. $\bar{1}.77815$
13. $\bar{1}.55630$
14. $\bar{1}.10037$
15. $\bar{2}.36173$
16. $\bar{2}.98227$
17. 5.89982
18. $\bar{3}.44716$
19. 6.00000
20. $\bar{5}.00000$
21. 3.81010
22. 3.92747
23. $\bar{3}.67788$
24. $\bar{6}.79525$
25. 0.93268
26. 2.85407
27. 2.21677
28. 3.92817
29. 4.49798
30. 5.26878

Exercise 29, page 121:

1. 30
2. 1,230
3. 759
4. .08562
5. .0008501
6. .0000005501
7. 247.74
8. 20.725
9. .046654
10. .00000101
11. .00000012142
12. .01357
13. 4,144.3
14. 13.098
15. .0029597

Exercise 30, page 122:

1. 18
2. 96
3. .18
4. 7.92
5. 736.8
6. 49,403
7. 2.9561
8. .0000144
9. 3,089,744
10. 672

Exercise 31, page 123:

1. 256
2. 367,060,000
3. 9.1895
4. 36,230
5. 2,511,900
6. 100,000
7. 2,951.2
8. 7,568.3
9. 100
10. 25
11. 2
12. 2.361

CHAPTER IX *Electromagnetism*

Electromagnetism is the production of magnetism or a magnetic field by means of a current flowing in a conductor. The production of a magnetic field by means of a current flowing in a conductor was discussed in section 85 of Chap. VIII on magnetism. This chapter gives further consideration to the magnetic effects due to the flow of currents in conductors.

104. Magnetic Field Around a Conductor. It was stated in section 85 that *an electric current gives rise to a magnetic field.* A current flowing in any conductor produces magnetic lines of force around the conductor and at *right angles* to it as indicated by the lines encircling the conductor of Fig. 72.

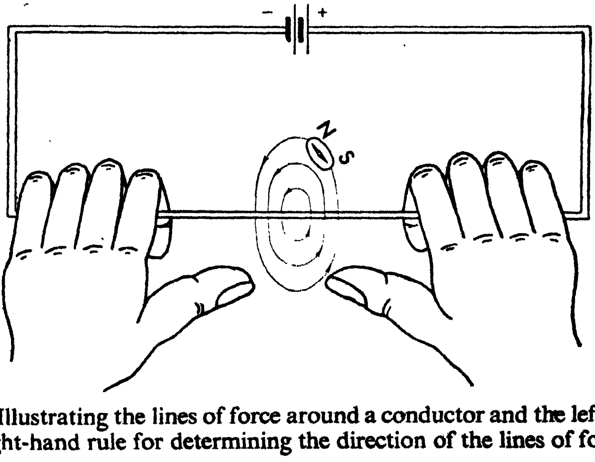


Fig. 72. Illustrating the lines of force around a conductor and the left-hand rule and the right-hand rule for determining the direction of the lines of force around a conductor.

Three questions immediately arise: (1) In which direction do the lines of force extend around a conductor? (2) What determines the direction of the lines of force? (3) Is there any simple rule to predetermine the direction of the lines of force?

The needle of a very small magnetic compass placed near the conductor of Fig. 72 assumes the position shown. The lines must extend around the conductor in a counterclockwise direction as the lines of force through a magnet extend from the south to the north pole.

Obviously, the direction of the lines of force must be determined by the direction of the flow of current through the conductor. It should also be self-evident that *reversing the connections to the battery would reverse the direction of current flow and the direction of the lines of force.* Such a reversal of battery connections would cause the lines of force to extend around the conductor in a clockwise direction. Two principal rules have been developed to determine the direction of the magnetic lines of force from the direction of current flow. These rules are given in section 106.

105. Direction of Current Flow. It will be noted that no mention of the actual direction of current flow has been made in regard to the circuit of Fig. 72. The direction of the lines of force was established by means of the small magnetic compass.

Early experimenters assumed that the current flowed out of the positive terminal of a battery into a conductor, through the conductor, and back into the battery through the negative terminal. This was indeed a most unfortunate assumption because it is now established that the opposite is true. The current actually *flows from the negative terminal* into the conductor and back to the positive terminal of the battery.

Teachers and authors have been reluctant to abandon the idea of current flowing from positive to negative for fear of the confusion that it might cause. Recent publications indicate a trend toward changing over to the correct direction of current flow.

Regardless of which direction we *say* the current is flowing, the lines of force in Fig. 72 are in the direction as shown.

106. Left-hand Rule. The *left-hand rule* may be used to determine the direction of the magnetic lines of force from the correct direction of current flow (from negative to positive). If the conductor of Fig. 72 is grasped with the left hand with the thumb pointing in the direction of current flow, the ends of the fingers encircling the wire will point in the direction of the lines of force encircling the conductor.

The right-hand rule was formerly devised to show the direction of lines of force when considering the assumed direction of current flow from positive to negative. In this instance, the wire is grasped with the right hand as shown in Fig. 72.

Both rules are illustrated in Fig. 72. It will be noted that the direction of the lines of force is the same, regardless of which rule is used.

107. Electromagnetism. The existence of a magnetic field about a conductor (section 104), due to current flowing in the conductor, has been the basis of many radio developments.

The creation of a magnetic field due to current flowing in a conductor is called *electromagnetism.*

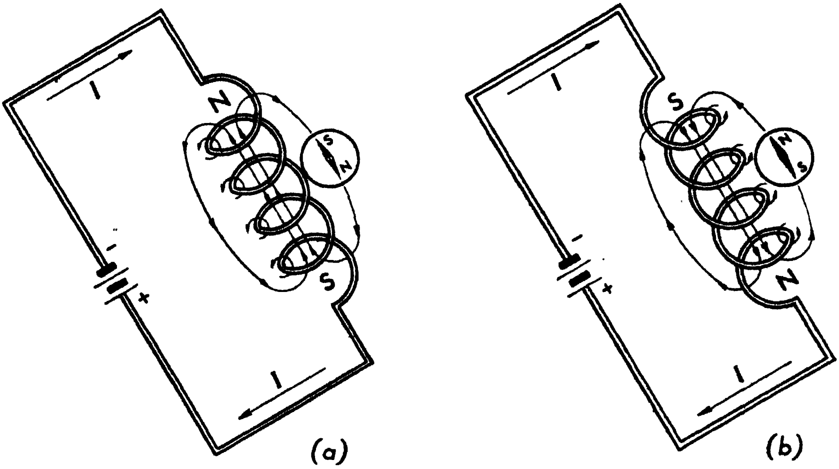


Fig. 73. a. Illustrating polarity of the magnetic field about a coil carrying current. *b.* Showing how the polarity and direction of lines of force of *a* may be reversed by rewinding the coil in the opposite direction (counter-clockwise).

A stronger magnetic field may be obtained if a conductor is formed into a coil as shown in Fig. 73. Each turn of wire around the coil has a magnetic field about it which encircles the turns of wire adjacent to it. The direction of the lines of force around each conductor can be checked by the left-hand rule to show that the lines of force around all turns are in the same direction.

The same rule may be used to trace the magnetic lines of force all of the way around each conductor, showing how a cancellation of lines exists between adjacent turns. The lines of force about two adjacent turns are in opposite directions at the point where the turns touch or are nearest each other. This causes most of the lines of force about the individual turns to combine and flow through the center of the coil and around the outside of the coil as shown by lines of force in Fig. 73.

The lines of force about a coil are distributed in a manner similar to the lines of force about a magnet. The ends of a coil, which has current flowing through it, have *magnetic polarity*. One end is a north pole and the other end is a south pole. The polarity of a coil is determined by:

1. *Direction of current through the coil.*
2. *Direction in which turns of coil are wound.*

The polarity of a coil may be *reversed* by reversing the connections to the supply voltage or by rewinding the turns of the coil in the opposite direction. The coil of Fig. 73 *a* is wound in a *clockwise* direction *from the top* of the coil *down* to the bottom of the coil. Figure 73 *b* shows how the polarity of *a* may be reversed by rewinding the same coil in a *counterclockwise* direction *from the top down.*

Small bars of magnetic material (not magnetized), placed in the positions shown in Fig. 73, would be magnetized with polarities as indicated. A bar of the same material placed inside the coil would assume the polarity of the coil.

Magnetic material with good magnetic properties, placed inside a coil, is called a *core* and greatly increases the attracting or repelling force at the ends of the coil.

108. Permeability. It is seen from the above section that the magnetic-conducting properties of a high-quality magnetic material are quite high as compared to that of air. The magnetic-conducting properties of materials vary over wide limits exactly like the electric current-conducting properties of materials vary over wide limits. In fact, the magnetic-conducting properties of magnetic materials range from a few thousand to nearly 100,000 times that of air. This may seem high but it is rather low when we compare it to the ratio of electric current-conductance in a good conductor and a poor conductor (insulator).

The ratio of the magnetic-conducting ability of a material as compared to air is called the *permeability* of the material. The symbol for permeability is the Greek letter mu (μ). The permeability of air is considered as *one*, which makes it convenient in comparing the various magnetic materials to each other.

Materials that are poor conductors of an electric current are called *insulators* because the current flow is so minutely low as to be neligible.

There are no "magnetic insulators" because air or even vacant space (vacuum) conducts magnetic lines of force quite readily. Magnetic lines of force may be *directed away* from an area by a highly magnetic-conducting material such as a heavy iron case around an audio transformer to minimize unwanted magnetic coupling from power transformers or other magnetic devices.

Often the most satisfactory means of eliminating or minimizing unwanted magnetic coupling between circuit elements is to arrange the physical layout of the equipment such that the offending items are as far apart as possible.[1]

The well-known statement that "in most instances, the most economical magnetic shielding is *distance*," is well worth remembering.

109. Headphones. Headphones are normally designed for a very low current and consist essentially of two coils, a U-shaped permanent magnet and a diaphragm. The two coils are wound with several thousand turns of very fine insulated wire about the size of a human hair. The coils are placed on the legs of the permanent magnet. The diaphragm is made of a material with magnetic properties and is placed over the free ends of the U-magnet with a slight clearance. A single headphone unit showing its internal construction is shown in Fig. 74.

The permanent magnet exerts a pull on the diaphragm at all times in order to prevent distortion and maintain a higher sensitivity.

The portion of the received signal which is an audio-frequency *alternating current*[2] alternately cancels out part of the magnetic field on one half of the cycle and increases the field on the next half of the cycle. This causes the pull on the diaphragm to vary or vibrate in exact accordance with the audio-frequency signal, creating an audible signal in the surrounding air which can be heard by the ear.

The alternate strengthening and weakening of the field of the permanent magnet by the signal passing through the coils is accomplished in accordance with the principles introduced in sections 104 and 107. The signal current flowing through the coils of the headphones tends to produce a magnetic field which alternately adds to or cancels a portion of the magnetic field of the permanent magnet.

---

[1] See sec. 118 for further discussion of magnetic shielding.

[2] Covered in next chapter on Alternating Current.

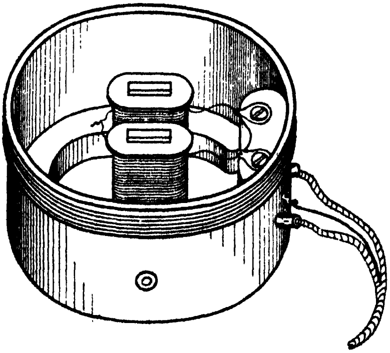


Fig. 74. View of single headphone showing its internal construction. (Cap and diaphragm removed.)

110. Speakers. Many of the earlier types of speakers closely paralleled the headphone unit in design and construction. The audio sound volume from such units was relatively low and unsatisfactory for most uses.

The dynamic type of speaker has long since replaced practically all other types of speakers.

Like the headphone, the dynamic speaker makes use of a magnet. Some employ a permanent magnet and others have a *field coil* to magnetize a soft iron core that maintains its magnetism only so long as direct current is flowing through the field coil.

Figure 75 shows a cut-away section of a typical speaker. A speaker consists primarily of a magnet and a coil attached to a cone. The cone is made in the shape of a funnel with a very light cylindrical coil form attached at the rear small end. A very few turns of wire, wound on the coil form, constitute the *voice coil* of the speaker. One pole of the magnet has a cylindrical opening large enough to accommodate the other pole, which is in the form of a cylinder. Enough clearance is allowed between the two poles to allow the voice coil to be suspended in the space in such a way that it is free to vibrate along with the cone without dragging against the surface of either pole of the magnet.

An alternating-current signal passing through the voice coil induces lines of force around the coil which react with the lines of force between the two poles of the magnet, causing the coil and cone to vibrate in accordance with the signal.

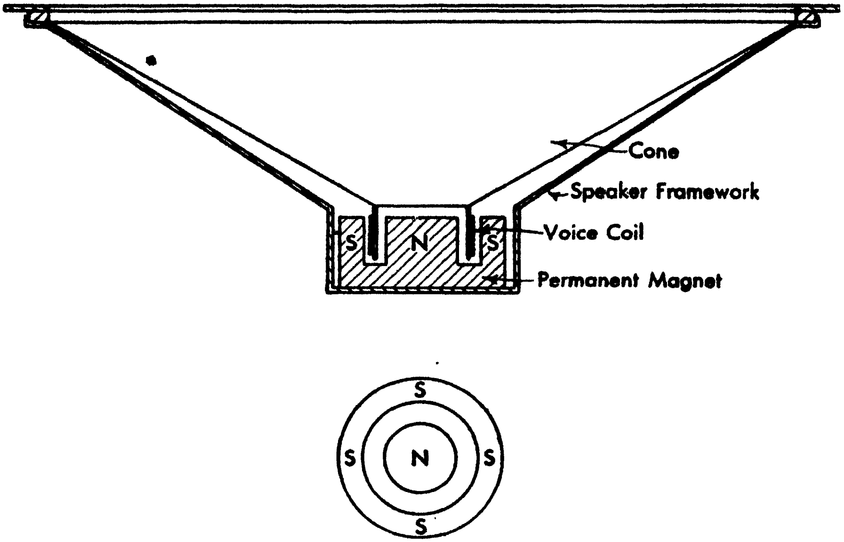


Fig. 75. Cut-away section of a permanent magnet dynamic speaker.

111. Relays. The relay is a most important application of electromagnetism. The principle of operation of a relay is simple. The principal parts of a relay consist of:

1. A coil wound around a ferromagnetic core
2. An armature
3. An armature spring
4. Contact points

All these parts are shown in Fig. 76.

The armature is held away from the core when no current is flowing through the relay coil. The armature is attracted to the core when the switch is closed. This moves contact arm no. 2 away from contact arm no. 1, which is stationary. Such a relay may be used to open or close any electrical circuit if the two contact arms are connected in series with the circuit.

Relays have many applications. They are used extensively in transmitters for circuit-switching operations. A broadcast-station studio system uses numerous relays to control studio microphones, signal lights, speakers, and other circuits.

Some relays require only a very small amount of current to operate them, and dry cells will furnish an adequate source of energy for their operation, while other relays require considerable current, making dry cells an unsatisfactory source of power. Relays are also designed and constructed such that they will operate from an alternating-current source of power.

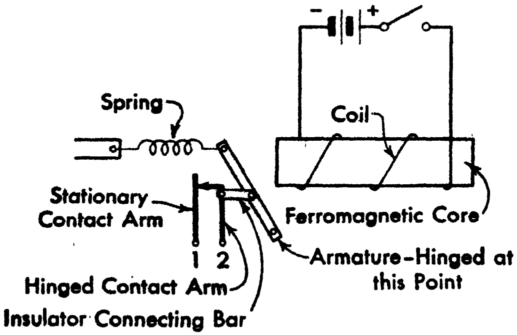


Fig. 76. Schematic of a simple relay.

Several commercial types of relays are shown in Fig. 77.

Relays are commercially available with many contact sequences and combinations. Some contact points may "break" while others may "make" when the relay coil is energized.

112. Magnetomotive Force. It has been stated than an electric current flowing in a conductor produces a magnetic force in the area around the conductor. This force is called *magnetomotive force* and is analogous to the electromotive force. The electromotive force of an *electrical circuit* is measured in *volts* and is considered as an electrical pressure that forces the current to flow around the circuit. The magnetomotive force of a *magnetic circuit* is measured in *gilberts* and is considered as the magnetic pressure that forces the magnetic lines of force (ϕ) around the magnetic circuit.

The magnetomotive force is directly proportional to the current flowing through the conductor. If a conductor is formed into a coil as in Fig. 73 or Fig. 76, the magnetomotive force is also directly pro-

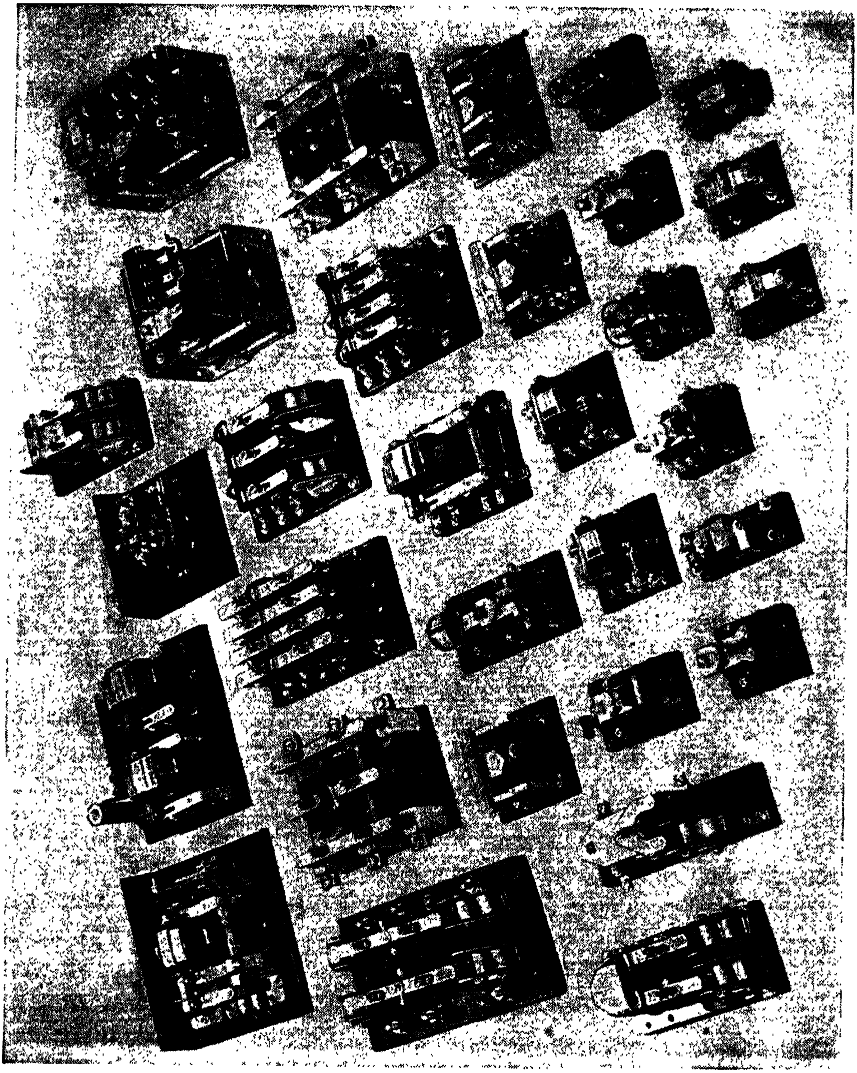


Fig. 77. Several types of commercial relays. (Courtesy, Ward Leonard Company.)

portional to the number of turns of the coil. Therefore, it may be said that:

$$F = .4\pi IN \tag{34}$$

where

F = magnetomotive force in gilberts
I = current in amperes
N = number of turns

The portion IN of equation 34 is called *ampere turns*.

Suppose that the relay coil of Fig. 76 has 300 turns and the current through the coil is 200 milliamperes. The ampere turns are $.2 \times 300 = 60$ and the magnetomotive force, $F = .4\pi\ .2 \times 300 = 75.42$ gilberts.

Exercise 32:

1. The field coil of a dynamic speaker has 5,000 turns. The current through the coil is 68 milliamperes. Determine ampere turns and magnetomotive force.
2. A current of .26 ampere flows in a coil of 2,800 turns. How much must the current be increased in order for the magnetomotive force to be three times as much as before if the coil has become damaged and 640 outer turns are removed?
3. The design of a certain coil requires 575 gilberts. How many ampere turns are required when a current of 120 milliamperes flows through the coil?

113. Magnetizing Force. The magnetomotive force F of Fig. 78 as expressed in equation 34 is the force necessary to force the total number of lines of force ϕ (maxwells) around the entire distance of the magnetic path l.

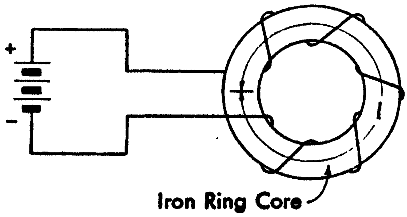


Fig. 78. **A coil uniformly wound on an iron core ring of uniform circular cross-sectional area and uniform magnetic properties.**

It is not only necessary to know the total amount of force acting around the entire path of the magnetic circuit but is of more importance to know what the force is *per unit length of the magnetic path.* This gives the *magnetizing force* which is represented by the letter H and is expressed in *oersteds* of magnetizing force.

The magnetizing force H is obtained by simply dividing F by l.

$$H = \frac{.4\pi IN}{l} \tag{35}$$

where l is length of the magnetic path in centimeters (2.54 centimeters = 1 inch).

Exercise 33:

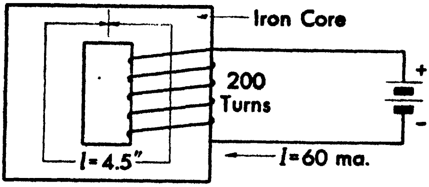


Fig. 79

1. Determine the magnetizing force of the magnetic circuit of Fig. 79.

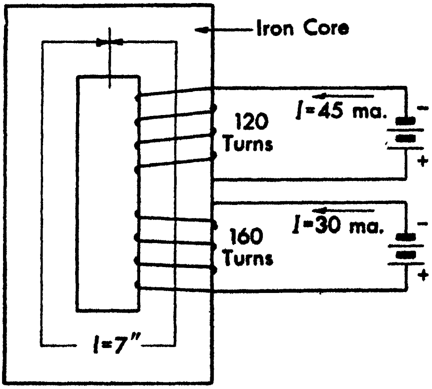


Fig. 80

2. Determine the magnetizing force of the magnetic circuit of Fig. 80.
3. What current in a coil of 485 turns will be required to provide a magnetizing force of 15 oersteds if the length of the magnetic path is 1 foot?

114. Reluctance. In the study of the flow of an electric current, it was learned that any material offers some opposition, called *resistance*, to a flow of electric current. Similarly, there is an opposition to the magnetic lines of force in a magnetic circuit. The opposition that a magnetic circuit offers to the magnetic lines of force is called *reluctance*[1] and designated by the script letter $\mathcal{R}$.

In determining the resistance of an electrical circuit, it was shown that the resistivity of various materials is not necessarily the same. The resistivity of materials varies over extremely wide limits. The permeability of a material in a magnetic circuit may be compared to the reciprocal of the resistivity of a material in an electrical circuit except that the permeability of materials does not vary with the materials over as great a range. In fact, the range of permeability is relatively small as compared to that of resistivity. Attention is called to the fact that magnetic permeability is the magnetic-conducting ability of a material while the electrical-conducting ability (conductance) of an electrical circuit is the *reciprocal of resistance.*

$$\mathcal{R} = \frac{l}{\mu A} \tag{36}$$

where

$\mathcal{R}$ = reluctance
l = length of magnetic path in centimeters
A = cross-sectional area in square centimeters
μ = permeability

It is self-evident from equation 36 that the *reluctance* of a material *varies directly as the length* and *inversely as the cross-sectional area and permeability* of the material. A small air gap in an otherwise continuous magnetic path through iron may offer a very high reluctance to the magnetic lines of force as compared to the reluctance offered by the iron even though the length of the magnetic path through the iron may be many times the length of the air gap. This is due to the fact that the permeability of air as stated in section 108 is one, while the permeability of any ferromagnetic material is usually several thousand times that of air.

---

[1] The unit of electrical resistance is the ohm, but there is no name for the unit of magnetic reluctance. The name oersted was formerly used but this name, by recent international agreement, is now used to name the unit of magnetizing force.

The total reluctance of a series magnetic circuit is equal to the *sum* of the reluctances of the various substances through which the magnetic lines pass.

Reluctances acting in series or parallel are treated in the same way that series and parallel resistors are treated. For a series magnetic circuit, the total reluctance is:

(37) $\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2$ and the total

reluctance for a parallel circuit is determined by the following expression:

(38) $$\frac{1}{\mathcal{R}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \frac{1}{\mathcal{R}_3} + \cdots$$

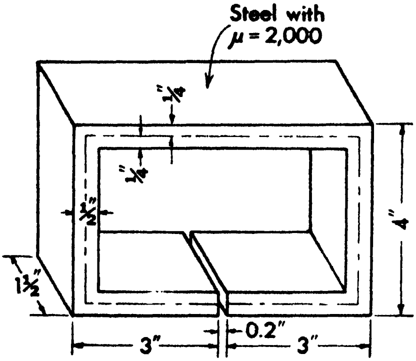


Fig. 81

In Fig. 81, the total reluctance is equal to the reluctance of the iron plus the reluctance of the air gap. The length of a magnetic path as shown in Fig. 81 is determined by considering the path to be halfway between the outside surface and inside surface as shown by the dotted line of the figure. The length of the portion of the path through the steel is:

$l_{\text{steel}} = 2(3 - \frac{1}{4}) + 2(4 - \frac{1}{4} - \frac{1}{4}) + (6.2 - \frac{1}{4} - \frac{1}{4})$

$= 5.5 + 7 + 5.7 = 18.2$ inches, or $18.2 \times 2.54 = 46.25$ centimeters

cross-sectional area $= (2.54 \times .5)(2.54 \times 1.5)$

$= 4.84$ square centimeters

$l_{\text{air}} = .2 \times 2.54 = .508$ centimeter

$\mathcal{R} = \frac{l}{\mu A} = \frac{46.25}{2{,}000 \times 4.84} = .004775$ reluctance of steel portion of magnetic circuit.

$\mathcal{R} = \frac{l}{\mu A} = \frac{.508}{1 \times 4.84} = .105$ reluctance of air-gap portion of magnetic circuit.

$.004775 + .105 = .109775$ total reluctance of the magnetic circuit.

The path of the flux through the steel is approximately ninety-one times as long as through air but the reluctance of the iron path is less than 5 per cent of the reluctance of the air path.

Exercise 34:

1. Determine the total reluctance of the magnetic path of Fig. 81 if the thickness of the core is increased from ½ inch to 1 inch and other dimensions remain as shown.

2. Determine the total reluctance of the magnetic path of Fig. 81 if the air gap is increased to ½ inch by cutting the required amount of iron off the core at the right-hand side of the air gap.

3. Determine the total reluctance of the magnetic path of Fig. 81 if an additional air gap is provided by sawing out a section ½-inch long immediately above the present air gap.

115. Ohm's Law for Magnetic Circuits. Ohm's law for the electric circuit states that the *current* flowing in the circuit is equal to the *electromotive force* (voltage) divided by the *resistance.* Similarly, for a magnetic circuit, the *lines of force* (ϕ in maxwells) are equal to the *magnetomotive force* (F in gilberts) divided by the reluctance ($\mathcal{R}$).

$$\phi = \frac{F}{\mathcal{R}} \tag{39}$$

which may also be expressed in the following forms:

$$\mathcal{R} = \frac{F}{\phi} \tag{40}$$

and

$$F = \phi\mathcal{R} \tag{41}$$

Assume that the coil of Fig. 82 has 6,000 turns, $l_1 = 6$ inches, $l_2 = \frac{1}{2}$ inch, cross-sectional core area = 2 square inches, and current

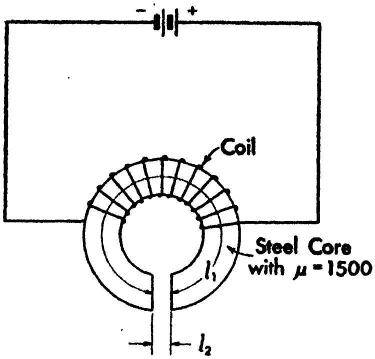


Fig. 82

is 300 milliamperes. The total lines of force may be determined from equation 39.

$$F = .4\pi IN = .4\pi .3 \times 6{,}000 = 2{,}262 \text{ gilberts}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu A} = \frac{6 \times 2.54}{1{,}500 \times 2(2.54)^2} = .000787 \text{ reluctance of steel path}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu A} = \frac{.5 \times 2.54}{2(2.54)^2} = .0985 \text{ reluctance of air path.}$$

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 = .000787 + .0985 = .099287 \text{ total reluctance of magnetic circuit.}$$

The reluctance of the path through the steel may be so small in comparison to the reluctance of the air gap, in some instances, that it may be neglected in the calculations of the total lines of force, as in this instance, without substantially affecting the calculated number of lines of force.

$$\phi = \frac{F}{\mathcal{R}} = \frac{2{,}262}{.0985} = 23{,}000 \text{ maxwells}$$

(disregarding reluctance of steel path)

Or,

$$\phi = \frac{F}{\mathcal{R}} = \frac{2{,}262}{.0993} = 22{,}800 \text{ maxwells}$$

(considering reluctance of both steel and air paths).

Exercise 35:

1. What would be the total number of lines of force in the above example, if the two faces of the core are pressed closer together so that the air gap is decreased to .0015 inch?

2. What cross-sectional area in square inches would be required for the core of Fig. 82 in order to have a total of 15,000 maxwells if $l_1 = 10$ inches, $l_2 = .002$ inch, $I = \frac{1}{2}$ ampere, and the coil has 4,000 turns?

116. Flux Density. One of the most important factors in magnetic-circuit calculations is the *flux density* or the number of lines for a given unit cross-sectional area at right angles to the direction of the magnetic lines of force.

The unit of flux density is the gauss, represented by the letter B. 1 gauss = 1 line per square centimeter.

$$B = \frac{\phi}{A} \tag{42}$$

Consider Fig. 82. The magnetizing force H increases in direct proportion to the current through the coil, causing magnetic lines of force to be set up around the magnetic path through the steel core and the air gap. The density, B, of the lines of force in the air through the air gap would increase in direct proportion to the magnetizing force were it not for the steel path through which the magnetic lines of force must also pass. The flux density of ferromagnetic materials does not increase in direct proportion to the magnetizing force. The corresponding *increase* in flux density for each *uniform step* of increase in magnetizing force becomes smaller and smaller with each additional step of increase in magnetizing force. A point is finally reached where further increases in H produce little or no increase in B. This is clearly shown by the B—H curves for various materials in Fig. 83.

117. Core Saturation. The curves of Fig. 83 should be thoroughly studied. It should be noted from these curves that a low flux density is relatively easy to obtain while high flux densities in the regions beyond the points where the curves begin to bend sharply require a relatively high magnetizing force.

The point at which a B—H curve bends sharply is called the *saturation point*. Increased flux density beyond the saturation point is hard to obtain, as discussed in section 116. The saturation point is not the point beyond which no additional lines of force may be added. It is the point beyond which additional lines of force are not practical to be obtained.

It was stated in section 108 that the permeability varied from one

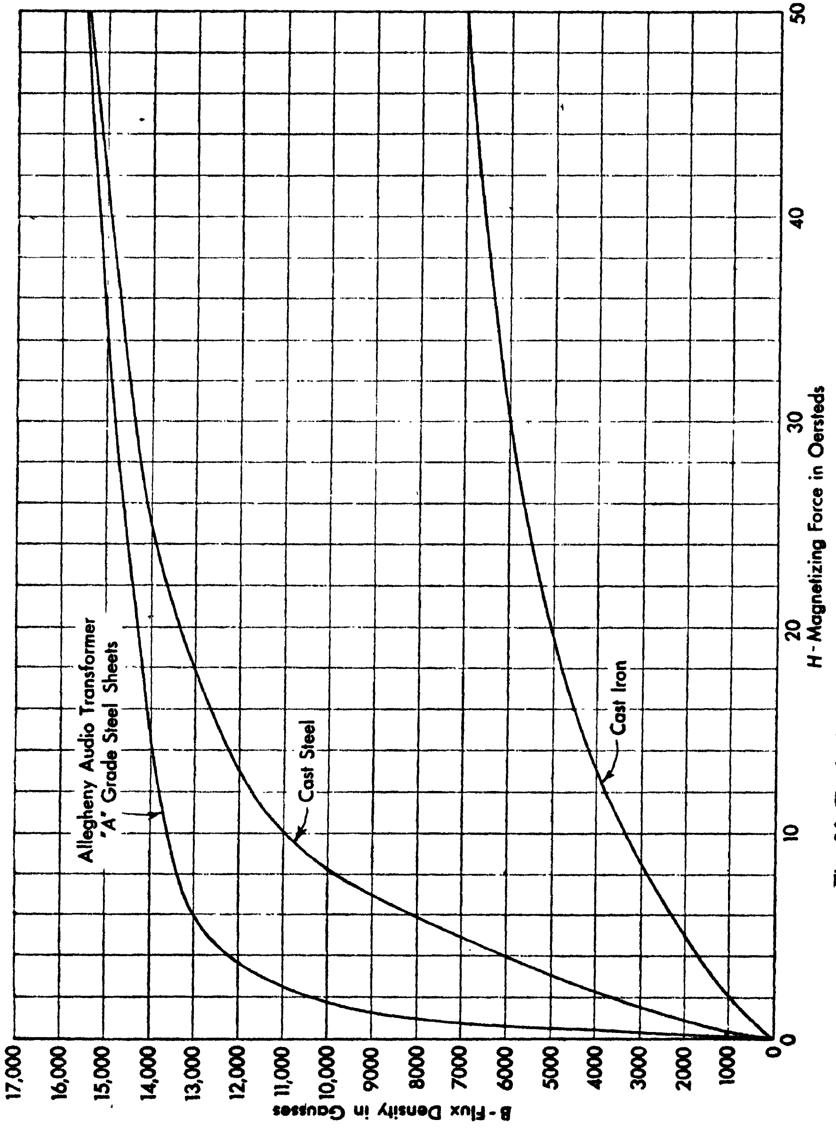


Fig. 83. Typical *B-H* curves for various ferromagnetic materials.

for air to nearly 100,000 for some of the special alloys. The permeability of any one material may also vary over wide limits for different values of magnetizing force.

$$\mu = \frac{B}{H} \tag{43}$$

where

μ = permeability

B = flux density in gausses per square centimeter

H = magnetizing force in oersteds

The permeability curves of Fig. 84 were plotted from the corresponding values of B and H of the curves of Fig. 83, substituted in equation 43. Inspection of the curves of Fig. 83 shows that the permeability of a material starts decreasing at approximately the saturation point.

The data shown in Fig. 83 and Fig. 84 are most helpful in selecting the proper core material as well as aiding in the determination of the proper number of turns, current, and so on in designing the many electromagnetic devices used in radio, such as relays, transformers, filter chokes, and speakers. Magnetic materials that have a fairly high permeability are used in electromagnetic devices whose *ampere turns* (coil current times coil turns) are low while devices whose ampere turns are high and whose operating current varies over considerable limits use magnetic core material with a lower permeability.

Some few devices are designed to operate at the bend or knee of the B—H curve of the magnetic core material for special applications. The swinging filter choke, not discussed in this text, is one of these devices.

118. Magnetic Shielding. The design of radio apparatus often requires that the circuit components be placed relatively close together because of the desirability and, at times, absolute necessity for compactness. This may require that other than normal precautions be taken in order to avoid objectionable magnetic coupling between the various circuit components.

Several transformer manufacturers make a line of transformers and filter chokes especially designed to minimize magnetic coupling from

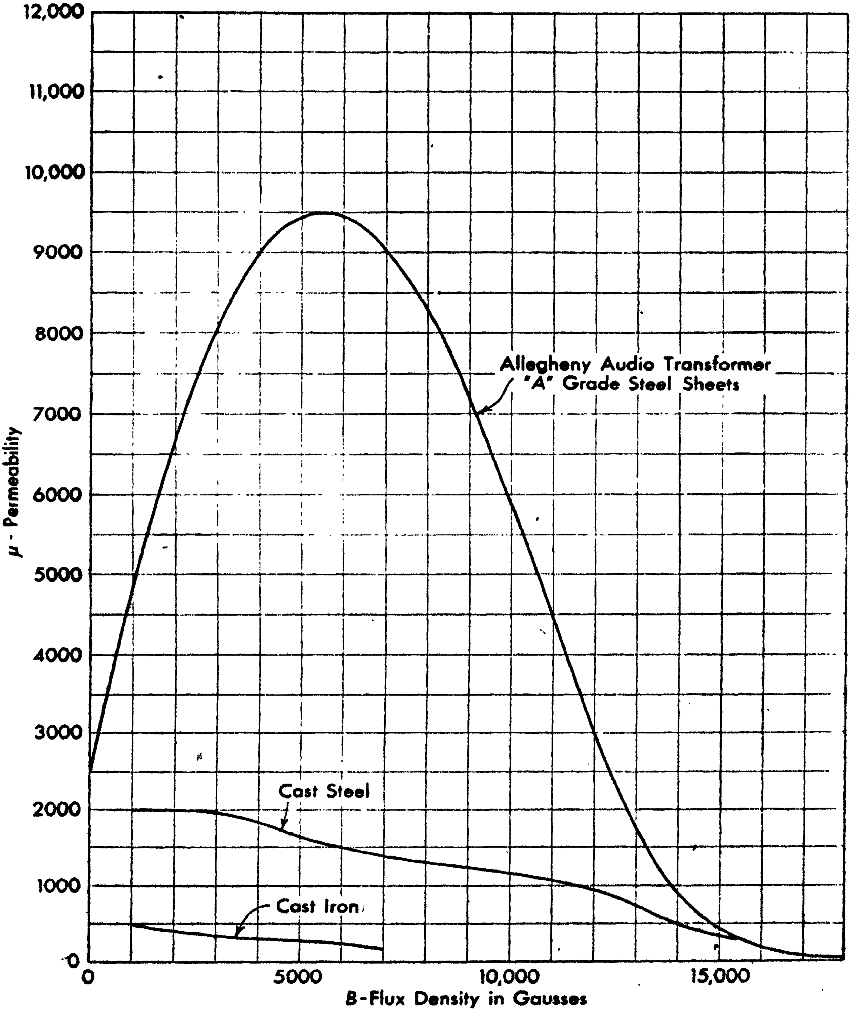


Fig. 84. Permeability curves obtained by the use of equation 43 and the corresponding values of B and H from the curves of Fig. 83.

one unit to another. The use of these units may be satisfactory in most instances if care is exercised in properly placing the ones most likely to give trouble as far apart as possible.

Proper magnetic shielding is quite important and, in this connection, a factor that should not be overlooked is the *shielding property* of the material.

Materials with a high permeability have a lower reluctance than those with a low permeability and should likewise be better for shielding purposes. This considers that the magnetizing force in the shielding material is such as to operate just below the knee of the *B—H* curve. If the magnetizing force is sufficiently strong to saturate the shielding material, the permeability as shown in Fig. 84 may decrease to such a low value that very little shielding will be obtained.

Most magnetic materials have a knee or bend in the *opposite direction* in their *B—H* curve for extremely *low values* of magnetizing force, and the permeability is relatively low. The shielding properties are likewise low.

From the foregoing discussion, it may be said that any shielding material is far less effective when the magnetizing force is very low or extremely high than it is when the magnetizing force is such that the operating point on the *B—H* curve is just below the knee of the top bend.

119. Residual Magnetism. In Chap. VIII, ferromagnetic materials were divided into two groups. In one are the ferromagnetic materials which tend to maintain their magnetism, when magnetized, thereby forming permanent magnets. All other ferromagnetic materials form a group that maintain their magnetism only so long as a magnetizing force is present, thereby forming only temporary magnets.

All the materials falling into the temporary magnet class, although called *temporary magnets*, have a trace of magnetism called *residual magnetism* remaining when the magnetizing force is removed. A magnetizing force, acting in the opposite direction, must be applied in order to reduce the residual magnetism to zero. The higher the residual magnetism, the greater the magnetizing force acting in the opposite direction must be in order to reduce the residual magnetism to zero.

If the magnetizing force, acting in the opposite direction to that of the original magnetizing force, is further increased after the residual magnetism has been reduced to zero, the material will be magnetized in the opposite direction (reversal of direction of magnetic lines of force). If the magnetizing force is removed, it will be found that the material now has residual magnetism with the lines of force in the opposite direction to the lines of force for the first residual magnetism.

120. Hysteresis. The action of the magnetic flux density in lagging behind the magnetizing force is called *hysteresis* and is graphically shown in Fig. 85.

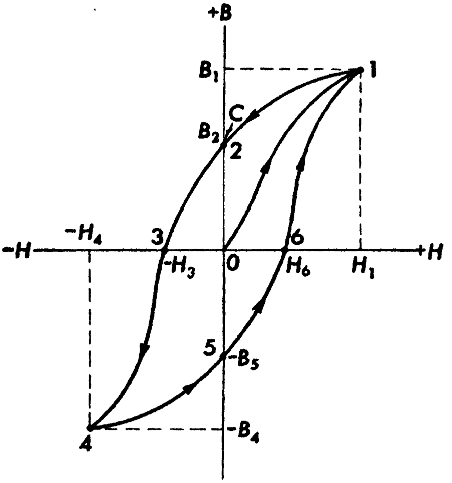


Fig. 85. A typical hysteresis loop for iron or steel used in electromagnetic devices.

The loop *0–1–2–3–4–5–6–1* of Fig. 85 is called the *hysteresis loop.* If the magnetizing force H is increased in the plus direction from zero to a value of H_1, the flux density will increase from a zero value to a value of B_1. The B—H curve for all values of H from *0* to H_1 and B from *0* to B_1 is shown by *0–1*. If the magnetizing force H is decreased, the flux density B will not decrease in the same proportion that it increased with H when H was increasing. The new B—H curve is shown by the curve of Fig. 85 between points *1* and *2*. At point *2*, H is zero, but B has decreased only down to a value of B_2. A magnetizing force (magnetizing force producing lines of force in the opposite direction) of value $-H_3$ is required to reduce the value of B to zero. Further increase of H in the minus direction from $-H_3$ to $-H_4$ produces a flux density B with lines of force in the opposite direction with a value of $-B_4$. In like manner, the lag of the magnetic flux density behind the magnetizing force may be traced from point *4* to *5* to *6* and back to *1*. Further variations of H from H_1 to $-H_4$ will cause the cycle to be repeated again from point *1* through the path *1–2–3–4–5–6–1*.

121. Hysteresis Losses. If it were not for the presence of residual magnetism, the *1–2–3–4* part of the curve of Fig. 85 would coincide with part *4–5–6–1* and part *0–1*, and no energy would be used. The hysteresis loop represents one complete cycle of magnetization, and the area bounded by the loop represents the power used in producing the cycle. This is considered as power lost since it is power used in counteracting residual magnetism. The power loss for each cycle is expressed by equation

$$P_h = .796 \frac{(BH)}{10^8} \qquad (44)$$

where

P_h = hysteresis-core loss in watt-seconds per cubic centimeter per cycle

(BH) = area of hysteresis loop from B—H graph where the flux density B is plotted in gausses and the magnetizing force H is plotted in oersteds

Example:

Find the hysteresis loss indicated by the hysteresis loop of Fig. 85 if the B scale is 5,000 gausses per centimeter, the H scale is 10 oersteds per centimeter, and the area of the loop is 20 square centimeters.

Solution:

$5000 \times 10 = 50{,}000$, the $(B$—$H)$ area of 1 square centimeter

$20 \times 50{,}000 = 1{,}000{,}000$, $(B$—$H)$ area for 20 square centimeters

$$P_h = .796 \frac{1{,}000{,}000}{10^8}$$

$$= .00796 \text{ watt-second per cubic centimeter per cycle.}$$

Another formula used for determining hysteresis-core loss, when the cycle is sinusoidal, was developed by Dr. Steinmetz. This formula is:

$$W_h = \frac{KB^{1.6}}{10^7} \tag{45}$$

where

W_h = hysteresis-core loss in watt-seconds per cubic centimeter per cycle

K = a constant as determined by the properties of the magnetic material

B = maximum flux density in gausses attained during the cycle

The value of K may vary from a value as low as .007 for silicon steel used in transformers to a value as high as .025 for hard cast steel. Other typical values of K are:

Cast iron	.012
Annealed steel sheets	.001

Example:

Consider a transformer core material to have a maximum flux density of 8,000 gausses and the value of K, .0007. The transformer has a volume of 65 cubic centimeters. Determine the core loss for 1 cycle per second; 60 cycles per second; and 5,000 cycles per second.

Solution:

$$W_h = \frac{.0007 \times 8000^{1.6}}{10^7}$$

$$8000^{1.6} = 1.6 \log 8000 = 1.6 \times 3.9031 = 6.24496$$

$$\text{Antilog } 6.24496 = 1{,}758{,}000$$

$$W_h = \frac{.0007 \times 1{,}758{,}000}{10^7} = .000123 \text{ watt-second per cubic centimeter per cycle}$$

$.000123 \times 65 = .00798$ watt loss for 1 cycle per second

$.00798 \times 60 = .479$ watt loss for 60 cycles per second

$.00798 \times 5{,}000 = 3.99$ watts loss for 5,000 cycles per second

Exercise 36:

1. What will be the loss in the hysteresis loop of Fig. 85 if B is plotted on a scale of 1 inch = 8,000 gausses, H is plotted on a scale of 1 inch = 12 oersteds, and the (B—H) loop area = 16 square inches?

2. Draw a hysteresis loop similar to Fig. 85 on graph paper with the width of one large square equal to 5,000 gausses along the vertical axis and 10 oersteds along the horizontal axis. Determine the approximate area of the loop by counting the squares and determine the hysteresis loss from equation 44.

3. What would be the loss in watt-seconds per cubic centimeter per cycle of a hard cast-steel transformer core whose maximum flux density is 7,000 gausses?

4. A transformer core with a volume of 275 cubic centimeters operating from the 60-cycle utility power source has a hysteresis loss of 3.3 watts per second. What is the loss in watt-seconds per cubic centimeter per cycle? What is the value of K if the maximum flux density is 9,000 gausses?

122. Magnetic-circuit Calculations. Magnetic-circuit calculations,[1] while similar to electric-circuit calculations,[2] are not so readily performed as are those for electric circuits. This is due to the fact that the permeability,[3] μ, and likewise, the reluctance, $\mathcal{R}$, change as the flux density B changes.

Magnetic-circuit calculations require curves or data, such as shown in Fig. 83 and Fig. 84, for the particular material or materials used for the path of the magnetic lines of force.

Various equations from this chapter are listed together in this section for the convenience of the student.

$$F = .4\pi IN \tag{34}$$

$$H = \frac{.4\pi IN}{l} \tag{35}$$

$$\mathcal{R} = \frac{l}{\mu A} \tag{36}$$

$$\mathcal{R} = \mathcal{R}_1 + \mathcal{R}_2 \tag{37}$$

$$\frac{1}{\mathcal{R}} = \frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} \tag{38}$$

$$\phi = \frac{F}{\mathcal{R}} \tag{39}$$

$$B = \frac{\phi}{A} \tag{42}$$

---

[1] Equations 39, 40, and 41.

[2] Equations 2, 3, and 4 in Chap. III.

[3] Figure 84.

(43) $$\mu = \frac{B}{H}$$

(44) $$P_h = .796 \frac{(BH)}{10^8}$$

(45) $$W_h = \frac{KB^{1.6}}{10^7}$$

where

F = magnetomotive force in gilberts

I = current in amperes

N = turns

H = *magnetizing force* in oersteds. (Formerly called *gilberts* per centimeter. Also formerly called *gausses.*)

l = *length* of magnetic path in centimeters

$\mathcal{R}$ = reluctance or opposition to magnetic lines of force. (Formerly called *oersted*, but has no name now.)

μ = permeability or ratio of the magnetic-conducting ability of iron or other material as compared to air

A = cross-sectional area of magnetic path in square centimeters

ϕ = total number of *lines of force* in maxwells

B = *flux density* in guasses per square centimeter

P_h = hysteresis-core loss in watt-seconds per cubic centimeter per cycle

W_h = hysteresis-core loss in watt-seconds per cubic centimeter per cycle (using formula developed by Dr. Steinmetz)

(BH) = Area of hysteresis loop from B—H graph where the flux density B is plotted in gausses, and the magnetizing force H is plotted in oersteds

K = a constant as determined by the properties of the material

REVIEW QUESTIONS

1. Discuss the magnetic field about a conductor.
2. What practical means do we have of determining the direction of the lines of force about a conductor?
3. What is electromagnetism?
4. Define permeability.

5. Give the principal parts of a headphone and explain the purpose of each.

6. What are the most essential parts of a speaker. Give the purpose of each part.

7. Why does a speaker give more volume than a set of headphones?

8. What are relays?

9. Why are relays used in radio circuits?

10. The magnetomotive force of a magnetic circuit may be compared to what force in an electrical circuit?

11. Give the unit and symbol of magnetomotive force.

12. Explain "ampere turns."

13. What is the difference between magnetomotive force and the magnetizing force?

14. What are maxwells?

15. What is an oersted?

16. What is a magnetic circuit?

17. How does the action of reluctance in a magnetic circuit compare with the action of resistance in an electrical circuit?

18. What is meant by flux density?

19. What is meant by core saturation?

20. How does an air-gap affect a magnetic circuit?

21. Discuss magnetic shielding at extremely low values of magnetizing force.

22. Explain residual magnetism.

23. Define hysteresis?

24. Is it particularly objectionable for hysteresis to be present in a magnetic circuit? Give reason for your answer.

ANSWERS

Exercise 32, page 135:

1. $IN' = 340$ ampere turns
   $F = 427$ gilberts
2. $I = .752$ ampere increase in current required
3. $IN = 457.6$ ampere turns

Exercise 33, page 136:

1. 1.318 oersteds
2. .042 oersted
3. .75 ampere

Exercise 34, page 139:

1. Reluctance = .05462
2. Reluctance = .26719
3. Reluctance = .37214

Exercise 35, page 140:

1. 2,292,000 maxwells
2. .0203 square inch

Exercise 36, page 148:

1. .01222 watt-second per cubic centimeter per cycle
3. .00355 watt-second per cubic centimeter per cycle
4. *a.* .0002 watt-second per cubic centimeter per cycle
   b. $K = .000941$

CHAPTER X *Alternating Current*

In the foregoing chapters we considered voltages which were substantially constant in value and produced an electrical pressure or electromotive force in a fixed direction. The flow of current resulting from such a voltage was constant and in one direction. Under such an arrangement of voltages and current, it is possible for the electrons in one part of the circuit eventually to traverse the entire distance around a closed circuit and return to their original position or starting point. It is not of especial concern to us how long it will take the electrons to travel completely around the circuit or if they ever complete the trip. We are chiefly concerned with how many electrons pass a given point within a certain time.

123. Alternating Current. If the electrons suddenly stop and move in the opposite direction, the effect is the same. The sudden change in the direction of travel of the electrons may be accomplished by reversing the connections to the battery, thereby reversing the polarity of the voltage applied to the terminals of the circuit. If these connections are continually reversed, thereby reversing the flow of the current, the current is said *to alternate.* Alternating current is not produced exactly in this manner.

For reasons made obvious in succeeding sections, it is desirable for the voltage not only to change in polarity, but also to rise and fall in value. It is desirable that these changes be periodic.

A flow of current is accomplished only through the presence of a voltage. Any change in the voltage will cause a corresponding change in the current. A periodic change in the voltage causes a periodic change in the current.

An alternating current is a current whose direction of flow *changes periodically.*

124. Alternation and Cycle. The rise of voltage or current from zero to some maximum value and back to zero again is called an *alternation.* The same principles that govern the flow of current in a circuit apply for an alternation. During the entire alternation, current is going in the same direction and is due to a voltage source which has plus and minus polarity terminals. The polarity of each terminal does not change during the entire alternation.

Immediately following the first alternation, the current repeats the above process, but in the opposite direction. This is due to a change in polarity of each of the voltage source terminals.

An alternation in one direction, with a subsequent alternation in the opposite direction, is called a *cycle.* During this period, the current goes through a complete cycle of changes, back to the original starting point or set of conditions. The current starts at zero and, while flowing in one fixed direction, increases to a maximum value, then decreases to zero again for one alternation (one-half cycle). During the next half of the cycle, the current flows in the opposite direction, while again increasing in value from zero to maximum and decreasing to zero.

125. Frequency. A cycle represents a complete set of changes in voltage or current. Further alternations produce additional cycles like the first. The number of times per second that the voltage or current goes through a cycle of change is called the *frequency.* The radio engineer is required to work with voltages and currents with frequencies as low as 30 cycles per second and as high as several million cycles per second.

Ordinary home electric service is alternating current with a frequency of 60 cycles. The human voice extends up in frequency to several thousand cycles. Frequencies, audible to the average human ear, extend up to about 17,000 cycles per second. The useful radio frequencies extend up to about 30,000,000,000 cycles, or 30,000 megacycles, per second.

It is necessary that the radio engineer become fully familiar with the behavior of alternating currents at the various frequencies encountered in radio work and with the characteristics of a-c circuits. Full mastery of a-c circuit solutions means that the greatest hurdle in learning radio has been cleared.

One can readily see that the problems of the radio engineer are quite different from those of an electrical engineer working with power. The power engineer deals primarily with only one frequency (60 cycles per second), while the radio engineer must be well versed in the behavior of a tremendously wide band of frequencies.

126. Angles. Two straight lines, meeting at a point, form an angle. Figure 86 shows various angles formed by two straight lines, meeting at a point.

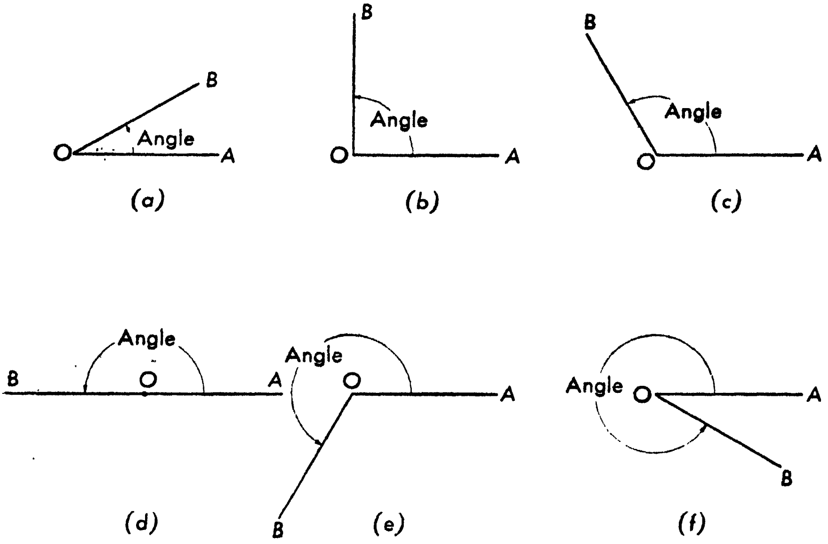


Fig. 86. Angles formed by two straight lines meeting at a point.

Angles are measured in degrees. If one line is perpendicular to the other, as in Fig. 86 *b*, the angle thus formed is called a *right angle* and contains 90 degrees. Side *OB* is rotated twice as far in Fig. 86 *d*, and the angle thus formed is 180 degrees. If side *OB* is rotated all the way around and back to *OA*, the angle thus formed is 360 degrees. If two complete revolutions are made, the angle thus formed is 720 degrees. The expressions 20 degrees, 78 degrees, 135 degrees, are written 20°, 78°, 135°, to keep from writing out the word "degrees." An angle less than 90°, such as Fig. 86 *a* is called an *acute* angle.

Quite often, the number of degrees in an angle may be unknown and still it may be necessary to designate or name the angle in some manner. Various methods are used. Each of the angles in Fig. 86 may be referred to as angle *AOB*. Usually, a letter or Greek alphabet symbol is used and is placed in the space between the two lines close to the point. The most commonly used Greek symbol is θ (theta). Other Greek symbols used quite often are α (alpha) and β (beta).

127. Triangles. The sides of the box in Fig. 87 *a* form four right angles, making a total of 360°. If the box is divided as in Fig. 87 *b*, two right triangles are formed. Each triangle has a total of 180° in the three angles formed by its three sides. All triangles, regardless of their size or shape, have a total of 180° in their three angles.

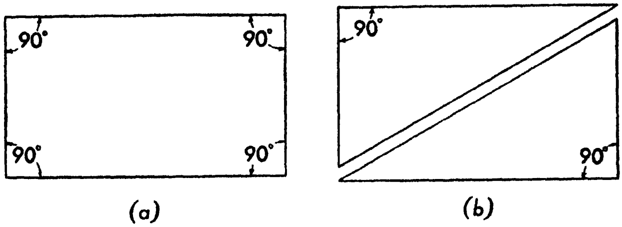


Fig. 87. Showing why any triangle has a total of 180° in the three angles formed by its three sides.

128. Trigonometry. Trigonometry deals with the mathematical relations of the sides and angles of triangles to each other. Trigonometry is used extensively in civil engineering as an aid in surveying. Other branches of engineering find the principles of trigonometry applicable to the solution of many engineering problems. More especially is trigonometry useful in the solution of problems where two or more forces or quantities, each producing a somewhat different effect, are acting to produce a common result.

A full knowledge of trigonometry is helpful, but not necessary, in the study of radio. The radio engineer needs to be concerned only with the simple fundamentals of trigonometry as covered in the following passages. These fundamentals form the background of alternating-current circuits and should be thoroughly covered.

129. Right Triangles. A right triangle is a triangle which has one right angle (90°). The sum of the two remaining angles is 90°, since the total number of degrees in all three angles is 180°.

In Fig. 86, the angle *AOB* was shown for line *OB* rotated around to various positions. In Fig. 88, angle *AOB* is designated as θ and the sides are designated *hypotenuse, adjacent side,* and *opposite side.* The hypotenuse is so named because it is opposite from the right angle formed by the other two sides. The hypotenuse is always the longest side. The adjacent side is so named because it is adjacent to the angle θ. The opposite side is so named because it is opposite angle θ. It is not practical to use these long names of designation for the sides of triangles when working out mathematical solutions of problems, so each side is usually designated by any single letter of the alphabet, as shown in Fig. 88. The sides are designated *a, b,* and *c.* It must be remembered that, regardless of what letter is used to designate the side *OB,* this side still remains the hypotenuse. The same holds true for the adjacent side and opposite side.

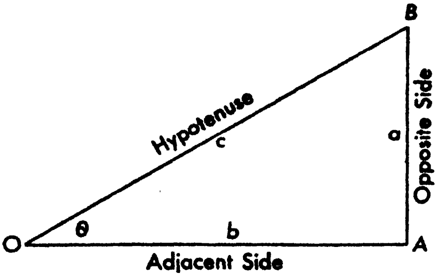


Fig. 88. Right triangle.

One of the most important facts about a right triangle is that the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$c^2 = a^2 + b^2 \tag{46}$$

and

$$c = \sqrt{a^2 + b^2}$$

The length of any of the three sides of a triangle may be determined from the above formula, if the other two sides are known.

Example:

The hypotenuse of a right triangle is 6 inches long, the adjacent side is 5 inches long. What is the length of the opposite side?

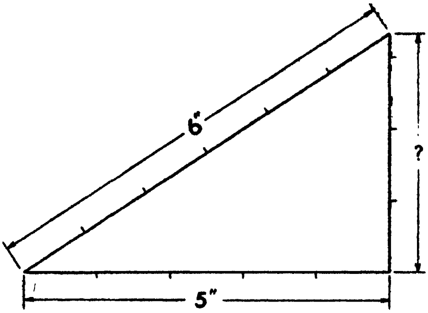
Solution:

The triangle is shown at right.
Since

$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$

then

$$a = \sqrt{c^2 - b^2} = \sqrt{(6)^2 - (5)^2}$$
$$= \sqrt{36 - 25} = \sqrt{11}$$
$$= 3.32 \text{ inches}$$



Exercise 37:

1. One of the angles of a right triangle is 30°. How many degrees in the other angle?
2. Find the length of the hypotenuse of a right triangle whose sides are 4 inches and 3 inches long.
3. The hypotenuse of a right triangle is 8 inches long and one of the sides is 4 inches long. Find the length of the other side.

130. Relation of Sides to Angles. Figure 89 shows how the adjacent side becomes shorter and the opposite side becomes longer (the hypotenuse remaining constant) as the angle becomes greater.

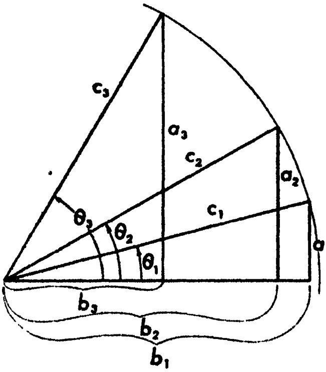


Fig. 89. Showing how a change in the angle θ changes the length of a and b.

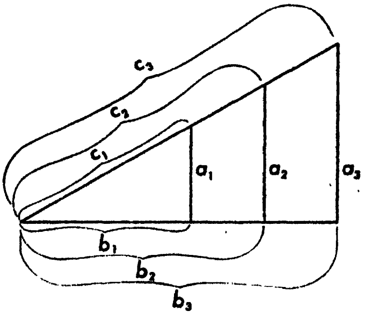


Fig. 90. Showing that the size of the triangle does not affect the ratios of the sides.

The length of a increases as the angle increases, showing that a definite relation exists between the length of a and the size of angle θ. Suppose that we consider the ratio of a divided by c or a/c. As the value of angle θ increases, the value of this ratio increases, until the ratio finally becomes 1 when a equals c which is at $\theta = 90°$. Other ratios readily become evident, such as b/c and a/b. The size of the the triangle does not affect the ratios of the sides. This fact is shown in Fig. 90.

$$\frac{a_1}{c_1} = \frac{a_2}{c_2} = \frac{a_3}{c_3} \tag{47}$$

$$\frac{b_1}{c_1} = \frac{b_2}{c_2} = \frac{b_3}{c_3} \tag{48}$$

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} \tag{49}$$

Example:

If $a_1 = 6$ inches, $c_1 = 10$ inches, $a_3 = 8$ inches in Fig. 90, what is the length of c_3?

Solution:

Since $$\frac{a_1}{c_1} = \frac{a_3}{c_3}$$

then

$$c_3 = \frac{c_1 a_3}{a_1} = \frac{10 \times 8}{6} = \frac{80}{6}$$
$$= 13.33 \text{ inches}$$

Exercise 38:

1. If $a_1 = 2$ inches, $c_1 = 4$ inches, and $c_2 = 5$ inches in Fig. 90, what is the length of a_2?

2. If $b_2 = 8$ inches, $c_2 = 10$ inches, and $b_1 = 6$ inches in Fig. 90, what is the length of c_1?

3. If $a_1 = 4$ inches, $b_1 = 6$ inches, $b_2 = 9$ inches, and $a_3 = 8$ inches in Fig. 90, what is the length of a_2 and b_3?

131. Trigonometric Functions. In Fig. 88, the sides of a right triangle were properly named hypotenuse, adjacent side, and opposite side. A right triangle is laid out with the opposite side 4 inches long and the hypotenuse 8 inches long, as in Fig. 91. The length of the adjacent side was computed from equation 46 and is found to be 6.93 inches long. Measurement of the acute angle at O shows it to be 30°.

The ratio of the opposite side to the hypotenuse is $\frac{a}{c} = \frac{4}{8} = \frac{1}{2}$. The length of the hypotenuse is kept constant and the length of a is increased to a new value, as in Fig. 89. The new length of a is measured, the new ratio of $\frac{a}{c}$ computed, and the value of the new angle measured with a protractor. A complete set of values of the ratio $\frac{a}{c}$ may be compiled for all values of the angle from 0° to 360° in this manner. Such tables are already in existence in many forms, not only for the ratio $\frac{a}{c}$, but for all of the other possible ratios which are: $\frac{b}{c}$, $\frac{a}{b}$, $\frac{c}{a}$, $\frac{c}{b}$, and $\frac{b}{a}$. Measurement of angles and sides of triangles graphically is not highly accurate. Actually, the values of trigonometric tables are calculated by higher mathematics. It would be quite confusing if these ratios were constantly referred to as $\frac{\text{opposite side}}{\text{hypotenuse}}$, etc., so names have been given to the ratios which are as follows:

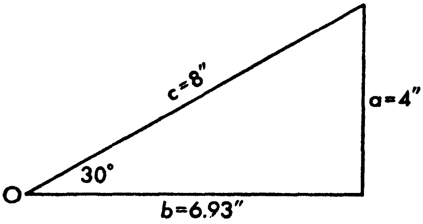


Fig. 91

$$\text{sine } \theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \tag{50}$$

$$\text{cosine } \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \tag{51}$$

$$\text{tangent } \theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \tag{52}$$

$$\text{cosecant } \theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{c}{a} \tag{53}$$

$$\text{secant } \theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b} \tag{54}$$

$$\text{cotangent } \theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a} \tag{55}$$

The last three are opposite ratio relations of the first three and will not be considered further, since they are of little relative importance in radio engineering, as compared to the first three.

Sine, cosine, and tangent are seldom spelled out in full, but abbreviated to read:

$$\sin\theta \qquad \cos\theta \qquad \tan\theta$$

Example:

Calculate the sin, cos, and tan functions of the angle θ in a right triangle whose adjacent side is 4 inches and opposite side is 3 inches.

Solution:

Since

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c &= \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

then

$$\begin{aligned} \sin\theta &= \tfrac{3}{5} = .60 \\ \cos\theta &= \tfrac{4}{5} = .80 \\ \tan\theta &= \tfrac{3}{4} = .75 \end{aligned}$$

Exercise 39:

1. Calculate the sin, cos, and tan functions of the angle θ in a right triangle whose hypotenuse is 7 inches and opposite side is 2 inches.

2. If $\sin\theta = \frac{6}{10}$, what is the length of the adjacent side?

3. If $\tan\theta = 1$, and the adjacent side is 3 inches, what is the length of the opposite side and the hypotenuse?

4. If $\cos\theta = .5$ and the hypotenuse is 5 inches, what is the length of the two sides?

132. Table of Trigonometric Functions. The more expensive slide rules are equipped with proper scales for determining trigonometric functions. While not too accurate, the values thus obtained are satisfactory in many instances, and are easily and rapidly determined.

Various tables of trigonometric functions are to be found in engineering, mathematical, and other technical books. Some give values, accurate to four or five places, for every minute, one-tenth, or one-hundredth degree. The present trend appears to favor dividing the degree into decimal parts rather than into minutes and seconds.

Less interpolation is required when the degrees are divided into small parts than when larger divisions of the degree are used. This text does not deal with the solution of alternating-current circuit problems requiring the extended use of trigonometry, therefore Table VI, which gives the trigonometric functions of sin θ, cos θ, and tan θ for every half degree from 0° to 90°, is simple yet fully adequate for the solution of the exercises in this chapter and Chapters XII, XIII, and XIV. This table has two principal uses which are as follows:

1. Given an angle—to find one or more functions.
2. Given a function—to find the angle.

133. Given the Angle—To Find the Function.

Example:

To find the sin, cos, and tan functions of an angle of 6°. The column marked "Degs." is followed until 6° is found. Immediately under this, in the sin row, is 0.1045, which is written: sin 6° = 0.1045. Immediately under the sin row is the cos row and the value under the column headed 6° is 0.9945, which is written: cos 6° = 0.9945. Tan 6° is found in the same manner and is equal to 0.1051, which is written: tan 6° = 0.1051.

Quite often it is necessary to find the functions of fractional angles. Functions of fractional portions of a degree other than even half degrees cannot be read directly from the table, but must be interpolated or calculated from the nearest value of the functions that are given in the table. Suppose that it is required to find the function of 6.2°.

Solution:

$$\begin{aligned} \sin 6.5^\circ &= 0.1132 \\ \sin 6.0^\circ &= \underline{0.1045} \\ \text{Difference of } .5^\circ &= 0.0087 \text{ difference in function} \end{aligned}$$

Required difference in degrees = .2°. The required difference in function is found by simple proportion:

$$\frac{.5}{.0087} = \frac{.2}{x}$$

$$x = \frac{.2 \times .0087}{.5} = .00348$$

$$\sin 6.2^\circ = .1045 + .00348 = .10798$$

$$\begin{aligned} \cos 6.5^\circ &= .9936 \\ \cos 6.0^\circ &= \underline{.9945} \\ \text{Difference of } .5^\circ &= -.0009 \text{ difference in function} \end{aligned}$$

Required difference in degrees = .2°. The required difference in function is found by simple proportion as above, except the difference is negative and must be subtracted from the function of 6° since the value of the cosine function is decreasing as the value of the angle is increasing.

$$\frac{.5}{-.0009} = \frac{.2}{x}$$

$$x = \frac{-.2 \times .0009}{.5} = -.00036$$

$$\cos 6.2^\circ = .9945 - .00036 = .99414$$

Inspection of the table shows that as the angle increases from 6° to 6.2°, the tangent function increases, so the procedure of interpolation is the same as for the sine.

$$\begin{aligned} \tan 6.5^\circ &= .1139 \\ \tan 6.0^\circ &= \underline{.1051} \\ \text{Difference of } .5^\circ &= .0088 \text{ difference in function} \end{aligned}$$

$$x = \frac{.2 \times .0088}{.5} = .00352$$

$$\tan 6.2^\circ = .1051 + .00352 = .10862$$

Exercise 40:

Find the sin, cos, and tan functions of the following angles from Table VI.

1. 15°
2. 44°
3. 87.5°
4. 7°
5. 25.5°
6. 4.3°
7. 77.7°
8. 23.4°
9. 52.8°
10. 52.85°

134. Given the Function—To Find the Angle. Angles and functions of angles are used extensively in the solutions of alternating-current problems. In most instances, the computations give the functions and the angles must be determined from tables.

Example:

Given a sin function of .8387. Find the angle and the cos and tan functions. This may be written as: $\sin^{-1} .8387$ or arc sin .8387, which means: "An angle whose sine function is .8387."

Look through the sin functions until the value .8387 is found and note the corresponding angle in degrees which is at the head of this column and is 57°. The cos and tan functions will be found in the same column immediately under the sin function.

$$\cos 57^\circ = .5446$$

$$\tan 57^\circ = 1.5399$$

Example:

$\theta = \sin^{-1} .7230$. Find the angle θ.

From the table:

$$\begin{aligned} \sin 46.5^\circ &= .7254 \\ \sin 46^\circ &= \underline{.7193} \\ \text{Difference of } .5^\circ &= .0061 \text{ difference in function} \\ \sin \theta &= .7230 \\ \sin 46^\circ &= \underline{.7193} \\ \text{Unknown difference in degrees} &= .0037 \text{ difference in function} \end{aligned}$$

Here again, a simple proportion is used to interpolate for the exact angle:

$$\frac{.5}{.0061} = \frac{x^\circ}{.0037}$$

$$x^\circ = \frac{.5 \times .0037}{.0061} = .3$$

$$\sin (46^\circ + .3^\circ) = .7230$$

$$\sin 46.3^\circ = .7230$$

Exercise 41:

Find the angles of the following functions from Table VI.

1. $\sin \theta = .3746$
2. $\cos \theta = .5225$
3. $\tan \theta = .5736$
4. $\sin \theta = .5878$
5. $\cos \theta = .5878$
6. $\tan \theta = .5878$
7. $\sin \theta = .9826$
8. $\cos \theta = .4195$
9. $\tan \theta = 1.1106$
10. $\cos \theta = .4307$

135. Signs in Different Quadrants. The signs of the trigonometric functions are not the same in all quadrants, as shown in Fig. 92. The sine function is positive in the first and second quadrants and negative in the third and fourth quadrants. The graph of the sine function in Fig. 95 bears out this fact as the portion from 0° to 180° is above the reference line and the portion from 180° to 360° is below the reference line.

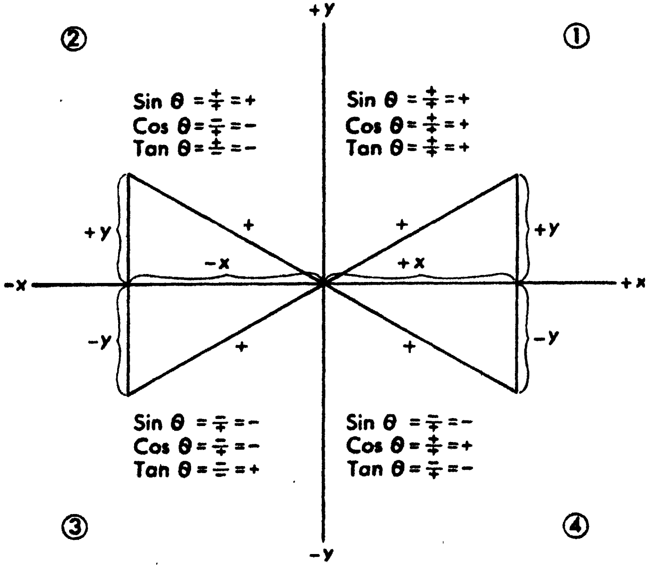


Fig. 92. Sign of functions in all four quadrants.

136. Sine, Cosine, and Tangent of Angle in Different Quadrants. Quite often, it is necessary to find the sine of an angle which is greater than 90°. The sine of an angle, in other than the first quadrant, is:

(56) Sine of any *angle* in the second quadrant = sin (180° − *angle*)

(57) Sine of any *angle* in the third quadrant = −sin (*angle* − 180°)

(58) Sine of any *angle* in the fourth quadrant = −sin (360° − *angle*)

The equations for cosine and tangent of angles in other than the first quadrant are similar to equations 56, 57, and 58 for sine. Use 180° − angle for second quadrant, angle − 180° for third quadrant, and 360° − angle for fourth quadrant, and use the proper sign for each quadrant as shown in Fig. 92.

Example:

Find the sine of 120°:

$$\sin 120° = \sin (180° - 120°) = \sin 60° = .866$$

Exercise 42:

Find the sin, cos, and tan functions of the following angles from Table VI:

1. 100°
2. 150°
3. 135°
4. 170°
5. 180°
6. 190°
7. 270°
8. 280°
9. 300°
10. 360°

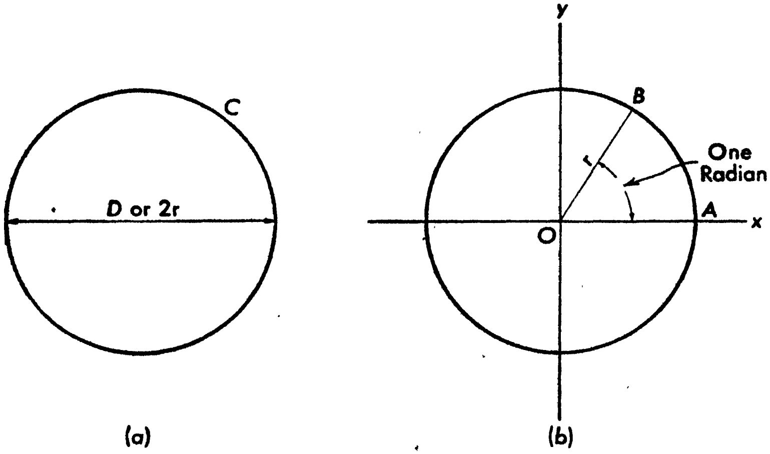


Fig. 93. a. Circumference *C*, diameter *D*, and radius *r* of a circle. *b.* Radian formed by distance $AB = OA = OB = r$.

137. Radian. In Fig. 93 *a*, the distance around the circle is denoted by the circumference and the distance across the circle by the diameter *D* or twice the radius *r*. The ratio of the circumference to the diameter is 3.1416+ and is denoted by the Greek symbol π (pi). This ratio is shown in equation 59.

(59) $$\pi = \frac{C}{D} = \frac{C}{2r}$$

Other forms of this equation are:

(60) $$C = 2\pi r$$

and

(61) $$2\pi = \frac{C}{r}$$

In Fig. 93 *b*, the angle *AOB* is so adjusted that the distance *AB* around the curved surface of the circle is equal to the distance *OB* (also *OA*) which is the radius of the circle. The angle thus formed is called a *radian*. The circumference, divided by the radius, gives the number of radians in a circle and is 2π or 6.28, as shown in equation 61.

(62) $$\frac{360^\circ}{2\pi} = 57.3^\circ \text{ in one radian}$$

(63) $$\frac{1^\circ}{57.3^\circ} = .0174 \text{ radians in one degree}$$

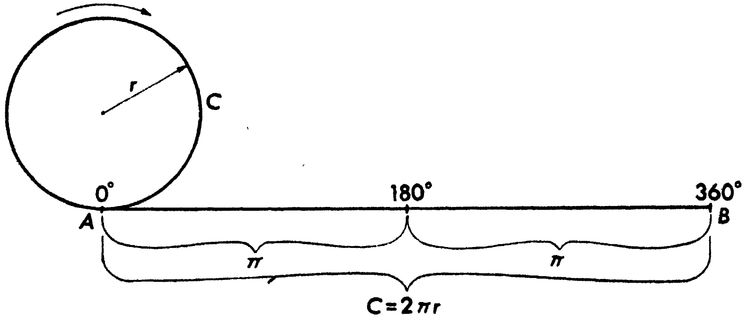


Fig. 94

If the circle in Fig. 94 is rolled to the right, it will have completed one revolution when it reaches point *B*. The distance *AB* represents the circumference *C* or its equivalent $2\pi r$. These facts are used in plotting curves of the functions of an angle. The distance along the *y* axis is laid off in units representing the value of the function.

138. Sine Wave Graph. If the wheel in Fig. 95 is rotated in a counterclockwise direction by means of the handle A, it will rise upward until point 3 is reached. Further rotation of the wheel will cause the handle to move downward, passing points 4, 5, 6, 7, and 8, reaching point 9. From point 9, the handle again moves upward, passing points 10 and 11 and reaching point 12, which coincides with the starting point 0. The distance traveled around the circumference of the wheel by the handle is shown along the x axis by points numbered 0 through 12. The up-and-down distance traveled by the handle is represented by the distance along the y axis between points 3 and 9. An exact plot of the movement of the handle may be made by projecting the y-axis values to the left parallel to the x axis, and the x-axis values vertically parallel to the y axis. The intersection of each set of lines, such as 1–1, 2–2, 3–3, gives a point on the curve. A line through all of these points forms a graph of the sine wave. The equation of this curve is:

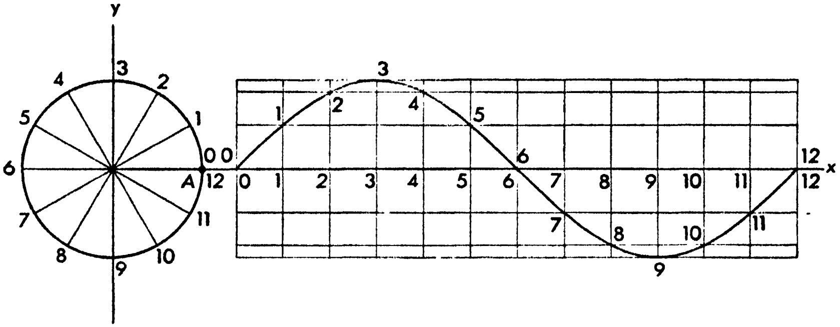


Fig. 95. Development of the sine curve.

$$y = \sin x \tag{64}$$

The sine wave may also be plotted from a set of data prepared from Table VI. Values of angles from 0° to 360° are assigned to x, and the sine function corresponding to each value of x is the value of y at that point. These data are shown in Fig. 96.

An inspection of the sine function values of Table VI shows that the absolute value of $\sin \theta$ is never more than 1. The spokes (radius) of the wheel in Fig. 95 may be any length and certainly more than 1.

Equation 64 holds true so long as the radius of the wheel or circle is unity. This equation may be rewritten to apply for any radius r and is:

$$y = r \sin x \tag{65}$$

where

r = radius in same units as y

x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
y	0	.5	.87	1	.87	.5	0	-.5	-.87	-1	-.87	-.5	0

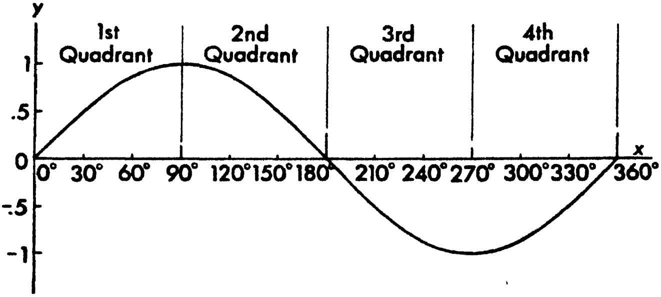


Fig. 96. $y = \sin x$.

139. Waves. In the early part of this chapter, alternations and cycles were covered. Figure 96 is a graph of one sine wave or cycle. The first half or alternation of this wave is positive and the last half is negative. If the wave is plotted further, another wave or cycle will be formed which will be exactly like the first. This is called a *periodic function.*

140. Alternating-current Wave. The variations or alternations of current, as covered in section 123 in the first part of this chapter, follow the variations as shown by the graph of the sine function in Fig. 95. The electrons flowing in the positive direction are zero in number at 0° and increase in number until the maximum value is reached at 90°. This increase in number of electrons or amperes of current is represented by an increase in the value of y. This is also called the *height* or *amplitude* of the wave. The value of current decreases as the angle increases from 90° to 180°. In fact, *all current flow completely ceases at 180°*. The next instant current starts flowing again and in increasing quantities from 180° to 270°, but in the *opposite* direction. At this point, the current starts decreasing in value and is zero again when 360° is reached. Note that the current is maximum in the positive direction at 90° and has the same maximum value but in the opposite direction at 270°.

Since the x axis is laid off in degrees and the y axis is laid off in current, equation 65 is written for current as:

(66) $i = I \sin \theta$

where

i = instantaneous current at any instant, and

I = maximum value of current

Exercise 43:

1. Plot a sine wave graph of an alternating current whose maximum current I is 10 amperes.

2. Plot a sine wave graph of an alternating current whose maximum current is 6 amperes.

141. Alternating-current Voltage Wave. The voltage wave rises and falls in amplitude in the positive direction and repeats the same procedure in the negative direction the same as the current wave does. The equation for the voltage wave is:

(67) $e = E \sin \theta$

where

e = instantaneous voltage at any instant, and

E = maximum value of voltage

However, the voltage wave and the current wave are not always exactly "in phase" ("in step") with each other in all parts of an a-c circuit. Quite often a circuit may contain several voltages that may not all rise and fall in amplitude at the same time (not in phase). One voltage may even be rising as another is decreasing in value, and the sign may not be the same. Since such waves are not rising and falling in amplitude together, they do not pass through zero at the same instant or reach maximum or minimum (maximum in the negative direction) at the same time. The time interval between the instant one wave passes through its maximum value and the instant the other wave passes through its maximum value is called the *phase relation* of the two waves. This time interval is measured in degrees and is called *angular difference* or *phase angle*.

142. Wave Additions. Radio a-c circuits in most instances are a maze of many voltage and current waves of various amplitudes and with different phase relations. Therefore, one should have a clear picture of how various voltage waves may be acting in a circuit, how various current waves may be acting in a circuit, or how combinations of both may be acting in a circuit. It is possible to show wave additions graphically. The following examples show various combinations of voltage waves and combinations of current waves. It should be kept in mind that *a voltage wave cannot be added to a current wave*, but the two may be multiplied to produce a power wave.

Examples:

1. *a.* Prepare a set of data from Table VI for $i = 3 \sin \theta$ and plot graph of one cycle.

b. Prepare a set of data from Table VI for $i = 2 \sin \theta$ and plot one cycle on same axis as *a*.

c. Combine current waves *a* and *b* graphically and plot resultant wave (labeled "wave *c*").

Data for Wave *a:*

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\sin \theta$	0	.5	.87	1	.87	.5	0	−.5	−.87	−1	−.87	−.5	0
$3 \sin \theta$	0	1.5	2.61	3	2.61	1.5	0	−1.5	−2.61	−3	−2.61	−1.5	0

Data for Wave *b:*

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360
$\sin \theta$	0	.5	.87	1	.87	.5	0	−.5	−.87	−1	−.87	−.5	0
$2 \sin \theta$	0	1	1.74	2	1.74	1	0	−1	−1.74	−2	−1.74	−1	0

Data for Wave *c:*

θ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
(*a*) + (*b*)	0	2.5	4.35	5	4.35	2.5	0	−2.5	−4.35	−5	−4.35	−2.5	0

Figure 97 shows the graph of these two waves which are designated as waves *a* and *b*. Wave *c* is a graph of the combination of waves *a* and *b*. The values of current for this wave are obtained by adding the values of current of each wave at each point along the *x* axis. At 90° the value of wave *a* is 3 amperes and wave *b* is 2 amperes. The current for the combined wave at this point is $3 + 2 = 5$ amperes. In adding the corresponding values of waves, the values must always be added algebraically. Quite often, the value of one may be negative at a certain point while the value of the other is positive at this point. In Fig. 97, waves *a* and *b* are in phase and increase and decrease together.

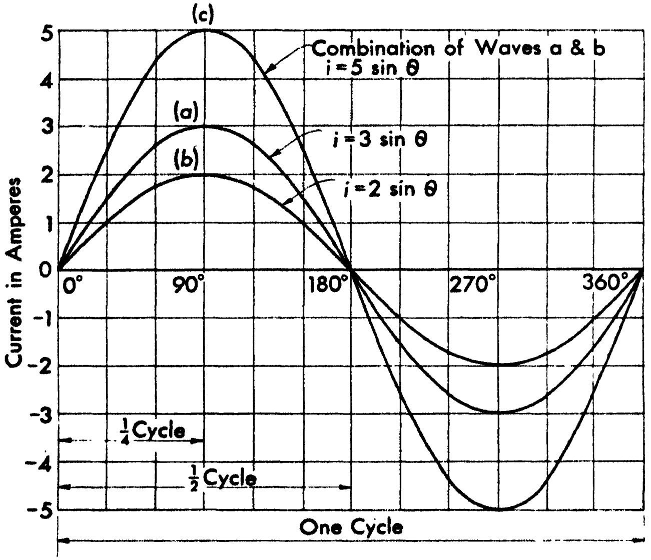


Fig. 97. Showing the addition of two sine waves which have the same frequency, are in phase, but have different amplitudes.

Sin θ is equal to 1 at 90°, therefore, the maximum or peak value occurs at this point and is equal to r from equation 65. The value of r is 3 for wave *a*, 2 for wave *b*, and 5 for wave *c*.

2. *a.* Plot the graph of a sine wave with a maximum value of 3 amperes.

b. Plot the graph of a sine wave with a maximum value of 3 amperes, the same frequency as *a*, and with a phase angle of 90° lag.

c. Combine current waves *a* and *b* graphically and plot resultant wave.

A graph of waves *a*, *b*, and *c* is shown in Fig. 98. The *maximum* value of *b* is reached 90° *after* the *maximum* value of *a* is reached, showing *b* as lagging *a*. If wave *b* had reached its maximum value *before* wave *a* reached its maximum value, then wave *b* would be *leading a.*

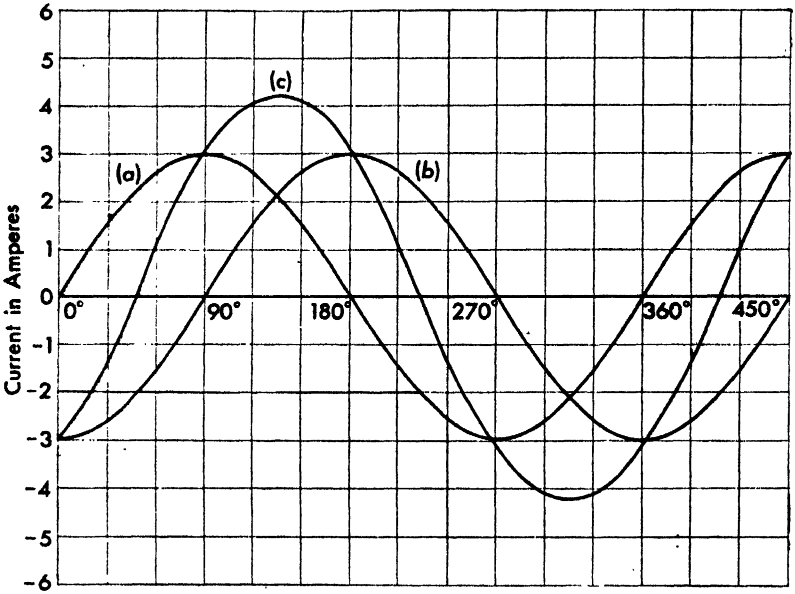


Fig. 98. In this figure, current wave (*b*) lags current wave (*a*) by 90°. Wave (*c*) is the resultant wave from the combination of (*a*) and (*b*).

3. *a.* Plot the graph of a sine wave with a maximum value of 3 amperes.

b. Plot the graph of a sine wave of twice the frequency of *a* and with a maximum value of 3 amperes.

c. Combine *a* and *b* and plot resultant wave.

These waves are shown in Fig. 99.

4. *a.* Plot the graph of a sine wave with a maximum value of 5 amperes.

b. Plot the graph of a sine wave of three times the frequency of *a* and with a peak value of 2 amperes.

c. Combine *a* and *b* and plot resultant wave.

These waves are shown in Fig. 100.

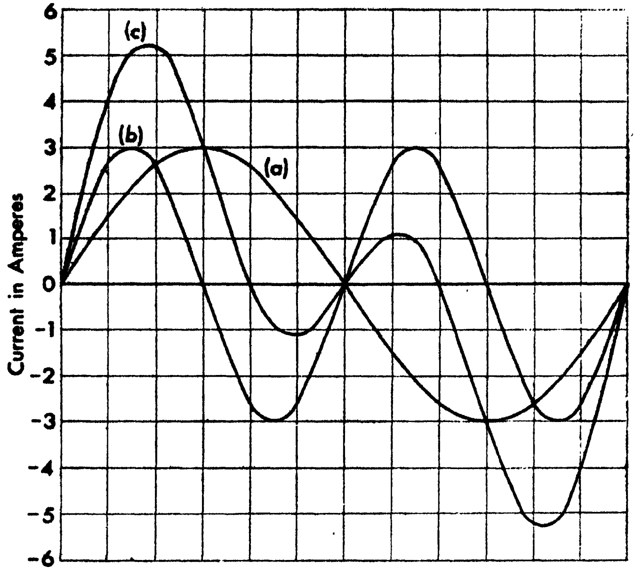


Fig. 99. In this figure, waves (*a*) and (*b*) have the same amplitude but the frequency of (*b*) is twice that of (*a*). (*c*) is obtained by adding the instantaneous values of (*a*) and (*b*) as was done in Fig. 97 and Fig. 98.

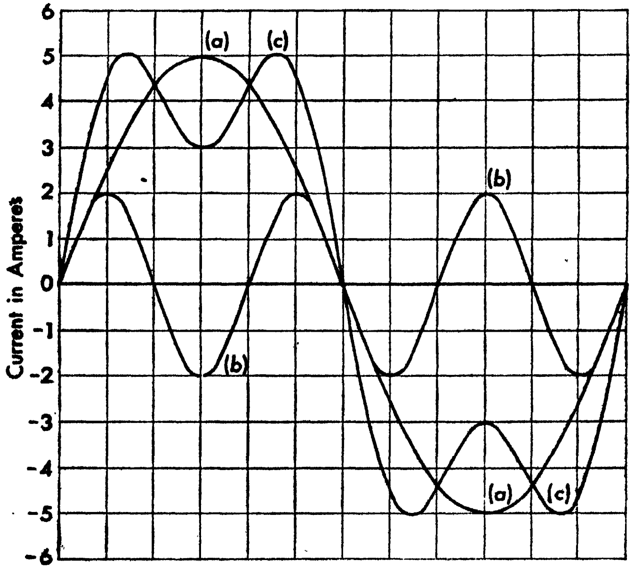


Fig. 100. In this figure, (*a*) and (*b*) have different amplitudes and (*b*) has a frequency three times that of (*a*). (*c*) was obtained by combining (*a*) and (*b*).

5. *a.* Plot the graph of a sine wave with a peak value of 6 amperes.

b. Plot the graph of a sine wave of twice the frequency of *a* and with an amplitude of 1 ampere.

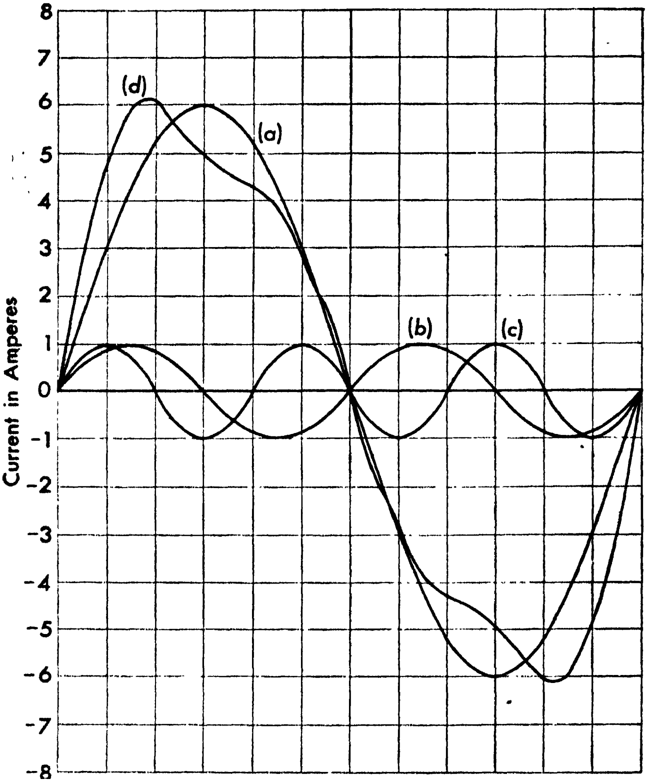


Fig. 101. In this figure, (*b*) and (*c*) have the same amplitude. The frequency of (*b*) is twice that of (*a*) and the frequency of (*c*) is three times that of (*a*). (*d*) was obtained by combining (*a*), (*b*) and (*c*).

c. Plot the graph of a sine wave of three times the frequency of *a* and with an amplitude of 1 ampere.

d. Combine *a*, *b*, and *c* and plot resultant wave.

These waves are shown in Fig. 101.

143. Complex Waves. A review of Figs. 97–101 indicates that many odd looking waves may be obtained by adding several waves. A wave formed by adding several sine waves is called a *complex wave.*

It should also be remembered that *any complex wave, no matter how complicated, is formed by the combination of several sine waves.*

Electrical waves produced by the human voice speaking into a telephone, radio, public-address system, or other microphone, are very complex. These waves change in amplitude and frequency with each spoken word.

Exercise 44:

All graphs of this exercise should be made on suitable graph paper.

1. Prepare a set of data from Table VI for waves *a* and *b* of Fig. 98 and plot waves. Prepare data for wave *c* and plot wave.

2. Prepare set of data from Table VI for waves *a* and *b* of Fig. 99 and plot waves. Prepare data for wave *c* and plot wave.

3. Prepare set of data from Table VI for waves *a* and *b* of Fig. 100 and plot waves. Prepare data for wave *c* and plot wave.

4. Prepare set of data from Table VI for waves *a*, *b*, and *c* of Fig. 101 and plot waves. Prepare data for wave *d* and plot wave.

5. *a.* Plot the graph of a sine wave with a peak of 6 amperes.

b. Plot the graph of a sine wave of twice the frequency of *a* and with an amplitude of 2 amperes.

c. Plot the graph of a sine wave of three times the frequency of *a* and with an amplitude of 4 amperes.

d. Combine *a*, *b*, and *c* and plot the resultant wave.

144. Generation of Alternating Current. If the permanent magnet in Fig. 102 is moved rapidly from position 1 to 3, a current will flow through the light bulb and a-c ammeter. The current will be maximum when the magnet is in position 2. The current increases from zero at position 1, to maximum at position 2, and decreases back to zero at position 3. If the magnet is rapidly moved in the opposite direction from position 3 to position 1, *the current again rises and falls* but flows in the opposite direction.

Alternating-current generators operate on the same principle as illustrated in Fig. 102, although their actual construction is somewhat different. The magnetic field is usually stationary and produced by direct current flowing through coils of wire wound around pole pieces of iron or other magnetic material. The coil is wound around another piece of magnetic material called the *armature.* The armature rotates in the magnetic field.

One or more sine waves are generated, with each revolution of the armature, as determined by the number of poles involved. One sine wave is generated with each revolution of the armature if the generator has one *set of poles* (one north and one south). Two sine waves are generated if the generator has two sets of poles, and so on.

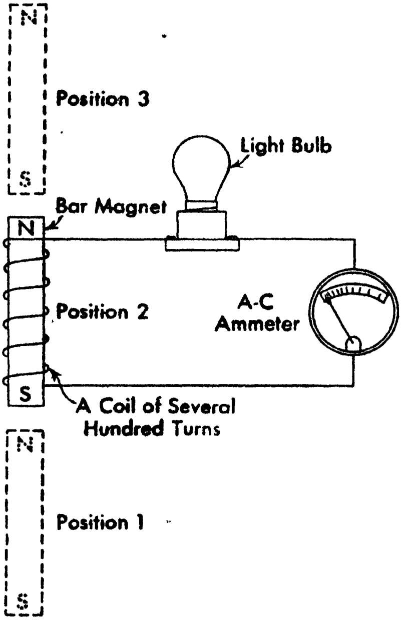


Fig. 102. Generation of alternating current.

145. Effect of Alternating Current and Voltage. In sections 123 and 124, at the beginning of this chapter, the student's attention was called to the fact that alternating current not only changes in direction of flow, but increases and decreases in value from zero to some maximum value.

In the study of direct current, it was found that a current flow of 1 ampere represents a movement of a definite number of electrons as long as the circuit is completed. A typical circuit is shown in Fig. 103 *a*. As long as the switch is closed, a steady current flows as shown by Fig. 103 *b*.

The battery is replaced by a generator as shown in Fig. 103 *c*. If the peak value of the alternating current, as shown in Fig. 103 *d*, is the only value of the current where enough electrons are flowing in the circuit to constitute a current of 1 ampere, and all during the remaining time the current is less than this value, then it is evident that this current is not equivalent to the direct current of Fig. 103 *b* that maintains a value of 1 ampere at all times. *An a-c ampere must produce the same heating effect as a d-c ampere to be equivalent to it.*

The same thing applies to a-c voltage. An a-c voltage of 1 volt cannot exert the same pressure as a steady d-c voltage of one volt if it is less than 1 most of the time and even zero at times.

It becomes apparent that there is a need for a measure of a-c voltage and current that compares directly with d-c voltage and current. If the a-c voltage is increased as in Fig. 103 *e*, the cur-

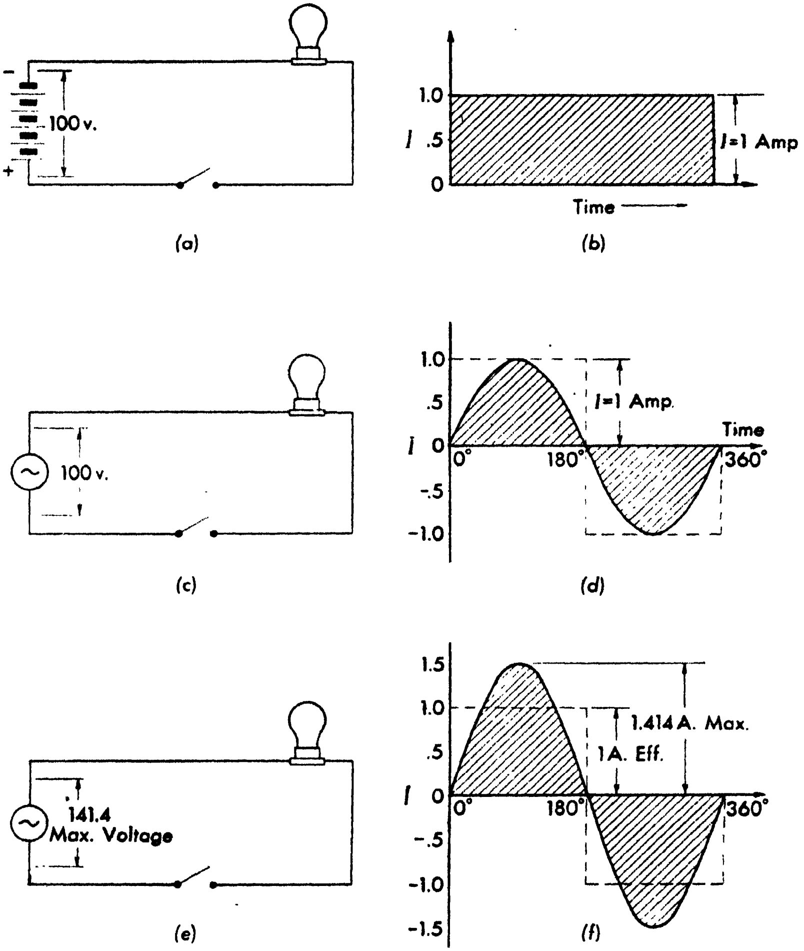


Fig. 103

(*a*) Direct current circuit
(*b*) Steady current flow of direct current circuit
(*c*) a-c circuit
(*d*) Alternating current flow of a-c circuit
(*e*) a-c circuit with peak voltage of 141.4 volts and effective voltage of 100 volts
(*f*) Effective current and peak current of circuit (e)

rent will increase. The current must be increased until its maximum value is 1.414 amperes before it will be equivalent to a d-c ampere of 1 ampere. Therefore, the *effective* value of an a-c current of 1.414 amperes is only 1 ampere. The exact ratio of the *maximum* current to *effective* current as determined mathematically is $\sqrt{2}$, which is approximately equal to 1.414. From the foregoing, the following relations are obtained:

An alternating current of 1.414 amperes has an effective value of 1 ampere and produces the same heating effect as 1 d-c ampere.

$$\frac{I_{eff}}{I_{max}} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = .707 \tag{68}$$

$$\frac{I_{max}}{I_{eff}} = \frac{\sqrt{2}}{1} = \frac{1.414}{1} = 1.414 \tag{69}$$

$$\frac{E_{eff}}{E_{max}} = \frac{1}{\sqrt{2}} = \frac{1}{1.414} = .707 \tag{70}$$

$$\frac{E_{max}}{E_{eff}} = \frac{\sqrt{2}}{1} = \frac{1.414}{1} = 1.414 \tag{71}$$

where

I_{eff} = effective current
I_{max} = maximum current
E_{eff} = effective voltage
E_{max} = maximum voltage

The operating a-c voltages and currents of radio equipment, specified by the manufacturer, are the effective voltages and currents, unless stated otherwise. Likewise, all a-c instruments read effective values of current and voltage unless otherwise indicated. In practice and in technical literature, voltages and currents are understood to be effective unless otherwise stated, a practice that is also followed in this text. The effective values of voltage and current are also called *root-mean-square* (*r.m.s.*) values.

Inspection of any sine wave such as Fig. 103*f* shows that the value of amplitude varies from zero to the maximum or peak value. It has just been shown that the effective value is .707 times the maximum value. The *average* value is even less than the effective value. If many vertical lines were erected from the x axis to the first half of the sine wave and all of the heights added and divided by the number of lines used, it would be found that the value $2/\pi$ (approximately equal to .637) would be obtained if an infinite number of lines were used (by calculus solution).

146. Ohm's Law for R in an A-c Circuit. Ohm's law is the same for resistance in an a-c circuit as in a d-c circuit, except that either maximum values of both current and voltage should be used or effective values of both current and voltage should be used.

Example:

The line voltage across a resistor is 110 volts. The peak current through the resistor is 3 amperes. What is the resistance of the resistor?

$$3 \times .707 = 2.121 \text{ amperes effective current}$$

$$R = \frac{E_{eff}}{I_{eff}} = \frac{110}{2.121} = 51.75 \text{ ohms}$$

Exercise 45:

1. Referring to Fig. 103 *e*, if the generator voltage $E_{max} = 150$ volts, and $I_{eff} = 2$ amperes, calculate the resistance of the lamp.

2. The filament of a vacuum tube requires 6.3 volts at 300 milliamperes. What is the resistance of the filament?

147. Alternating-current Power in Resistances. In a direct-current circuit, the power P is equal to EI. In an alternating-current circuit, the power is continually changing from one instant to the next. The power continually varies from zero to some maximum value. The wave form is sinusoidal (sine-wave) with a frequency twice that of the current or voltage. Such a wave is shown in Fig. 104. The axis for the power wave is shown as $X - X$.

The *voltage* and *current* in a *resistance* are always in phase; therefore, the instantaneous power is:

$$p = ei \tag{72}$$

The actual or average power is:

$$P_{avg} = E_{eff}I_{eff} \tag{73}$$

or

$$P_{avg} = \frac{E_{max}}{\sqrt{2}} \times \frac{I_{max}}{\sqrt{2}} = \frac{E_{max}I_{max}}{2} = \frac{P_{max}}{2}$$

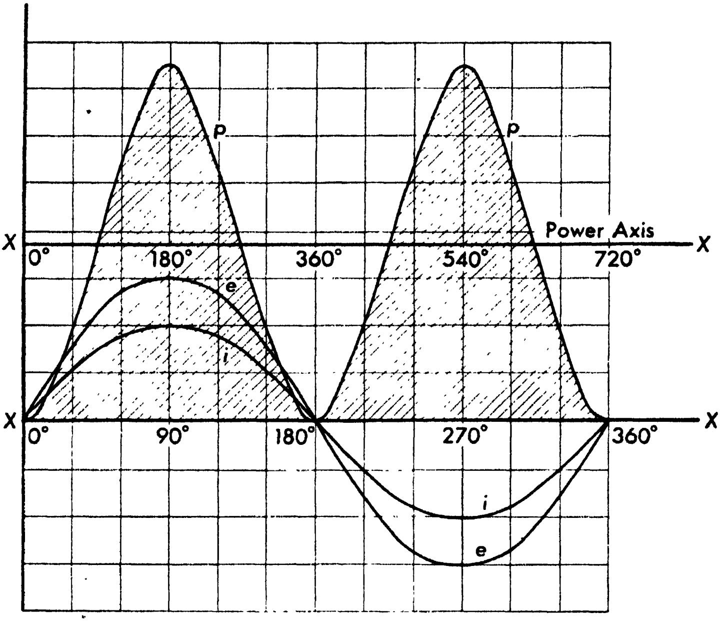


Fig. 104. a-c voltage, current and power waves for a resistance circuit.

and the maximum power is:

(74) $P_{max} = E_{max}I_{max}$

where:

p = instantaneous power in watts

P_{max} = maximum power in watts

P_{avg} = average power in watts

Power, unless otherwise stated, is considered to be average power. This text follows that practice.

Example:

Two amperes of current are flowing through a resistor. The instantaneous peak or maximum power is 25 watts. What is the voltage drop across the resistor?

$$I_{max} = 2 \times 1.414 = 2.828 \text{ amperes}$$

$$E_{max} = \frac{P_{max}}{I_{max}} = \frac{25}{2.828} = 8.83 \text{ volts}$$

$$E_{eff} = .707 \times 8.83 = 6.24 \text{ volts}$$

Exercise 46:

1. The voltage drop across a resistor, dissipating a power of 8 watts, is 5 volts.

a. What is the peak current?

b. What is the maximum instantaneous power?

2. If the current through a 60-watt light bulb is 1.414 amperes, what is the peak line voltage?

3. A 100-watt soldering iron is plugged into a house-current wall receptacle supplied with 120-volt alternating current.

a. How much current will flow through the soldering iron?

b. What is the instantaneous peak power?

4. If 8 Christmas-tree lamps are connected in series across a 120-volt a-c house-current line, what current will flow in the circuit if each lamp uses 5 watts of power?

5. The maximum instantaneous power that a resistor will safely dissipate is 40 watts. What current will flow through the resistor when operating at one-half rating when the voltage drop across the resistor is 5 volts?

148. Vectors. A force F is applied to the weight in Fig. 105 *a.* The action of the force is in the right-hand direction as indicated by the arrow. If the force F is enough to move the weight, it will move in the direction of the arrow.

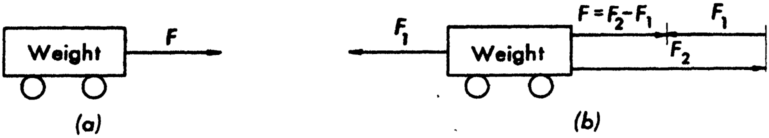


Fig. 105. a. A force F acting on weight in the direction of arrow. *b.* A force F, acting on weight in the left hand direction and force F_2 acting on the weight in the right hand direction.

The weight in Fig. 105 *b* is acted upon by a force F_1 in one direction and a force F_2 in the opposite direction. The weight will naturally move in the direction in which the greater force is acting. The *direction* in which a force is acting is indicated by an arrow pointing in that direction. The *amount* of force acting in any direction is indicated by the *length* of the arrow in that direction. In Fig. 105 *b*, the F_2 arrow is longer than F_1, therefore, the force F_2 is greater than F_1. The resultant force F is equal to $F_2 - F_1$ and in the direction of F_2.

The various forces indicated in Fig. 105 are *vector quantities.*

A vector quantity is a quantity which has both magnitude and direction.

Vectors may be used to represent various things. In the study of radio, we are particularly interested in the use of vectors to represent the directions and magnitudes of currents, voltages, resistances, reactances,[1] and impedances.[2]

149. Addition of Vectors. The vector quantities of Fig. 105 are acting in a horizontal plane but vectors *may act in any direction* as in Fig. 106 *a*.

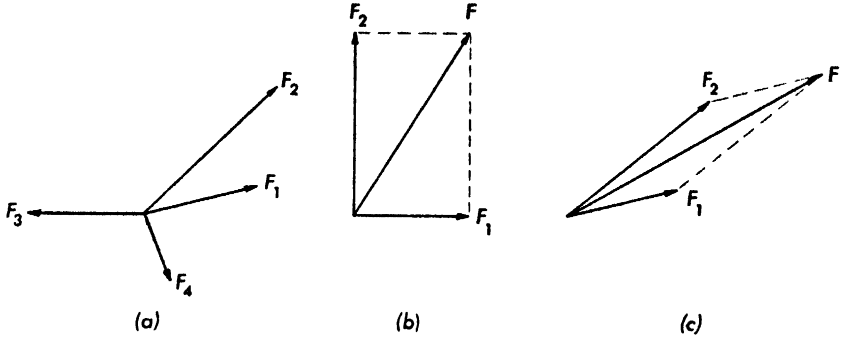


Fig. 106. a. Vectors may have any magnitude and direction. *b* and *c*. Graphical addition of vectors.

Only vector quantities which are *parallel* in direction may be added algebraically.

There are two general methods by which vectors may be added: (1) *analytical;* and (2) *graphical.*

The analytical method makes use of mathematical calculations, while the graphical method makes use of actual vectors scaled on graph paper with proper magnitudes and directions.

150. Graphical Addition of Vectors. In Fig. 106 *b* and *c*, vectors F_1 and F_2 were added graphically by completing the *parallelograms* (dotted lines). The vector F in each case is the resultant. If more than two vectors are to be added, any two may be added as in Fig. 106 *b* and *c*, then the result of these two may be added in a like manner to another vector. This resultant may be added to still another vector and so on until all vectors are added as in Fig. 107.

The parallelogram method may be shortened by using the method shown in Fig. 108. This method eliminates the use of so many lines and "subtotal" vectors which are confusing when a number of vectors

---

[1] Discussed in Chaps. XII and XIII.

[2] Discussed in Chap. XIV.

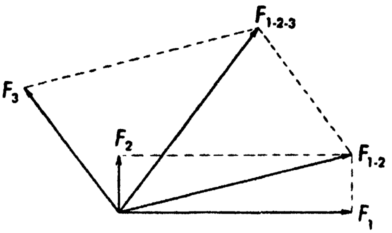
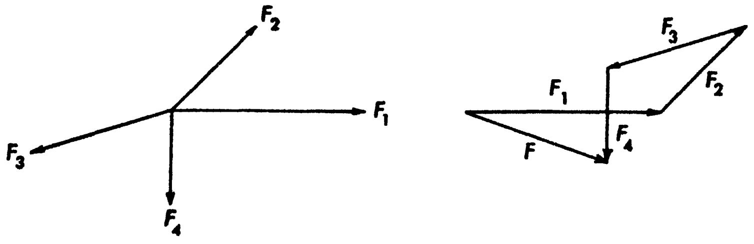


Fig. 107. Parallelogram method of graphically adding vectors. $F_{1\text{-}2}$ is the resultant of F_1 and F_2. $F_{1\text{-}2\text{-}3}$ is the resultant of $F_{1\text{-}2}$ and F_3, or the result of vectors F_1, F_2 and F_3.

are added. Figure 108 *b* shows the addition of the various vectors shown in Fig. 108 *a*. Vector F_1 in *b* is drawn in a line parallel to the corresponding F_1 vector in *a* and with the same magnitude. Vector F_2 in *b* is started at the *arrowhead end* of F_1 and drawn with the same *magnitude* and *direction* as the F_2 vector of *a*. Each succeeding vector is drawn in the same manner. The *resultant vector* is drawn from the starting point of the first vector to the ending point or arrowhead of the last vector.



(a) *(b)*

Fig. 108. Vectors shown in *a* are added in *b*. Vectors *b* are drawn with the same magnitude and direction as corresponding vectors in *a*.

Exercise 47:

All graphs of this exercise should be made on suitable graph paper.

1. Let $F_1 = 5$ pounds, making a 0° angle with the horizontal and $F_2 = 8$ pounds, making a 90° angle with the horizontal, in Fig. 106 *b*. Draw vector diagram to scale and determine *F graphically*.

2. Let $F_1 = 5$ pounds at 30° angle with horizontal and $F_2 = 8$ pounds

at 60° angle with horizontal in Fig. 106 *c*. Draw vector diagram to scale and determine *F graphically*.

3. Let $F_1 = 10$ pounds with 0° angle, $F_2 = 2$ pounds with 90° angle, and $F_3 = 6$ pounds with 135° angle in Fig. 107. Draw vector diagram to scale and determine *F graphically*.

4. Let $F_1 = 20$ pounds at 0°, $F_2 = 8$ pounds at 30°, $F_3 = 12$ pounds at 190°, and $F_4 = 6$ pounds at 270° in Fig. 108 *a*. Draw vector diagram to scale and determine *F graphically* by the parallelogram method. Draw vector diagram to scale and determine *F graphically* by the method shown in Fig. 108 *b*.

151. Vector Components. In the use of rectangular coordinates as in Fig. 109, any point such as *E* is located by measuring off a certain number of divisions along the *x* axis and a certain number of divisions along the *y* axis. Both of these measurements are projected as shown.

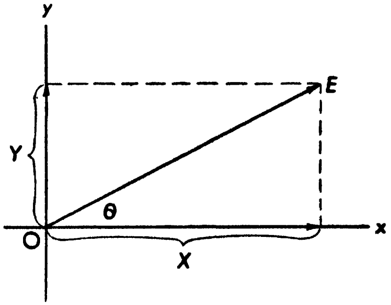


Fig. 109. Vector *E* with its *X* and *Y* components.

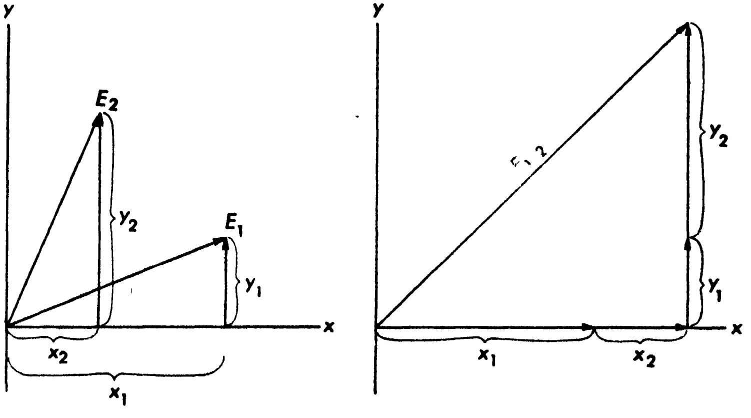
The intersection of these projections locates point *E*. A line drawn from the origin, *O*, to *E* is the resultant of the *component* vectors *X* and *Y*. From previous sections, we know that the *x* and *y* components are obtained from the following relations:

$$x = E \cos \theta \tag{75}$$

and

$$y = E \sin \theta \tag{76}$$

152. Addition of Vector Components. In section 149, it was stated that only vectors which are parallel in direction may be added algebraically. Vectors E_1 and E_2 in Fig. 110 cannot be added algebraically since they are not parallel in direction. Each vector is resolved into its x and y components by the use of equations 75 and 76. The x component of E_1 is parallel in direction to the x component of E_2. Likewise, the y-axis components are also parallel in direction. Therefore, the following rules may be formulated:



(a) (b)

Fig. 110. a. Vectors E_1 and E_2 and their corresponding x and y components. *b.* The resultant E_{1-2} obtained by the addition of all x and y components.

Rule 1: Vectors that are to be added may be resolved into their vertical and horizontal component vectors.

Rule 2: The horizontal component vectors may be added algebraically to obtain a resultant horizontal vector.

Rule 3: The vertical component vectors may be added algebraically to obtain a resultant vertical vector.

Rule 4: The resultant vector may be obtained by the use of the value of the resultant horizontal vector in equation 75 or the use of the value of the resultant vertical vector in equation 76 or from the relation: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Example:

A current vector, I_1, with a magnitude of 10 amperes, makes an angle of 20° with the horizontal. A current vector, I_2, with a magnitude of 15 amperes, makes an angle of 80° with the horizontal. A current vector, I_3, with a magnitude of 6 amperes, makes an angle of 110° with the horizontal. These three currents are shown in Fig. 111. Determine: (*a*) horizontal and vertical component of each current: (*b*) resultant current and angle it makes with horizontal.

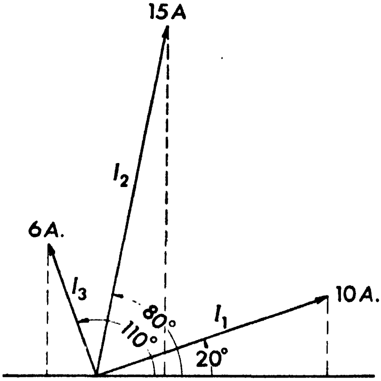


Fig. 111

$$
\begin{aligned}
I_{1x} &= 10 \cos 20^\circ = 10 \times .9397 = 9.397 \text{ amperes}\\
I_{2x} &= 15 \cos 80^\circ = 15 \times .1736 = 2.61 \text{ amperes}\\
I_{3x} &= 6 \cos 110^\circ = 6 \times (-.342) = -2.05 \text{ amperes}\\
I_{\text{total } x} &= 9.397 + 2.61 - 2.05 = 9.957 \text{ amperes}\\
I_{1y} &= 10 \sin 20^\circ = 10 \times .342 = 3.42 \text{ amperes}\\
I_{2y} &= 15 \sin 80^\circ = 15 \times .9848 = 14.79 \text{ amperes}\\
I_{3y} &= 6 \sin 110^\circ = 6 \times .9397 = 5.638 \text{ amperes}\\
I_{\text{total } y} &= 3.42 + 14.79 + 5.638 = 23.85 \text{ amperes}
\end{aligned}
$$

$$\theta = \tan^{-1} \frac{23.85}{9.957} = \tan^{-1} 2.397 = 67.3^\circ$$

$$I = \frac{I_{\text{total } y}}{\sin \theta} = \frac{23.85}{\sin 67.3^\circ} = \frac{23.85}{.9225} = 25.9 \text{ amperes}$$

Exercise 48:

Determine the *x*-axis and *y*-axis components of the following vectors:

1. 10 amperes at 15°
2. 5 amperes at 25°
3. 20 amperes at 50°
4. 7 amperes at 90°
5. 9 amperes at 135°
6. 6 amperes at 180°
7. 100 volts at 200°
8. 150 volts at 230°
9. 500 volts at 280°
10. 3,000 volts at 300°

Exercise 49:

Determine the magnitude and direction of the resultant vector in each of the following problems.

1. 15 amperes at 30°; 5 amperes at 60°
2. 4 amperes at 15°; 12 amperes at 80°
3. 6 amperes at 40°; 6 amperes at 60°; 10 amperes at 80°
4. 10 volts at 25°; 25 volts at 58°; 75 volts at 100°
5. 110 volts at 65°; 50 volts at 170°; 80 volts at 210°

153. Current and Voltage Vectors. It has been shown that the instantaneous values of voltages and currents in a-c circuits change from one instant to the next. It was shown in equations 66 and 67 that the instantaneous value of current or voltage is equal to the product of a *rotating vector* times the sine of the angle of rotation. This is the y-axis component of the vector.

The use of vectors to represent the values of voltages and currents is very helpful in the study of electrical circuits.

Vectors are useful in graphically showing the phase relation between various voltages, various currents, or currents and voltages.

154. Phase and A-c Power. Section 147 covered the conditions of power and phase in an a-c circuit containing resistance only. It was pointed out that the phase angle between the current through a resistor and the voltage drop across the resistor is zero. This is always true. The phase angle between the voltage and *current in all other (inductance and capacitance) circuit elements is 90° and they use no power.*.

Power in an a-c circuit is

$$P = EI \cos \theta \tag{77}$$

where

P = average power in watts

E = effective voltage in volts

I = effective current in amperes

θ = phase angle between the voltage and current

From trigonometry, the multiplication of a quantity by cos θ gives the x-axis or horizontal component. In a-c circuits, it is not sufficient to multiply the current by the voltage to obtain the actual power, as this gives only the *apparent power.* The apparent power in Fig. 112 is equal to EI. The actual power is equal to $EI \cos \theta$ or $E_1 I$, since E_1 is obtained by $\frac{E_1}{E} = \cos \theta$ or $E_1 = E \cos \theta$. Cos θ is called the *power factor,* since it is the factor by which the apparent power is multiplied to obtain the actual power. It can never be more than 1.

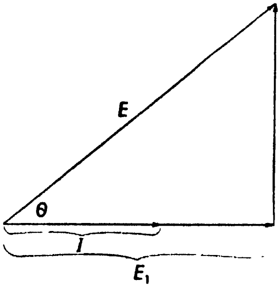


Fig. 112. Vector diagram of current and voltage with a phase angle θ. $P = EI \cos \theta$.

Since resistance is the only circuit element that uses power, and voltage and current are in phase only in a resistance, the voltage E_1 must represent the voltage drop across the equivalent resistance in the circuit. *Only the in-phase component of voltage is serving to produce power since it is the only portion of the voltage acting in a line parallel with the current.*

Example:

A circuit has a voltage of 120-volt alternating current and 10 amperes with a phase angle of 35°. Determine: (*a*) the apparent power; (*b*) the actual power; (*c*) the in-phase voltage component; (*d*) the peak power.

a. $P_{app} = EI = 120 \times 10 = 1{,}200$ watts, apparent power

b. $P = EI \cos \theta = 120 \times 10 \times \cos 35°$
$= 1200 \times .8192 = 981$ watts, actual power

c. $E = 120 \cos \theta = 120 \times .8192 = 98.1$ volts

d. $P_{max} = E_{max} I_{max} \cos \theta$
$= 120 \times \sqrt{2} \times 10 \times \sqrt{2} \times .8192$
$= 1200 \times 2 \times .8192$
$= 1962$ watts

Exercise 50:

1. The line voltage is 110-volt alternating current, the current is 7 amperes, and the phase angle between the current and voltage is 22°. Determine peak power and in-phase voltage component.

2. A circuit uses 100 watts at 2 amperes from a 120-volt alternating-current line. What is the phase angle between the voltage and current? What is the in-phase component of voltage?

3. A radio receiver uses a peak power of 125 watts from the 120-volt alternating-current house current with a power factor of .9. Determine: (*a*) the phase angle; (*b*) the current; (*c*) the in-phase voltage.

REVIEW QUESTIONS

1. What is an alternating current?
2. What is a cycle?
3. How many alternations in one cycle?
4. What is meant by frequency?
5. Discuss angles and degrees.
6. Give the total number of degrees in the three angles of any triangle.
7. Define trigonometry.
8. What is a right triangle?
9. Which of the three sides of a right triangle is called the hypotenuse?
10. What are trigonometric functions? Name and explain the three most important ones.
11. Give several reasons for using a table of trigonometric functions.
12. How many degrees in a radian?
13. How many radians in a circle?
14. What is a quadrant?
15. Describe a sine wave graph.
16. What is a wave?
17. How are waves added?
18. What is a complex wave?
19. Define an effective a-c ampere.
20. Discuss the relation between maximum voltage and effective voltage.
21. Give the expression for a-c power in a resistance.
22. Why is a basic knowledge of trigonometry helpful in the study of alternating current?
23. What are vectors?
24. Explain the statement: "Added vectorially."
25. What are vector components?
26. Can vectors be used to represent voltages and currents?
27. What is meant by phase?
28. Explain: (*a*) Power factor; (*b*) Apparent power; (*c*) Actual power.
29. Give the limits over which the power factor may vary.

ANSWERS

Exercise 37, page 157:

1. 60°
2. 5 inches
3. 6.93

Exercise 38, page 158:

1. $a_2 = 2.5$ inches
2. $c_1 = 7.5$ inches
3. $a_2 = 6$ inches
   $b_3 = 12$ inches

Exercise 39, page 160:

1. $$\sin\theta = \frac{2}{7}$$
   $$\cos\theta = \frac{6.71}{7}$$
   $$\tan\theta = \frac{2}{6.71}$$
2. 8
3. Opposite side 3 inches
   Hypotenuse 4.24 inches
4. 2.5 and 4.33 inches

Exercise 40, page 162:

	Sin	Cos	Tan
1.	0.2588	0.9569	0.2679
2.	0.6947	0.7193	0.9657
3.	0.9990	0.0436	22.90
4.	0.1219	0.9925	0.1228
5.	0.4305	0.9026	0.4770
6.	0.0750	0.9972	0.0752
7.	0.9770	0.2130	4.5864
8.	0.3971	0.9178	0.4327
9.	0.7965	0.6046	1.3175
10.	0.7971	0.6039	1.3198

Exercise 41, page 164:

1. 22°
2. 58.5°
3. 29.84°
4. 36.0°
5. 54.0°
6. 30.45°
7. 79.3°
8. 65.2°
9. 48.0°
10. 64.49°

Exercise 42, page 165:

	Sin	Cos	Tan
1.	0.9848	−0.1736	−5.6713
2.	0.5000	−0.8660	−0.5774
3.	0.7071	−0.7071	−1.0000
4.	0.1736	−0.9848	−0.1763
5.	0.0000	−1.0000	0.0000
6.	−0.1736	−0.9848	0.1763
7.	−1.0000	0.0000	Infinity
8.	−0.9848	0.1736	−5.6713
9.	−0.8660	0.5000	−1.7321
10.	0.0000	1.0000	0.0000

Exercise 45, page 179:

1. 53 ohms
2. 21 ohms

Exercise 46, page 181:

1. *a.* 2.262 amperes
   b. 16 watts
2. 60 volts
3. *a.* .8333 ampere
   b. 200 watts
4. .333 ampere
5. 2 amperes

Exercise 48, page 186:

	X axis	*Y axis*
1.	9.659 amperes	2.588 amperes
2.	4.5315 amperes	2.113 amperes
3.	12.856 amperes	15.32 amperes
4.	0	7 amperes
5.	−6.363 amperes	6.363 amperes
6.	−6 amperes	0
7.	−93.97 volts	−34.2 volts
8.	−96.42 volts	−114.9 volts
9.	86.8 volts	−492.4 volts
10.	1,500 volts	−2598 volts

Exercise 49, page 187:

1. 19.5 amperes at 37.3°
2. 14.17 amperes at 65.0°
3. 21.1 amperes at 63.7°
4. 99.7 volts at 84.7°
5. 99.4 volts at 136.5°

Exercise 50, page 189:

1. Peak power, 1428 watts
In-phase voltage component, 102 volts
2. Phase angle, 65.4°
In-phase component of voltage, 50 volts
3. *a.* 25.8°
b. Effective current, .578 ampere
c. 108 volts

CHAPTER XI *Electrical Instruments*

Electrical instruments play an important part in the design, manufacture, operation, and servicing of radio apparatus. A modern broadcast transmitter has many instruments (often called meters) mounted in its face panel, all important to the engineer in properly operating and servicing the transmitter. In fact, the receiver serviceman would be at a complete loss without instruments with which to measure currents, voltages, circuit continuity, or with which to trace the signal through the various circuits.

Instruments are also required to measure circuit values or to *view the wave form* of the signal.

155. Electrical Measurements. The student should readily appreciate the absolute necessity for having electrical measuring instruments. It is of equal importance that the instruments not seriously disturb the circuits being measured. For example, an *ammeter* used for measuring current is placed *in series with the circuit* and should have a relatively low internal resistance. If the ammeter has as much resistance as the circuit, the current will be cut to half the original value when the ammeter is placed in the circuit.

In voltage measurements, the instrument is placed *across* (in parallel with) *the voltage* potential that is to be measured. It is important that the voltage does not change appreciably when the *voltmeter* is shunted across the voltage source.

In general, measurements should be made with instruments that cause as little disturbance of the circuits as possible, and these disturbing effects should be well within the prescribed allowable limits.

156. D'Arsonval Movement. Figure 113 is a simplified sketch of the *D'Arsonval movement*. Figure 114 shows a phantom view of the D'Arsonval movement of a commercial instrument and some of the details of construction. This type movement is widely used in d-c measuring instruments and even some a-c measuring instruments.

The coil of very fine insulated wire is free to swing on two pivots. The pointer is attached to the coil assembly and swings across the scale when the coil swings on its axis. The two fine hairsprings (one at each pivot point) tend to hold the pointer at the zero point at the left-hand end of the scale and at the same time act as flexible conductors to the two ends of the coil.

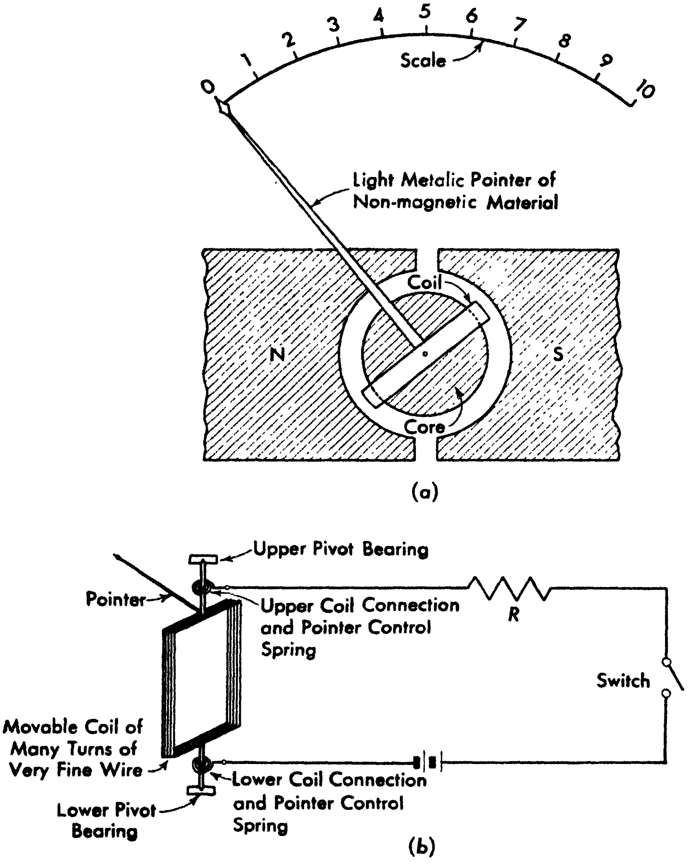

0
1
2
3
4
5
6
7
8
9
10
Scale
Light Metalic Pointer of Non-magnetic Material
Coil
N
S
Core
(a)
Upper Pivot Bearing
Pointer
Upper Coil Connection and Pointer Control Spring
R
Switch
Movable Coil of Many Turns of Very Fine Wire
Lower Coil Connection and Pointer Control Spring
Lower Pivot Bearing
(b)

Fig. 113. a. Simplified sketch of the D'Arsonval movement used extensively in electrical indicating instruments. *b.* Sketch showing how connections to the coil are made through the two hair springs.

When current is allowed to flow through the coil, lines of force are

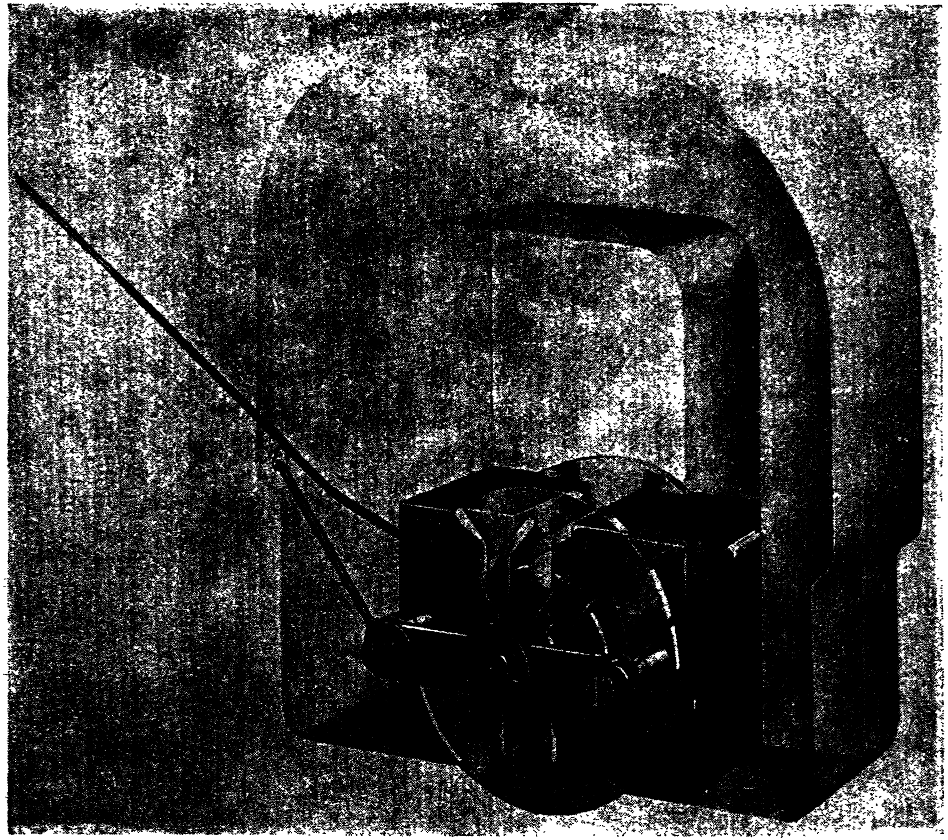


Fig. 114. Phantom view of the D'Arsonval movement of a commercial instrument showing some of the details of construction. (Courtesy, Weston Electrical Instrument Company.)

set up around the coil, creating magnetic polarity in the coil as shown in Fig. 73 of Chap. IX. The position of the coil with the pointer at zero is such that the magnetic poles of the coil (due to current flowing in the coil) act in opposition to the poles of the permanent magnet (like poles repel and unlike poles attract), causing the coil assembly to rotate on its axis against the opposing force of the control springs.

From section 107, a reversal of the direction of the current through a coil reverses the magnetic polarity of the coil. Instruments using the D'Arsonval movement have the proper polarity required for a "left to right" swing of the pointer marked on the instrument terminals. The terminal that is to receive the positive polarity is marked + (plus) and the terminal that is to receive the negative polarity is sometimes marked − (minus) and, in some instances, left blank. If the polarity of the voltage or current to the instrument is reversed, the instrument pointer swings against the left stop pin with which most instruments are supplied.

The core is centered in the space between the pole faces such that the coil is free to rotate in a uniform air gap. The deflection of the pointer is directly proportional to the current through the coil for all parts of the scale. Therefore, *the scale of a d-c type D'Arsonval instrument is uniform regardless of the range of current or voltage for which the instrument is designed.*

157. Current Measuring D-c Instruments. Instruments designed to measure direct current have scales graduated in amperes, milliamperes, or microamperes. They are called *ammeter*, *milliammeter*, and *microammeter*. Of these three types of instruments, the milliammeter is the most common in radio work. Direct currents encountered in most radio circuits are less than 1 ampere and are often only a few milliamperes or even less than 1 milliampere.

As pointed out in section 155, all current-measuring instruments are placed in series with the circuit whose current is to be measured and must therefore have low internal resistance.

158. Voltage Measuring D-c Instruments. The scales of instruments designed to measure direct-current voltage are usually graduated in volts or kilovolts. Such instruments are called *voltmeters*.

The D'Arsonval movement shown in Figs. 113 and 114 and described in section 156 is used in the construction of *voltage measuring* instruments as well as *current measuring* instruments. It will be observed that a D'Arsonval movement instrument, although designed and calibrated to measure voltage, is still a *current-operated* instrument.

Since a voltmeter is placed directly across the voltage to be measured, its resistance must be sufficiently high to keep from short-circuiting the voltage or cause it to drop appreciably.

159. Direct-current Instrument Movement Types. The radio industry requires d-c measuring instruments of many different ranges of currents and voltages. Most manufacturers prefer to manufacture a few standard type movements and construct all their many d-c measuring instruments from these relatively few types of movements, as car manufacturers construct many body styles of cars from one standard chassis.

The "1-milliampere movement" is one type that has been very popular with several manufacturers for a number of years.

Naturally, the physical size and other factors, as well as the current rating of the coil, are taken into consideration in designing a new type movement.

160. Direct-current Milliammeters. A commercial type milliammeter is shown in Fig. 115. The instrument case is usually Bakelite. If a metal case is used, it is made of nonferrous metal in order not to affect the magnetism of the permanent magnet.

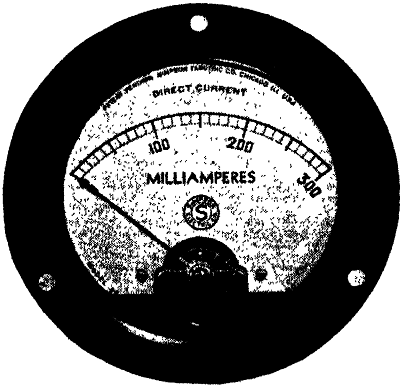


Fig. 115. A commercial type milliammeter. (Courtesy, Simpson Electric Company.)

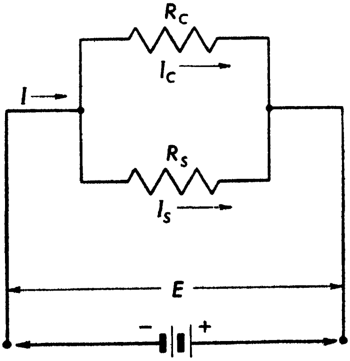


Fig. 116. Circuit of a milliammeter with a movement coil of R_c resistance, paralleled by a shunt with a resistance of R_s.

The range of a 1-milliampere movement or any other movement may be extended by paralleling a suitable resistance across the terminals of the instrument. Milliammeters of many different current ranges may be made by simply changing the value of this resistor. Such a resistor is called a *shunt* since a portion of the current to be measured is shunted around the coil of the instrument.

If the range of a 1-milliampere movement is extended to 50 milliamperes, 1 milliampere flows through the coil and 49 milliamperes will flow through the shunt when the instrument is measuring a current of 50 milliamperes.

In any instrument supplied with a shunt, the current through the coil for full-scale deflection of the pointer is the amount of current necessary to cause the pointer to have a full-scale deflection before the shunt was added. Hence, the current through the coil of a 1-milliampere movement is 1 milliampere for the full scale deflection of the pointer regardless of what the printed scale may show at that point. If the instrument is provided with a shunt that will have 99 milliamperes flowing through it when the pointer is at full scale, the range of the milliammeter will be 100 milliamperes and the scale will be graduated in even steps from zero to 100 milliamperes.

161. Shunts. The circuit of Fig. 116 shows the coil of a D'Arsonval movement paralleled with a shunting resistance.

In this circuit.

R_c = resistance of instrument coil

R_s = resistance of shunt

I = total current in amperes

I_s = current in amperes through the shunt

I_c = current in amperes through the instrument coil

From an inspection of the circuit of Fig. 116:

$$E = I_c R_c \tag{78}$$

$$I_s = I - I_c \tag{79}$$

and

$$R_s = \frac{E}{I_s}$$

(80) $$R_s = \frac{I_c R_c}{I - I_c}$$

(81) $$R_c = \frac{R_s(I - I_c)}{I_c}$$

Example:

Suppose that the internal resistance of a 1-milliampere milliammeter is 50 ohms. What shunt resistance will be required to extend the full-scale range to 10 milliamperes?

Solution:

Substituting in equation 80:

$$R_s = \frac{I_c R_c}{I - I_c} = \frac{\cancel{10^{-3}} \times 1 \times 50}{\cancel{10^{-3}}(10 - 1)} = \frac{50}{9} = 5.56 \text{ ohms}$$

In designing shunts, one should select a material that does not change its resistance appreciably with a change of current through it or with a change of temperature (caused by the flow of current or from atmospheric changes in temperature).

The shunt is usually incorporated into the design of the instrument and is mounted inside the instrument case along with the other parts of the instrument.

It is not only possible to extend the range of a low milliampere movement to higher milliampere ranges but it is also possible to select shunts such that the range may be extended to read in amperes.

162. Direct-current Voltmeter. It was stated in sections 155 and 158 that voltmeters are placed across or in parallel with the source of voltage to be measured. It was further stated in section 158 that *a voltmeter is in reality a milliammeter* that is calibrated and operated in such a way that its scale *indicates voltage.*

A voltmeter consists of *a milliammeter* and *an accompanying series resistor* which limits the current through the milliammeter to its full-scale value when a predetermined voltage (shown on the scale as the maximum or full-scale value) is applied to the voltmeter (including movement and resistor).

163. Multipliers. The series resistor of a voltmeter is called a *multiplier*. For low range voltmeters, this resistor is usually built into the instrument case, as is the practice with milliammeter shunts. When the range extends to several thousand volts, the multiplier may be as large or larger than the instrument and must be used externally.

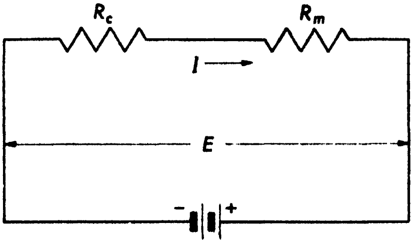


Fig. 117a. Voltmeter circuit showing instrument coil resistance R_c and multiplier resistance R_m.

From the circuit of Fig. 117a, the total resistance

$$R = R_c + R_m \tag{82}$$

where

R_c = instrument coil resistance

R_m = multiplier resistance

$$I = \frac{E}{R}$$

$$I = \frac{E}{R_c + R_m} \tag{83}$$

where

I = voltmeter current

E = maximum scale reading of voltmeter

$$R = \frac{E}{I}$$

$$R_c + R_m = \frac{E}{I}$$

$$R_m = \frac{E}{I} - R_c \tag{84}$$

Example:

Suppose that it is desired to use a 1-milliampere movement whose resistance is 50 ohms for a voltmeter with a full-scale range of 500 volts. Determine the resistance of the multiplier that will be needed.

Solution:

Substituting in equation 84:

$$\begin{aligned} R_m &= \frac{E}{I} - R_c \\ &= \frac{500}{.001} - 50 \\ &= 500{,}000 - 50 \\ &= 499{,}950 \text{ ohms} \end{aligned}$$

The lower the current necessary to operate a movement, the less current required by the voltmeter from the voltage source being measured. As the current requirement in a voltmeter design *decreases*, the multiplier resistance *increases*. It would be ideal if a voltmeter required no current at all, but this is not possible with a D'Arsonval type instrument because current, however small, is necessary to cause the pointer to deflect.

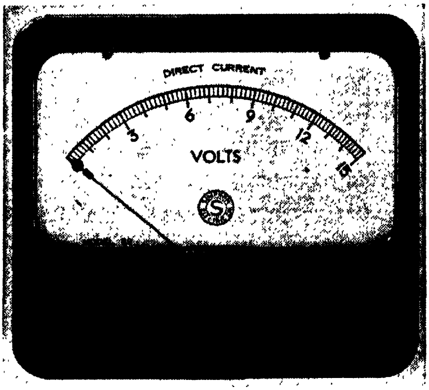


Fig. 117b. Direct-current voltmeter. (Courtesy, Simpson Electric Company.)

An indication of the merit of a voltmeter, as far as low current for operation is concerned, is its *ohms-per-volt* rating. The ohms-per-volt rating is determined from equation 85.

$$\text{ohms per volt} = \frac{R_c + R_m}{\text{full-scale voltage } E} \tag{85}$$

164. Care of D'Arsonval Type Instruments. One should always be certain that a d-c D'Arsonval type instrument is properly polarized to avoid having the pointer slam against the left-hand pointer stop which will happen if current is passed through the instrument in the opposite direction.

This, or any other type instrument that has a permanent magnet, should not be mounted or stored near the magnetic field from a permanent-magnet speaker or other magnetic device. This is very likely to weaken the magnetic field of the instrument magnet and change the calibration of the instrument.

Permanent-magnet type instruments should *never be mounted on a steel or other magnetic type panel* unless especially calibrated for such mounting, otherwise the readings may be as much as 50 per cent in error.

The D'Arsonval type instrument is rugged and will withstand normal handling. Dropping is almost sure to necessitate returning the instrument to the factory for repairs. An instrument may survive a 25 per cent overload or even a 100 per cent overload without burning out the coil, but excessive current through the coil heats the control springs with a resultant loss of spring action from the spring steel and change of calibration. Excessive current also causes the pointer to slam against the right-hand pointer stop pin which may break or severely bend the pointer.

D'Arsonval-movement instruments are manufactured with various accuracies. Milliammeters and voltmeters for general use have an accuracy of approximately 2 per cent. In many radio circuits, instruments are used more for an indication of current or voltage rather than for an exact measurement of the actual current or voltage involved. Laboratory standard instruments have an accuracy as high as 0.1 per cent.

165. Galvanometer. A type of D'Arsonval instrument not yet mentioned is the *galvanometer*. A galvanometer is used entirely as a current-indicating device. The pointer usually, but not always, has its zero or "at rest" position at the center of the scale, and the direction of deflection is determined by the direction of the current through the instrument. The scale is *not calibrated* but is graduated in even divisions in both directions from the center point.

The instrument is usually provided with two shunts to provide three steps of sensitivity (including use of movement without any shunt).

A galvanometer is used with a Wheatstone bridge (see sections 166 and 167) as a current null-indicating device and with other laboratory devices where current indicating instruments are required.

Exercise 51:

1. What shunt resistance will be required to extend the range of a 1-milliampere movement to 50 milliamperes full scale if the internal resistance of the movement is 50 ohms?
2. Repeat problem 1 for a 1-milliampere movement with 27 ohms internal resistance.
3. A 50-microampere movement was used to construct a 0–10-milliampere milliammeter. What is the internal resistance of the movement if a shunt of ½-ohm resistance was used?
4. What multiplier resistance will be required to construct a 0–100-volt voltmeter from a 0–1-milliampere movement of 50 ohms resistance? What is the ohms-per-volt rating of the instrument?
5. What is the internal resistance of a 0–5-milliampere milliammeter which requires a 99,995-ohm multiplier to convert it into a 0–500-volt voltmeter? What is the ohms-per-volt-rating?
6. Which type voltmeter gives the higher ohms-per-volt rating: (*a*) high current movement, or (*b*) low current movement?

166. Measurement of Resistance. One of the simplest methods of measuring resistance is by means of a *comparison with a known standard.* For instance, the unknown width of a table may be determined by the use of a 1-foot rule, a yardstick, or a tape measure, all of which are graduated in inches since the inch is a standard unit of measure.

An unknown resistance in a circuit with an unknown voltage source may be determined by a comparison with a known resistance if the current is known. The value of the unknown resistance is determined by substituting known resistances of various sizes in the circuit in the place of the unknown resistor until the same current flows in the circuit. From Ohm's law, the resistance value of the known resistor required to cause the same current flow as the unknown resistor must be equal to the resistance value of the unknown resistor.

A more common device used to determine an unknown resistance by means of a comparison to a standard is the *Wheatstone bridge.* The Wheatsone bridge provides a very accurate method of determining resistance, but it is used more as a laboratory instrument than as a servicing device.

Another device for determining resistance is the ohmmeter. It is less complicated to operate than the Wheatstone bridge, and gives the desired answer that can be read directly from a printed scale graduated in ohms. The ohmmeter is less accurate than the Wheatstone bridge, but its ease of operation and adaptability to servicing all types of radio equipment have made its use universal.

167. Wheatstone Bridge. A circuit of the Wheatstone bridge is shown in Fig. 118. From this circuit:

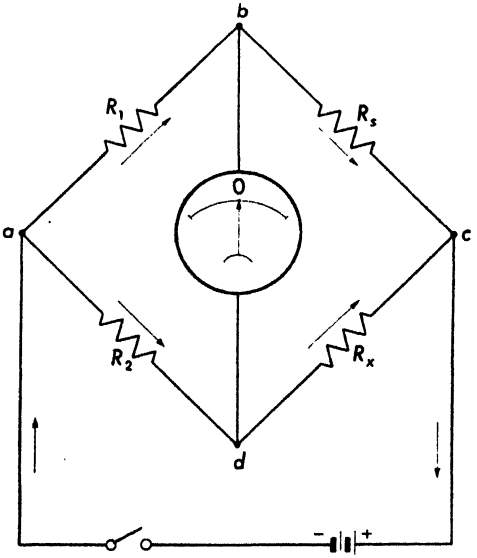


Fig. 118. Circuit of the Wheatstone Bridge.

$$\frac{R_1}{R_2} = \frac{R_s}{R_x}$$

(86) $$R_x = \frac{R_s R_2}{R_1}$$

where the values of resistors R_1 and R_2 are selected to give a suitable balance of the bridge which is determined by the value of the range of unknown resistance to be measured.

R_s = standard resistor of a known resistance. This resistor is variable in resistance in both large and small steps which are also known.

R_x = resistor of unknown resistance

From the circuit of Fig. 118, the value of the unknown resistance R_x is determined as follows:

1. The unknown resistor R_x is connected between terminals *c* and *d* of the bridge.
2. Switch is closed.
3. The value of the standard resistor R_s is varied until a balance is obtained (when the galvanometer pointer is at the zero point).
4. The value of the unknown resistance is determined from equation 86.

$$R_x = R_s\frac{R_2}{R_1}$$

where the value of $\frac{R_2}{R_1}$ is a ratio by which the value of R_s (when bridge is balanced) must be multiplied in order to get the value of the unknown resistor R_x. The $\frac{R_2}{R_1}$ ratio is adjustable in steps which are shown on the face of the bridge.

168. Ohmmeter. The circuit of Fig. 119 is the circuit of a simple ohmmeter. It consists of a battery, a variable resistor, and a low-

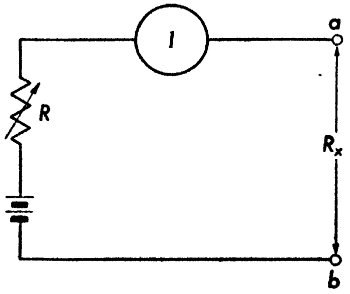


Fig. 119. Circuit of a simple ohmmeter.

current range instrument. The instrument should be one of 1 milliampere or less at full scale and is calibrated directly in ohms instead of current. The purpose of the variable resistor is to limit the current through the instrument to its full-scale value.

With no external resistance at R_x the terminals *a* and *b* are connected directly together and the variable resistor *R* is adjusted for a full-scale deflection of the instrument, corresponding to the zero resistance point on the scale of Fig. 120. This adjustment is considered as a calibration of the ohmmeter and should be checked from time to time as the battery deteriorates and its voltage drops.

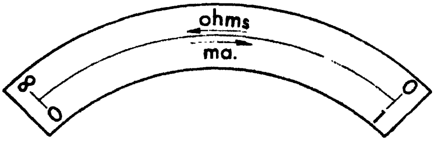


Fig. 120. Showing how the scale of a 0–1-ma. milliammeter may be calibrated in ohms for the ohmmeter circuit of Fig. 119.

If an unknown resistor R_x is inserted in the circuit at points *a* and *b* of Fig. 119 (with battery voltage and value of *R* unchanged from calibration point), the instrument pointer will be at a position less than full-scale deflection since the total resistance of the circuit was increased.

Suppose that a 1½-volt flashlight cell is used in connection with a 0–1-milliampere milliammeter. From Ohm's law the value of the variable resistor *R* for full-scale deflection should be:

$$R = \frac{E}{I} = \frac{1.5}{.001} = 1500 \text{ ohms}$$

Now suppose that an unknown resistor is placed in the circuit and the pointer deflects to .6 milliampere.

$$R = \frac{1.5}{.0006} = 2{,}500 \text{ ohms, total circuit resistance}$$

2500 − 1500 = 1,000 ohms, which is the value of the unknown resistance. A value of 1,000 ohms may be placed on the instrument scale (Fig. 120) opposite to the .6-milliampere point. The entire scale may be graduated in ohms in this manner.

It is obvious that a very small current variation at the low end of the scale would indicate a very large resistance change. For this reason the low end of the scale of such an instrument is not very accurate or useful. Commercial type instruments use a somewhat different and more complicated circuit and several scale ranges which make it possible to measure much higher values of resistance and with a greater accuracy than is possible with an instrument using the circuit of Fig. 119.

169. Volt-ohmmeter. It is desirable in servicing radio equipment to have an instrument or testing device that is portable and currents, voltages, and resistances can all be measured with the one instrument. Such devices are manufactured under various trade names but are popularly called *volt-ohmmeters*. Such a device is shown in Fig. 121.

Exercise 52:

1. A bridge is used to measure an unknown resistance. The value of the unknown resistance is found to be 50 ohms. The standard resistor R_s of the bridge is a precision wire-wound 500-ohm resistor. What is the $\frac{R_2}{R_1}$ ratio?

2. Complete the calibration of the ohmmeter portion of the scale of Fig. 120 from data in section 168 for 500; 1,000; 3,000; 5,000; 10,000; 30,000; 50,000; 100,000; and 500,000 ohms.

170. Measurement of Power. Direct-current power in radio circuits is usually determined by the use of voltmeters and milliammeters, the power in watts being determined from the product of the voltage and current readings.

Instruments using the volt-ampere method of measuring power are manufactured for the measurement of d-c power and are called *wattmeters*.

Instruments used to measure and record the amount of d-c power or a-c power used by dwellings and commercial establishments are called *meters*.

171. Alternating-current Measurements. It will be recalled from a study of Chap. X that alternating current alternates in direction. If a d-c instrument, such as described in the preceding sections of this chapter, is placed in an a-c circuit, it will not indicate the proper current or voltage, as the case may be. The positive half of each cycle tends to cause the pointer to deflect in one direction, while the negative half of the cycle tends to cause the pointer to deflect in the oppo-

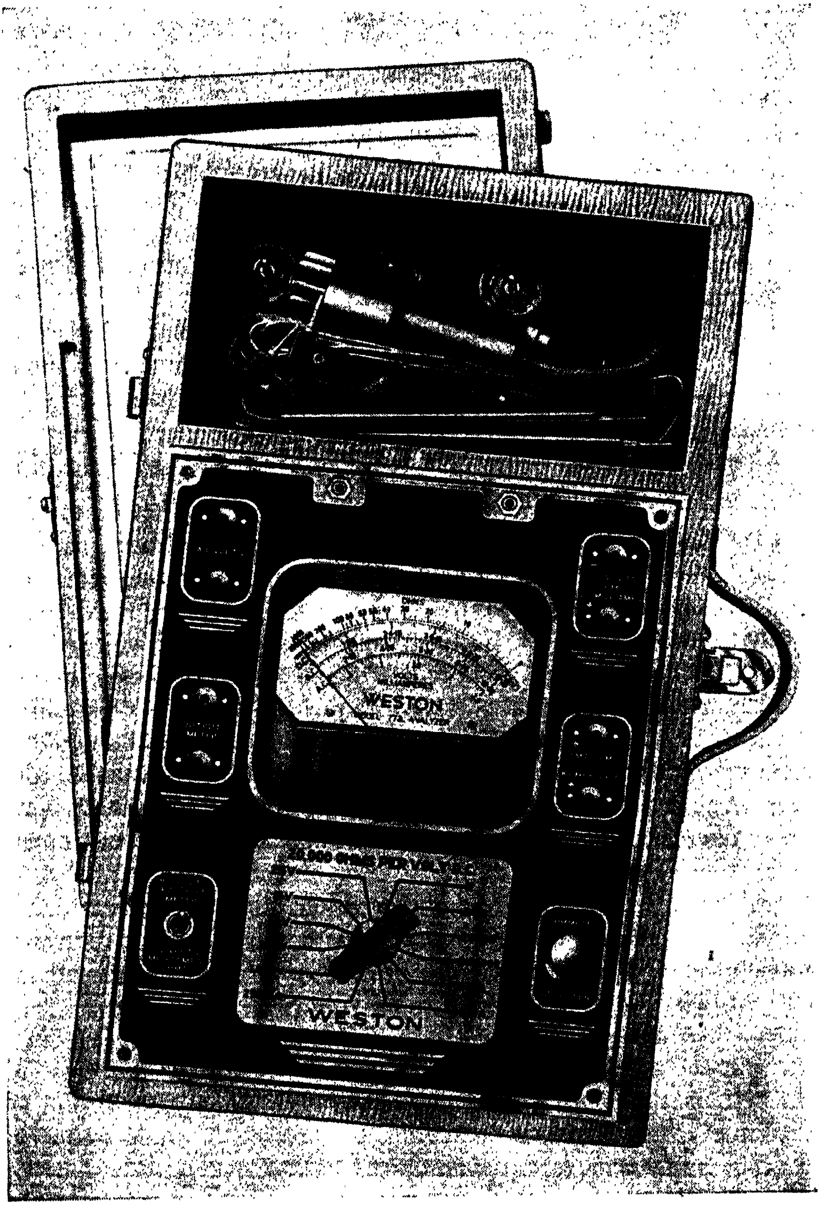


Fig. 121. Weston 772 Analyzer for measuring various currents, voltages, and resistances. (Courtesy, Weston Electrical Instrument Company.)

site direction. These two opposite actions on the pointer would cause the pointer to shift from one side to the other if the frequency of the alternations were low. At the power-line frequency of 60 cycles the pointer would stand practically motionless.

From the above discussion, it is evident that d-c instruments read the average value since the torque acting on the movement, at any instant, is directly proportional to the current flowing through the movement coil at the same instant. The average value of one cycle of alternating current is zero.

The D'Arsonval movement may be used to measure alternating current only if the a-c wave is rectified (changed to pulsating direct current) or from its heating effect in specially constructed thermal a-c instruments.

Alternating-current instruments discussed in the following sections read *effective* current and *effective* voltage.

172. The Effect of Frequency. The type of instrument used to measure alternating currents and voltages depends on the frequency. The *dynamometer* and *iron-vane* type instruments are used extensively to measure 60-cycle power-frequency currents and voltages but are not suitable for measuring radio-frequency voltages and currents.

Accurate measurement of radio-frequency voltages is extremely difficult. Low values of radio-frequency voltages as encountered in receiver circuits are usually measured by means of a vacuum-tube voltmeter. A D'Arsonval instrument is used in the *vacuum-tube* voltmeter circuit but the r-f current does not flow through the instrument coil.

Instruments that measure r-f voltages directly have been built but are not very satisfactory. Fortunately, with work in connection with transmitter and antenna circuits, one is more concerned with r-f currents and resistances than with r-f voltages.

Hot-wire and *thermocouple* type (D'Arsonval movement with a thermocouple unit) instruments are used to measure r-f currents and the *rectifier* type (D'Arsonval movement with an a-c rectifier unit) is used to measure frequencies extending up through the audio-frequency band of frequencies (approximately 20–17,000 cycles).

173. Electrodynamometer Type Instrument. In certain respects, the electrodynamometer type instrument is similar to the D'Arsonval type instrument. The pointer is attached to a moving coil, as in the D'Arsonval type instrument, but two coils replace the permanent magnet to be found in the D'Arsonval type instrument.

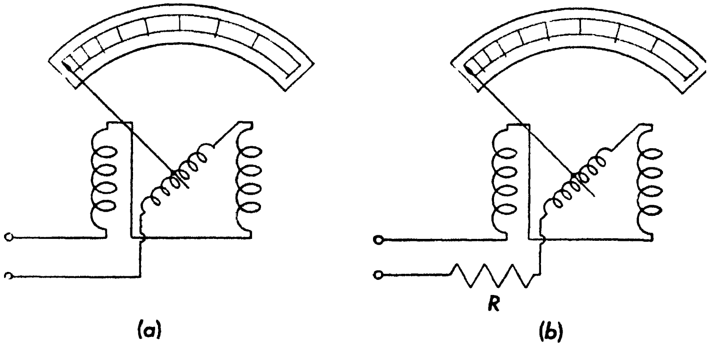


Fig. 122. Electrodynamometer type instruments. *a.* Ammeter. *b.* Voltmeter.

A sketch of the essential parts of the electrodynamometer type instrument is given in Fig. 122. The movement coil rotates in the magnetic field created by the current flowing through the two stationary coils. The two stationary coils are placed in positions relative to the movable coil approximately as shown in Fig. 122 and are wound in such a direction that their fields are additive.

This type instrument can be used to measure direct current as well as alternating current.

The electrodynamometer type instrument has a moving coil similar to the D'Arsonval type instrument. The moving coil in each of these instrument types can carry only a fractional portion of an ampere since the moving coil must be as light as possible, and because the spiral springs serving as flexible connecting leads to the coil cannot carry a very large current without being damaged.

It is a comparatively simple matter to equip the D'Arsonval type instrument with suitable shunts. It is possible to provide satisfactory shunts for electrodynamometer type instruments when the current range is moderate. The problems involved in providing proper shunts for electrodynamometer type instruments increase as the current range is increased. For this reason, the electrodynamometer type instrument is not used extensively for measuring medium or high values of alternating currents.

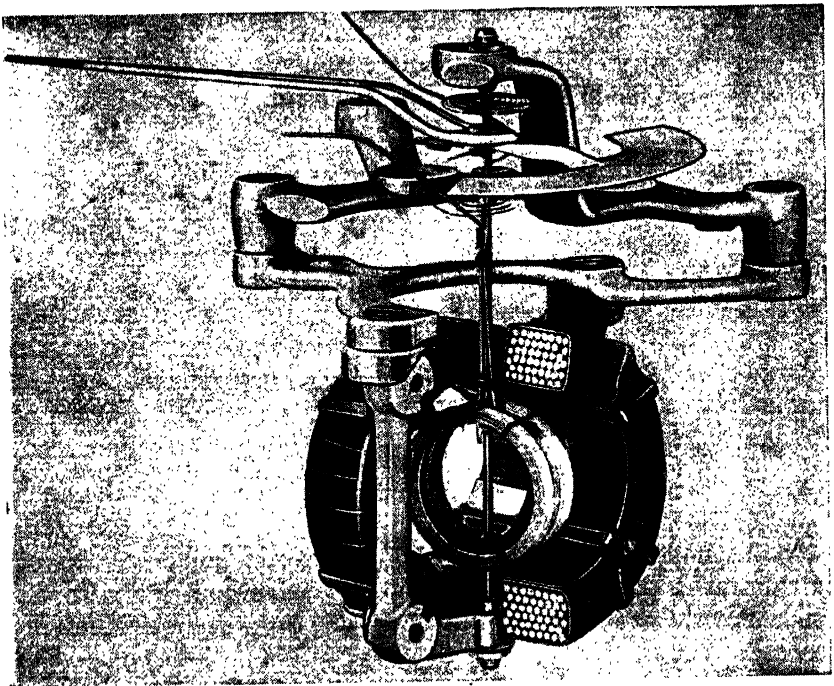


Fig. 123 a. Coil and pointer element for General Electric type P-3 electrodynamometer voltmeter shown in Fig. 123 *b*. A portion of one of the outside coils has been cut away to give a better view of the center assembly. (Courtesy, General Electric Company.)

174. Iron-vane Type Instruments. An instrument that is especially adaptable to commercial a-c measurements is the *iron-vane* type instrument. It is rugged, simple of construction, having no moving current coil, and reasonably accurate.

A sketch of the iron-vane type instrument is shown in Fig. 124. Actual commercial instruments may differ from this sketch somewhat, depending upon the manufacturer, but the principle of operation is the same.

The current to be measured flows through the fixed coil, creating a magnetic field around the coil. The lines of force from this field pass through the two soft iron bars (*a* and *b*) which become magnetized and stay magnetized as long as current is flowing through the coil. The lines of force through bar *a* are in the same direction as the lines of force through bar *b;* therefore, the magnetic poles of *a* and *b* im-

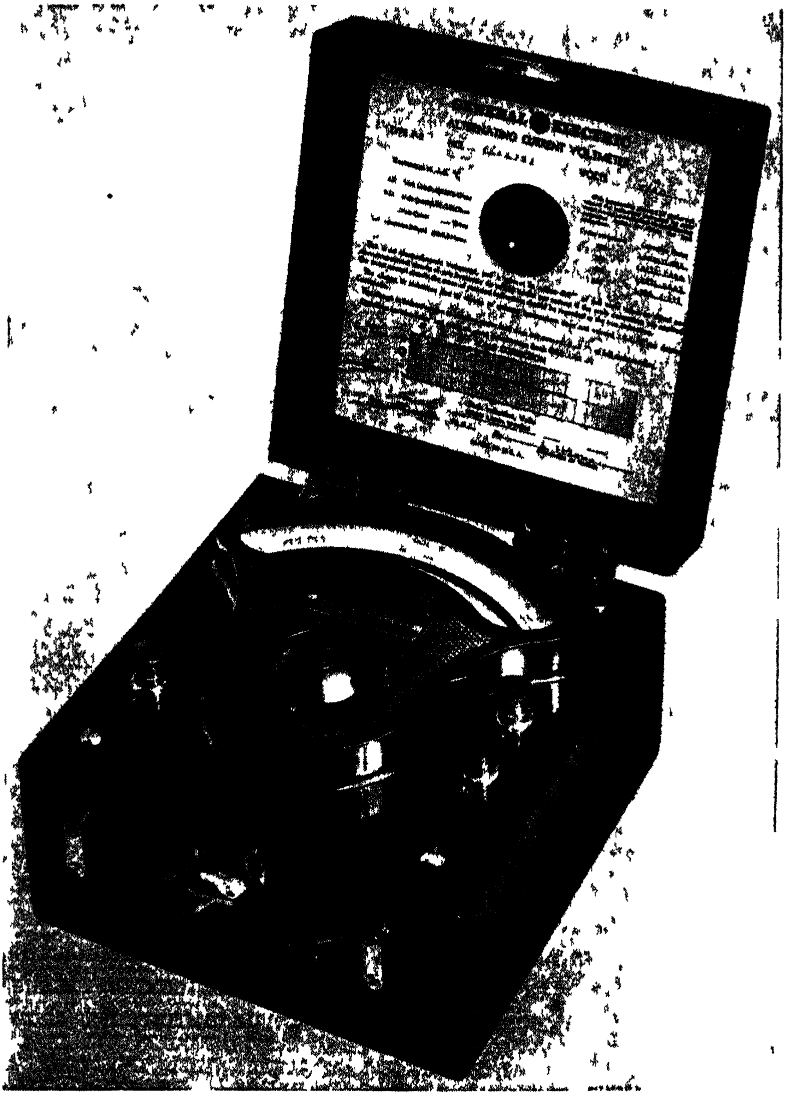


Fig. 123 b. General Electric type P-3 electrodynamometer voltmeter. (Courtesy, General Electric Company.)

mediately adjacent to each other are of like polarity. This causes the two bar magnets to repel each other. Since the current flowing through the coil is alternating, the direction of the lines of force is in the opposite direction during the negative half of the cycle. This reverses the polarity of the magnets, but since the polarity of both are changed they will still oppose each other.

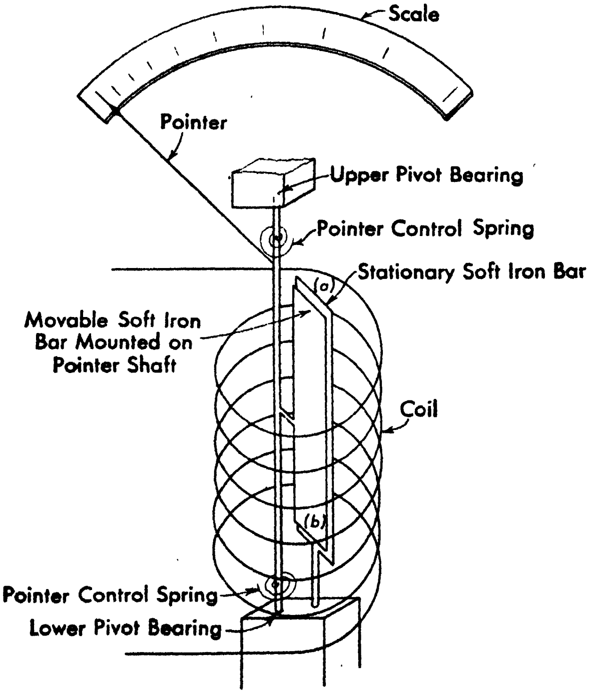


Fig. 124. Iron-vane type instrument.

Bar *b* is free to move on a pivot as shown in Fig. 124. An indicating pointer and proper coil control springs are also mounted on the shaft assembly of bar *b*. A commercial iron-vane type instrument is shown in Fig. 125.

175. Hot-wire Type Instrument. The *hot-wire* type instrument was used extensively for r-f current measurements in the early days of radio. This type of instrument utilizes the expansion of a heated wire to operate a pointer across a graduated scale. The expanding wire is

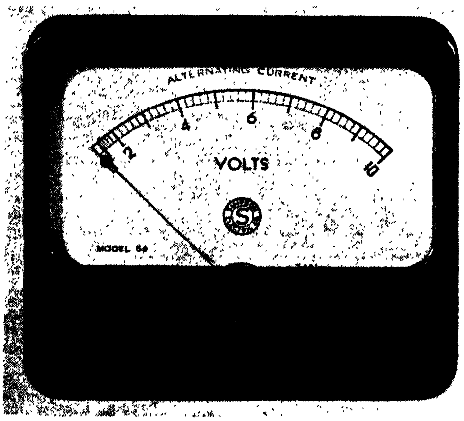


Fig. 125. Commercial iron-vane type alternating-current voltmeter. (Courtesy, Simpson Electric Company.)

heated by passing the r-f current, under measurement, through it. Obviously, such an instrument has many objectionable features.

176. Thermocouple Type Instrument. Another type of thermal instrument is the *thermocouple.* Like the hot-wire type instrument, its operation depends upon the heating effect of the current being measured.

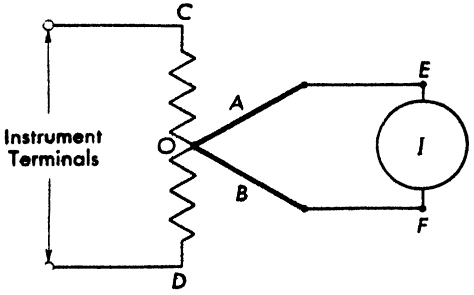
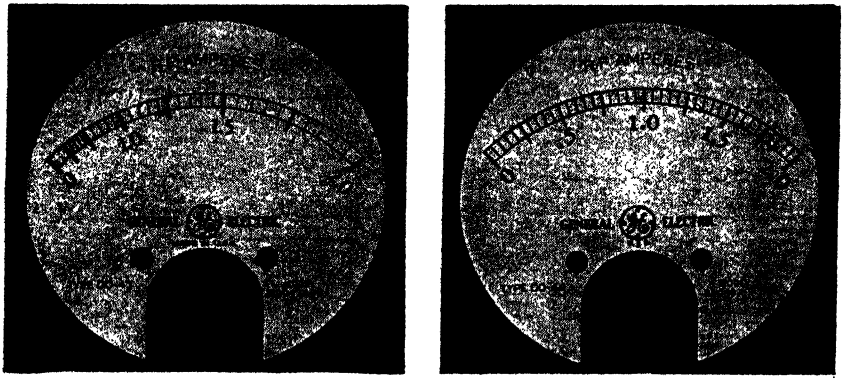


Fig. 126. **Sketch showing the essential parts of a thermocouple instrument.**

The essential parts of a thermocouple type instrument are shown in the sketch of Fig. 126. Two unlike metal wires, *A* and *B*, form a *junction* at point O. The current being measured flows through the resistance wire *C—D*, heating the junction point O, inducing a low d-c voltage between the opposite ends of the unlike metal wires at points *E* and *F*. The voltage between points *E* and *F* is low, and the available d-c current is low; therefore, the D'Arsonval type d-c instrument, *I*, connected in series with the thermocouple circuit, has a low current range.

177. Rectifier Type Instrument. A type of instrument used extensively by broadcast stations, telephone companies, and other communications companies is the *rectifier*. Audio-frequency alternating currents, such as those produced by speech and music, are changed from alternating to pulsating direct current by means of a copper oxide rectifier unit. This current is, in turn, measured with a sensitive D'Arsonvai type instrument.

178. Alternating-current Instrument Scales. The scale divisions of a-c instruments covered in the previous sections are not uniform as is the case with d-c D'Arsonval type instruments. The scale divisions of some of the available rectifier type instruments are nearly linear, while the divisions of the other type a-c instruments increase in width across the scale as shown in Figs. 125 and 127(*a*).



(*a*) (*b*)

Fig. 127. Commercial thermocouple type radio frequency ammeter scales. *a.* Typical crowded scale of a regular thermocouple instrument. *b.* Typical expanded type scale of thermocouple instrument of special design. (Courtesy, General Electric Company.)

Hot-wire and thermocouple type instruments operate from the *heating effect* of the current which increases as the *square of the current*. This causes the deflection of the pointer to tend to increase as the square of the current. Such a scale is crowded at the lower end and spread out at the upper end. Expanded scale thermocouple-type instruments have been developed to offset this objectionable feature. Such an instrument does not use a D'Arsonval movement with a uniform air gap as shown in Fig. 113 but one with a nonuniform air gap which is obtained by slanting the pole faces. A comparison of the standard scale and the expanded scale is shown in Fig. 127. The pointer deflection of an electrodynamometer type instrument also tends to increase as the square of the current since the same current flows through both fixed coils (Fig. 122) whose fields are additive.

An instrument whose pointer deflects as the square of the current flowing through the instrument has a scale that is crowded at the lower part of the scale and is therefore not so accurate on the lower part of the scale as on the higher part of the scale. Since the pointer deflection increases as the *square of the current,* the scale values increase as the *square root* of the *pointer deflection.* Therefore, it is possible to *square each current value* as shown in Fig. 128 and obtain a corresponding scale that is uniform. Such a scale is called a *current-squared* scale. An instrument supplied with such a scale is called a *current-squared instrument.* The corresponding actual current scale as shown in Fig. 128 is usually not supplied with such instruments, and it is not possible to take the square root of the values on the uniform current-squared scale to obtain the actual current values unless the full-scale current range of the instrument is known. For that reason, a current-squared instrument is usually called a *current-squared galvanometer.*

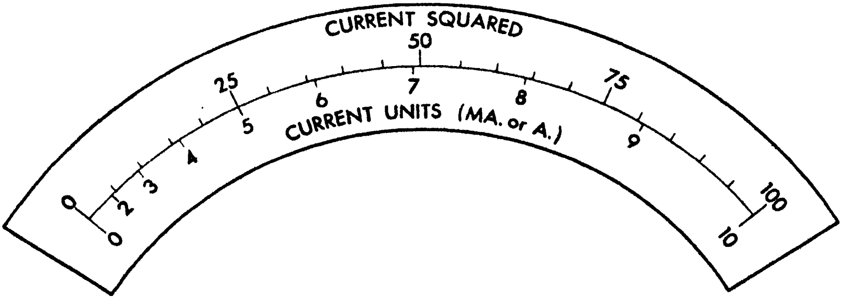


Fig. 128. Showing the relation between the actual current scale and the current squared scale.

Changing the scale alone of an instrument does not increase its sensitivity; therefore, placing a current-squared scale on an instrument does not change its sensitivity.

179. Conclusions. All the a-c instruments covered in this text will also indicate on direct current, but this does not necessarily mean that the a-c scale calibration will hold true for direct current.

The electrodynamometer principle is often used in the manufacture of d-c instruments, but the iron-vane, hot-wire, and thermocouple types are not used to measure direct current for obvious reasons. The average thermocouple type instrument is usually calibrated at 60 cycles. Considerable errors are experienced with such instruments when measuring r-f currents. The better grade thermocouple type instrument such as used by the modern broadcast station is calibrated for the operating frequency.

180. Other Electrical Measuring Equipment. Many devices are manufactured for use in servicing, testing, and operating radio equipment. They use one or more instruments covered in this text as well as tubes and other apparatus not included.

Of the many devices, the tube checker, audio-frequency oscillator, radio-frequency oscillator, and *cathode-ray oscilloscope* are the most common.

REVIEW QUESTIONS

1. What are electrical instruments?

2. Give the principal working parts and their function of a D'Arsonval type instrument.

3. Outline the principal differences between a current measuring instrument and a voltage measuring instrument.

4. What are the following: (*a*) A Milliammeter; (*b*) Shunt; (*c*) Voltmeter; (*d*) Multiplier; (*e*) Galvanometer?

5. Give and explain two methods used to measure resistance.

6. Why will a D'Arsonval type d-c milliammeter not measure alternating current?

7. Would 500 ma. alternating current damage a 100 ma. d-c D'Arsonval type milliammeter? Explain answer.

8. Why are the scales of regular a-c type instruments not linear like the scales of d-c type instruments?

9. What is the difference between meters and instruments?

10. Explain the principle of operation of the iron-vane type a-c instrument.

11. Why are electrodynamometer type and iron-vane type a-c instruments not suitable for measuring radio frequency currents?

12. What type instruments do we use to measure r-f currents?

13. What is a current-squared scale?

14. How does the expanded scale thermocouple type instrument differ from the regular thermocouple type instrument?

15. What are the advantages of having expanded scales for r-f current instruments?

ANSWERS

Exercise 51, page 202:

1. 1.02 ohms
2. .551 ohms
3. 99.5 ohms
4. 99,950 ohms; 1,000 ohms per volt
5. 5 ohms; 200 ohms per volt
6. Low-current movement type

Exercise 52, page 206:

1. 1/10

CHAPTER XII *Inductance*

The list of early scientists who contributed materially to present-day knowledge of radio would not be complete without the names of Lenz and Henry. These men made extensive studies of induced currents and their directions and the *inductance properties* of circuits.

181. Lenz's Law. In Chap. IX on Electromagnetism it was shown that current flowing through a coil induced a magnetic field about the coil (sections 104 and 107). If the circuit is broken, the magnetic field is said *to collapse*. If the voltage source is removed from the circuit such as in Fig. 129 when switch *S* is closed, the magnetic field surrounding the coil will collapse, causing a current to continue to flow and in the same direction through the coil. The intensity of the magnetic field about the coil is greatest at the instant the switch *S* is closed, and the value of the current flowing through the coil due to the magnetic energy surrounding it is greatest at this instant. The value of the current decreases to zero as the intensity of the magnetic field decreases to zero.

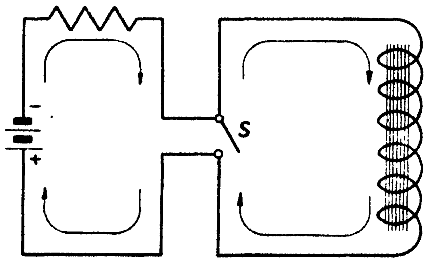


Fig. 129. Circuit showing how the magnetic energy surrounding a coil tends to maintain the current and in the same direction when the battery voltage is removed by closing switch *S*.

If a resistor is inserted in the circuit between one end of the coil and one terminal of the switch, it will limit the flow of current such that a longer time will be required for the induced current to decrease to zero when the switch is closed.

The action of the magnetic field about a coil with current flowing through it tends to *maintain* the *current flow.* The induced current flow due to the collapse of the magnetic field about the coil of Fig. 129 when switch *S* is closed tends to create a magnetic field about the conductor just as did the current flow previously from the battery. The polarity of this magnetic field and the direction of the lines of force are the same as for the battery current *since the direction of the current is the same.* This action, naturally, tends to oppose any change in the existing field.

Lenz formulated a law of these phenomena which in effect says: *The direction of an induced current is such that its magnetic field opposes any change in the magnetic field which produced it.*

182. Inductance. The action of the magnetic-field energy, surrounding a coil with current flowing through it, in an electrical circuit associated with it, is similar to the action of a flywheel on an engine.

If we try to stop the flywheel of an engine suddenly, there is a strong *inertia reaction* which tends to keep the flywheel turning. If we tried to speed up the flywheel suddenly we would meet with the same opposition. If the battery current of the circuit in Fig. 129 is suddenly stopped by closing switch *S*, the magnetic-field energy surrounding the coil tends to keep the current flowing in the circuit through the coil. This is called the *inductance property* of a circuit, which not only opposes a *decrease* but an *increase* in current as well.

Inductance may be defined as the property of a circuit which tends to prevent any change in the current flowing in the circuit.

The inductance property of a circuit is divided into two classes: *self-inductance* and *mutual inductance.*

183. Self-inductance. It was shown in section 107 that a stronger magnetic field may be obtained if a conductor with current flowing through it is formed into a coil. Since forming a conductor into a coil increases the magnetic field, without an increase of current, the inductance property of the conductor is also increased.

184. Determination of Self-inductance. It should be realized that inductance is a property of a conductor, coil, or circuit, and that the property exists whether current happens to be flowing at the moment or not.

The self-inductance of a straight conductor is slight in comparison to the self-inductance that the same conductor would have if it were formed into a coil. The self-inductance that the conductor will have when formed into a coil can vary over considerable limits depending upon the *kind of coil* formed. Such factors as size of wire, number of turns, length and diameter of coil all affect the self-inductance of the coil.

Many formulas have been devised for calculating the self-inductance of coils of many shapes and sizes to care for the many radio receiver, transmitter, and other applications. One point of practical interest is that these formulas show that the self-inductance varies as the *square of the turns*. If the number of turns is *doubled*, the resultant self-inductance is *four* times greater. Likewise, if the number of turns is *halved*, the resultant inductance is now *one-fourth* the original value. The above statements are based on the assumption that all magnetic lines of force around each turn also encircle all other turns. (Further explanation given in section 186.)

In practice, the student will find that the self-inductance of a coil is simply referred to as *inductance*.

185. Unit of Inductance. The unit of inductance is the *henry* and was named in honor of the noted American experimenter, Joseph Henry, who discovered electromagnetic induction. Inductance is denoted by the letter *L* and the henry is defined as *the inductance of a circuit when a current change of 1 ampere causes an induced voltage of 1 volt.*

Power-supply filter chokes for receivers and transmitters contain an iron core and usually range in value from 5 to 30 henries of inductance. Antenna coils and tuning coils of receivers and transmitters are much lower in value; usually a fractional portion of a henry. Such values of inductance are referred to as *millihenries* (1,000MH. = 1H.) or *microhenries* (1,000,000 μH. = 1H.).

Several typical inductances are shown in Figs. 130*a* and 130*b*. An iron core increases the linkage of the magnetic lines of force between the turns, thereby increasing the inductance of a coil. Several lines drawn inside a coil as shown in Fig. 129 indicate a coil with an iron

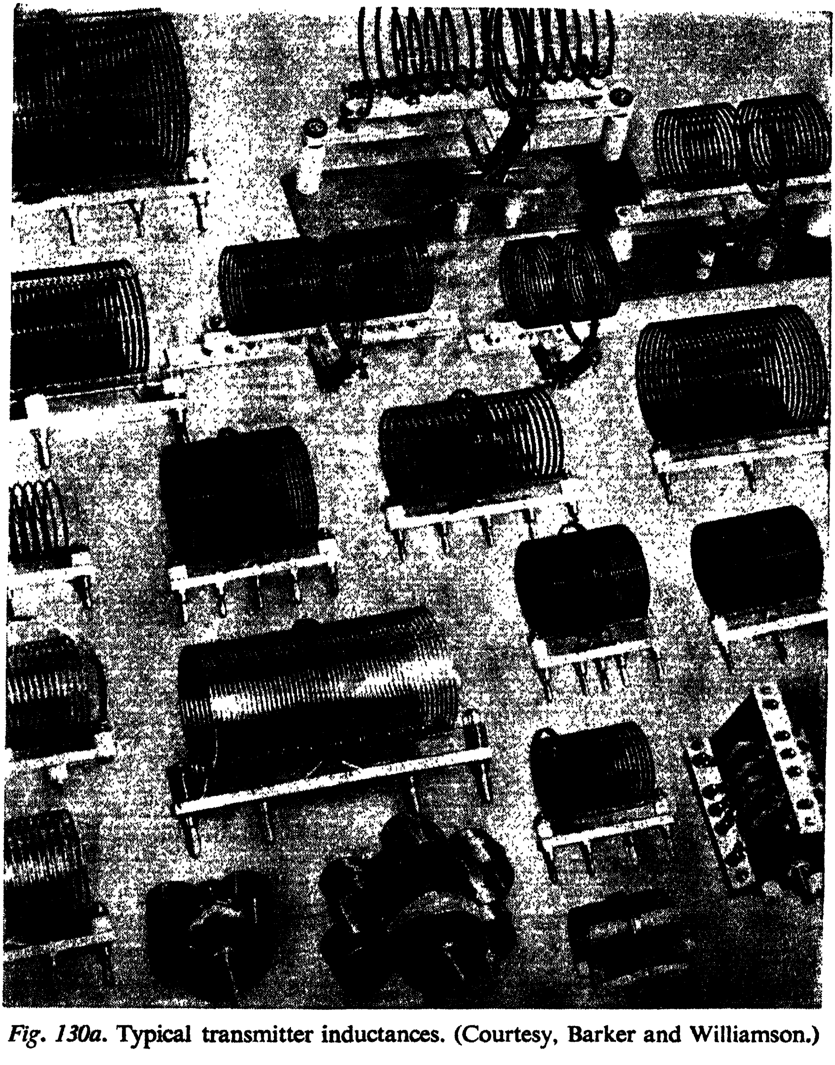


Fig. 130a. Typical transmitter inductances. (Courtesy, Barker and Williamson.)

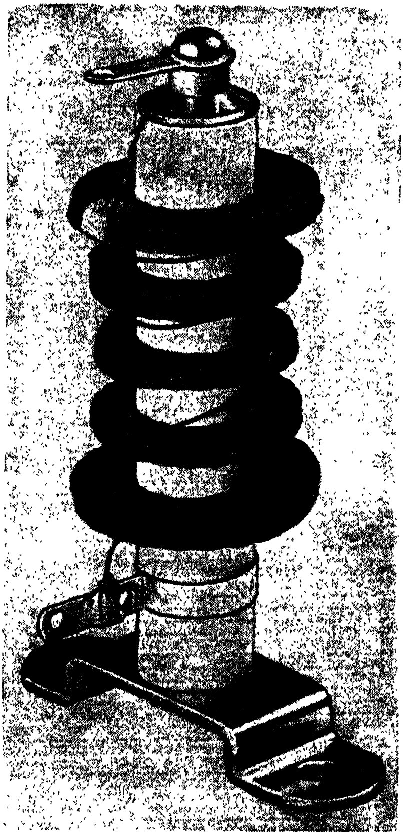


Fig. 130b. Typical inductance used in transmitter circuits to oppose the flow of radio-frequency currents (commonly called radio-frequency choke coil). (Courtesy, National Company.)

core. (The lines may be drawn just outside the coil instead of through the center of the coil.)

186. Mutual Inductance. If two coils are placed end to end, as in Fig. 131*a* and connected as shown so that the winding of one is a continuation of the other, they are said to be connected *series-aiding*. If the two coils are identical, we now have one coil with double the original number of turns. From formulas for calculating the inductances of coils referred to in section 184, the total inductance should be four times that of a single coil. Only is this true if all magnetic lines of force of each coil encircle all turns of the other coil.

Two coils which are placed in close proximity to each other are said to be *coupled* (magnetically) if part of the magnetic lines of force of each coil encircle the other. The degree to which the magnetic lines of force from one coil encircle or cut all turns of the other coil is called the *coefficient of coupling, k.*

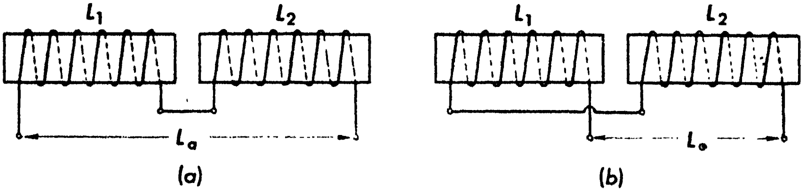


Fig. 131. a. Two coils connected series-aiding. *b.* Two coils connected series-opposing.

The added inductance gained by coupling two coils together is called *mutual inductance* and is designated by the letter M.

The two coils in Fig. 131*b* are connected *series-opposing.*

187. Determination of Mutual Inductance. The mutual inductance between two coils depends upon several factors. The most important factors are their proximity, permeability of coupling medium (air, iron), and the physical properties of the coils.

Since there is equal coupling from each coil to the other, the total mutual inductance due to coupling two coils together is $2M$. The total inductance L_a for two coils connected series-aiding is:

$$L_a = L_1 + L_2 + 2M \tag{87}$$

and the total inductance L_o for two coils connected series-opposing is:

$$L_o = L_1 + L_2 - 2M \tag{88}$$

while the coefficient of coupling, k, is:

$$k = \frac{M}{\sqrt{L_1 L_2}} \tag{89}$$

where

L_1 = self-inductance of coil one

L_2 = self-inductance of coil two

L_a = total inductance of coils one and two connected series-aiding

L_o = total inductance of coils one and two connected series-opposing

M = mutual inductance due to the magnetic coupling between the two coils

k = the coefficient of coupling between coil one and coil two

It is possible to have practically unity coupling, in the order of $k = .99$, when a properly designed iron or iron-alloy core is used, while the coefficient of coupling seldom reaches as high as $k = .6$ when air is the medium for the magnetic lines of force.

188. The Transformer. Two coils placed near each other as shown in Fig. 131 form a *transformer.*

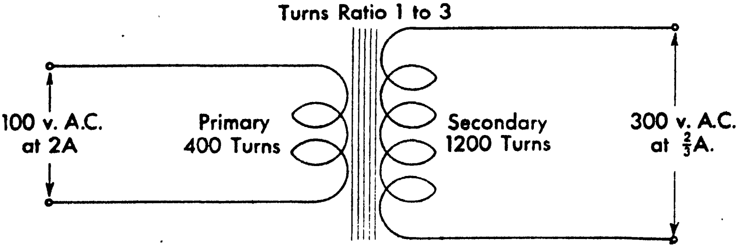


Fig. 132a. Iron-core transformer.

The usual definition of a transformer to students of electricity is that a transformer is a device for increasing or decreasing a-c voltage. For radio students, this description of the action of the transformer is not sufficient. It is also a device that breaks the metallic continuity of a circuit. Many radio circuits use transformers to *isolate* two d-c circuits but, at the same time, provide a path for alternating current from one circuit to the other (by magnetic induction from one winding to another). Such transformers may or may not have a "step-up" or "step-down" voltage ratio.

It should be obvious from the study of preceding sections that direct current in the primary winding does not induce current in the secondary winding unless the value of the direct current is changing.

189. Transformer Properties. A transformer does not step-up or step-down *power.* Any difference in the value of the power in the two windings of a transformer such as shown in Fig. 132*a* is due to power losses in the transformer. The two coils of the transformer of Fig. 132*a* are called *primary* and *secondary.* Power goes into the primary winding and comes out the secondary winding. The secondary voltage is

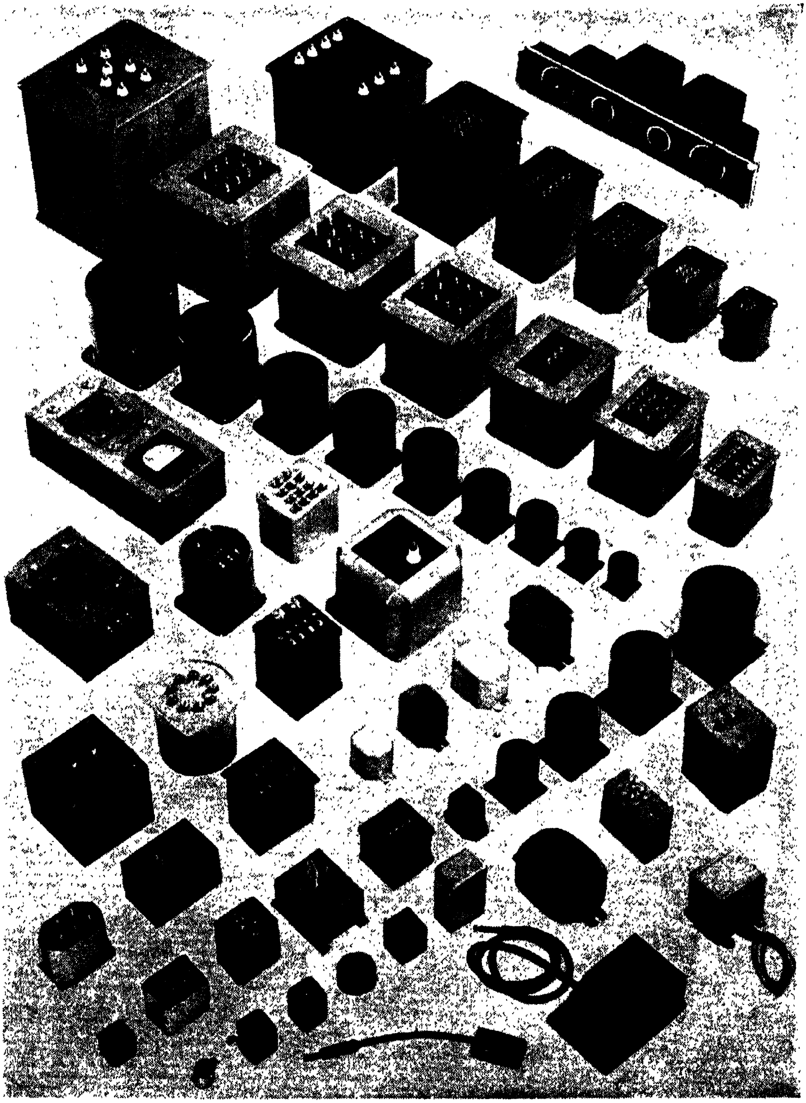


Fig. 132b. Typical radio transformers of various sizes and for the many varied radio applications. (Courtesy, United Transformer Company.)

determined by the *turns-ratio* between the two windings and the primary voltage.

$$\frac{1200}{400} = 3 \text{ to 1 step-up from primary to secondary}$$

$$3 \times 100 = 300\text{-volt secondary voltage}$$

Another method of obtaining the same result is to calculate the turns per volt in the primary and divide into the number of turns in the secondary, thus:

$$\frac{400}{100} = 4 \text{ turns per volt}$$

$$\frac{1200}{4} = 300\text{-volt secondary voltage}$$

Since the voltage was stepped up, the current was stepped down for the secondary power to equal the primary power. The foregoing discussions naturally neglect losses.

A transformer such as the power transformer of a receiver may have several secondary windings. Two or three secondary windings may be provided for tube filament lighting voltages as well as a high-voltage center-tapped winding to provide voltage for the rectifier tube plates.

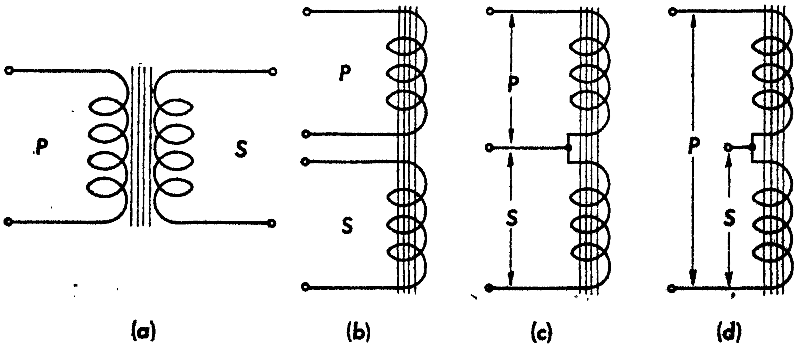


Fig. 133. Several ways of sketching a two winding transformer.

Figure 133 shows several ways of sketching essentially the same transformer. When the primary and secondary are one continuous winding as in Fig. 133 *c* and *d*, the transformer is called an *autotransformer*.

190. Transformer Losses. Transformer losses are principally due to leakage flux, resistance of the windings, and core losses.

The magnetic lines of force from the primary coil that fail to encircle or cut all turns of the secondary are called *leakage flux.* It is impossible to have a coil that does not have some resistance. It was learned earlier that current flowing through resistance dissipates power in that resistance. Core losses are due to *hysteresis* and *eddy currents.* Hysteresis of iron and other core materials was covered in sections 120 and 121. Eddy currents are magnetic lines of force which form small circles rather than traverse the magnetic path through the core as intended. Such local action in the core opposes the magnetizing force of the primary winding. Eddy-current losses increase with the size of the core. Eddy currents in any size core may be held to a very low value by using a core that is formed by a stack of thin laminations (sheets) instead of a solid core. The laminations are minutely spaced by treating the surfaces with some nonmagnetic material such as varnish.

191. Inductances Connected in Series-parallel. The resultant inductance for inductances connected in series or parallel is determined by equations similar to those for determining the resulting resistance of resistors connected in series or in parallel. For series-connected inductances, the total inductance is simply the sum, thus:

$$L_{total} = L_1 + L_2 + L_3 + \dots \tag{90}$$

The resulting inductance for several inductances connected in parallel is:

$$\frac{1}{L_{total}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots \tag{91}$$

Exercise 53:

1. A 1,000-turn radio-frequency choke coil for a low-frequency receiver has 100 millihenries. (*a*) What will the inductance be if the number of turns is reduced 50 per cent? (*b*) How many turns would one need to remove from the coil to reduce its inductance to 50 millihenries?

2. A 50-millihenry radio-frequency choke coil is connected in series with a 75-millihenry radio-frequency choke coil. (*a*) What is the mutual inductance if the coefficient of coupling is .4? (*b*) If the coils are connected series-aiding, what is the value of L_a? (*c*) What is the value of L_o if the coils are connected series-opposing?

3. $L_1 = 80$ millihenries, $L_2 = 60$ millihenries, and $M = 50$ millihenries. Determine the coefficient of coupling.

4. The 30-henry filter-choke coil of a power supply is rated at 50-milliampere maximum allowable current. Another 30-henry filter-choke coil with a maximum rating of 75 milliamperes is placed in parallel with the filter choke in order to use the power supply to furnish voltage to a low-power transmitter requiring 100 milliamperes. What is the total inductance of the two coils operating as stated?

5. The filaments of a transmitter are lighted from a transformer which steps the 110-volt power-line voltage down to 10 volts. (*a*) What is the turns-ratio from primary to secondary? (*b*) How many turns per volt are required for the secondary winding if the primary has 440 turns?

6. It is desired to wind the coil of a power-supply transformer for a receiver which requires secondary windings of the following voltages: 600 volts, 5 volts, and 6.3 volts. How many turns will be required for each secondary winding if the primary has 400 turns?

192. Rate of Change of Current. In Chap. X on Alternating Current, it was shown that a sine wave is continually changing in value. A sine wave of alternating current is shown in Fig. 134. An inspection of this figure not only shows that the current is continually changing but that the *rate of change* is also changing. For instance, it can be seen that the current change at the instant the current reaches its peak value for either the positive or negative half of the cycle is *zero*. At that instant, the increase in current has stopped, and the next instant the current starts decreasing. Since the wave is symmetrical about the zero axis,

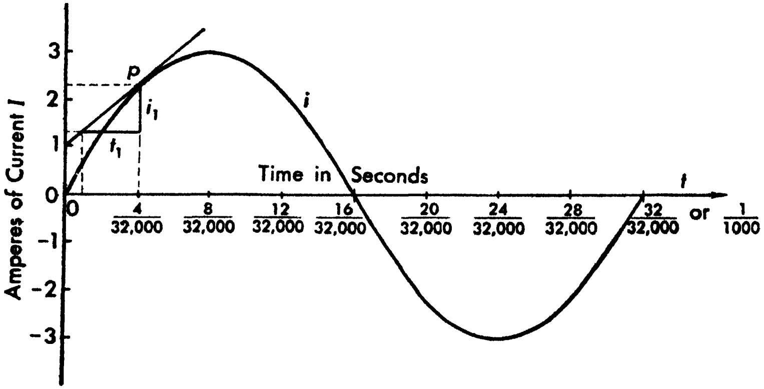


Fig. 134. Rate of change of current for a sine wave whose frequency is 1000 cycles per second.

the rate of change must be *greatest* at the instant the curve *crosses the zero axis.*

These facts can be proved mathematically. Draw a tangent to the curve at any point such as *p*. With this line as the hypotenuse, complete a right triangle by forming the two sides with lines parallel to the time axis *t* and the current axis *I*. The current change i_1 is approximately 1 ampere, and the time change t_1 is approximately $\frac{3}{32{,}000}$ second. The time rate of change is

$$\frac{i_1}{t_1} = \frac{1}{\frac{3}{32{,}000}} = \frac{32{,}000}{3} = 10{,}670$$

amperes per second.

At the instant the current is zero, the rate of change is a maximum and is equal to the full velocity of the radius vector times the maximum current. The radius vector completes 2π radians for each revolution and *f* revolutions per second. The *maximum rate of change of current is therefore* $2\pi f I_{max}$. In this instance, the maximum rate of change is $2\pi 1{,}000 \times 3 = 18{,}840$ amperes per second.

193. Phase Relation of *E* and *I* for Inductance. The value of the voltage of self-induction in a coil at any instant is equal to the *rate of change of current in amperes* at that instant *multiplied by the inductance of the coil in henries.* Like the voltage drop across a resistor, this voltage drop across an inductance is opposite in polarity (180° out of phase) to the supply voltage.

Since the voltage of self-induction depends upon the time rate of change of the current it reaches its *maximum value* when the rate of change of current is maximum which is *at the instant the current is zero.* Likewise, this *voltage is zero* when the rate of change of current is zero which is *at the instant the current is maximum.* From the foregoing, the phase angle between the current and the self-induction voltage is 90°. This is shown in Fig. 135.

From Lenz's law, the voltage of self-induction is in such a direction that it causes an induced current which opposes the change in current. Therefore, the voltage of self-induction *lags* the current 90° and the *supply voltage leads* the current 90°.

The rotating vectors of current and voltage for an inductance may be likened to two spokes of a wheel 90° apart. The spokes are obviously 90° apart regardless of the position of rotation of the wheel and the same spoke is always ahead.

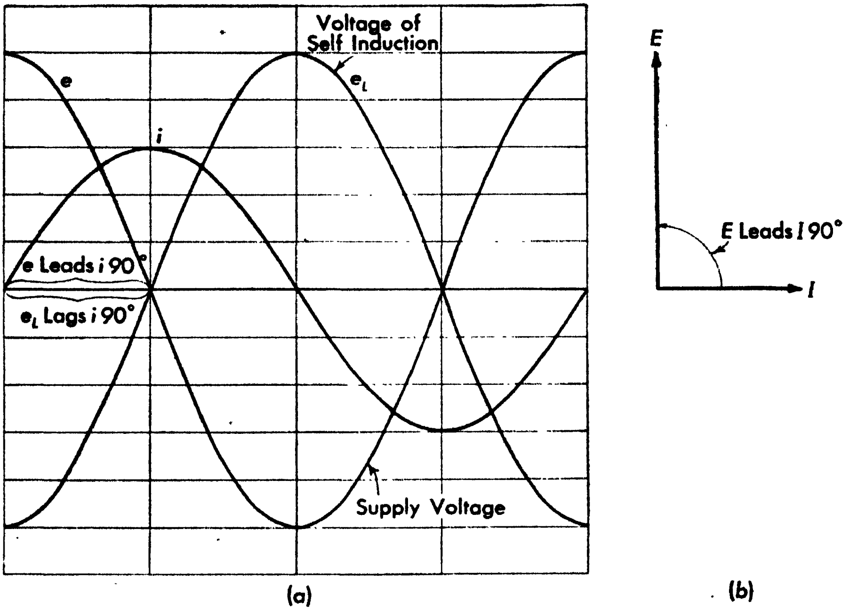


Fig. 135. Phase relations between supply voltage, current, and self-induction voltage of an inductance.

194. Inductive Reactance. From the discussion of inductance, it was stated that inductance is that property of a circuit which tends to oppose a *change* of current. The faster the change, the greater the opposition. The definition of the unit of inductance, the henry, limits the current to one change *per second.* It is necessary that there be a unit of measure of the opposition of an inductance, to the change in current, which takes into account the *frequency of the change.*

Inductive reactance is such a unit and is determined from the expression:

$$X_L = 2\pi fL \tag{92}$$

where

X_L = inductive reactance measured in ohms
2π = radians in one revolution of the radius vector
f = frequency in cycles
L = inductance in henries

From Ohm's law:

$$E = IX_L \tag{93}$$

Exercise 54:

1. A tuning coil of a broadcast transmitter has a maximum current of 20 amperes at a frequency of 1,000 kilocycles. What is the maximum rate of change of current? What is the maximum voltage of self-induction if the coil has an inductance of 50.6 microhenries?
2. What is the inductive reactance of the coil in the above problem?
3. A receiver power-supply choke coil has 30 henries of inductance. What is its inductive reactance at the power frequency of 60 cycles?
4. Would the filtering action of the filter choke of problem 3 be decreased or increased if the power frequency were increased to 100 cycles? How much?

195. Power in an Inductance. It will be remembered that the slight resistance of connecting wires connecting several resistors in series-parallel combinations was neglected in order to avoid complicating the resistance problems. The resistance of inductances will be neglected for the present for the same reason. The resistance action and reactance action found in a coil are separate. The resistance of a coil can be treated as an external resistance connected in series with the inductance.

That portion of Fig. 135 showing the supply voltage and current in an inductance is reproduced in Fig. 136. The power at any instant may be obtained by multiplying the voltage and current at that instant. An instantaneous power curve is shown in Fig. 136. This curve was obtained by multiplying all points on the curve of instantaneous voltage by corresponding points on the curve of instantaneous current. From equation 72 in Chap. X on Alternating Current:

$$p = ei$$

where

p = instantaneous power
e = instantaneous voltage
i = instantaneous current

An inspection of Fig. 136 will show that the positive and negative loops of the power wave are exactly equal to each other. The *positive loops* represent the *power put into the coil* from the power source. The *negative loops* represent the *power the coil returns* to the power source. The coil, being considered perfect with no losses, returns all the power to the source in this instance.

The student should carefully review section 147 on "Alternating-current Power in Resistances" and compare Fig. 136 with Fig. 104.

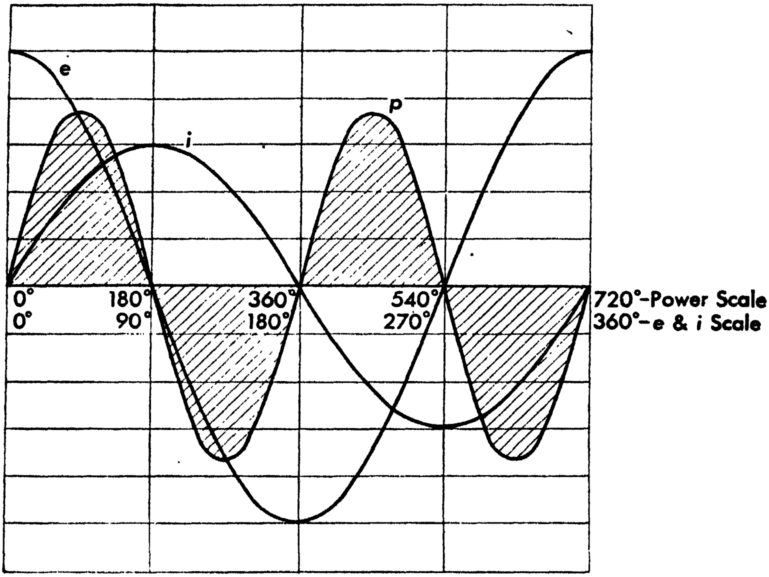


Fig. 136. Voltage, current, and power waves for an inductance showing that the power used by a pure inductance is zero.

The student's attention is directed to the fact that in both instances (power in a resistance and power in an inductance), the frequency of the power wave is *twice* that of the current or voltage.

From equation 77, section 154, the average power is:

$$P = EI \cos \theta$$

For an inductance:

$$P = EI \cos 90^\circ = EI \times 0 = 0$$

Exercise 55:

1. What will be the maximum instantaneous power in a 12-megacycle transmitter tuning coil of 30 microhenries inductance if the peak r-f voltage across the coil is 10,000 volts? How many maximum instantaneous points of power will there be for one cycle of current and at what points along the current axis will they occur? What will be the maximum rate of change of current?

2. The coil of problem 1 is inductively coupled to an antenna coupling coil of similar inductance. The mutual inductance is 8 μH. What is the coefficient of coupling?

REVIEW QUESTIONS

1. State Lenz's law.
2. Define and discuss the property of inductance.
3. What is meant by: (a) Self-inductance; (b) Mutual inductance?
4. Give and define the unit of inductance.
5. The inductance of a coil increases in what ratio or proportion to the number of turns?
6. Why does the inductance property of a coil not oppose direct current?
7. What is meant by the coefficient of coupling between two coils?
8. Explain: (a) "Series-aiding"; (b) "Series-opposing."
9. What is a transformer?
10. Explain: (a) "Step-down"; (b) "Step-up"; (c) "Turns ratio."
11. Is it possible to get a step-up of power from the primary to the secondary of a transformer?
12. Discuss fully the rate of change of current and its effect upon inductance.
13. Discuss fully the difference between inductance and inductive reactance.
14. Why is the phase relation between the voltage across an inductance and the current through the inductance 90°?
15. Does the voltage lead or lag the current for an inductance? Why?
16. Why is the average power in an inductance zero?

ANSWERS

Exercise 53, page 227:

1. *a.* 25 millihenries
   b. 293 turns
2. *a.* 24.5 millihenries
   b. 174 millihenries
   c. 76 millihenries
3. .722
4. 15 henries
5. *a.* 11 to 1
   b. 4 turns per volt
6. 2,180 turns for 600-volt winding
   18.2 turns for 5-volt winding
   22.9 turns for 6.3-volt winding

Exercise 54, page 231:

1. 125,600,000 amperes per second; 6,355 volts
2. 318 ohms
3. 11,304 ohms
4. Increased. Inductive reactance would be increased 7,536 ohms

Exercise 55, page 232:

1. 44,100 watts; four: 45°, 135° (negative), 225°, 315° (negative); 332,000,000 amperes per second
2. .267

CHAPTER XIII *Capacitance*

In this chapter we study another important property of the electrical circuit called *capacitance*. A circuit element that has capacitance properties is called a *condenser*. Like inductance, the capacitance is also an indication of an electrical circuit's property to store electrical energy. It was shown in the previous chapter that an inductance stores energy in the magnetic field surrounding the coil. A condenser stores energy in the *electric field* of the condenser.

196. The Condenser. Any two conductors that are in close proximity, but insulated from each other, form what is essentially a condenser. The most popular conception of a condenser is that the two conductors are metal plates parallel to each other. A condenser may have only the two plates or it may have two sets of plates with a dozen or more plates in each of the two sections. Some condensers have plates that are *air spaced* and others use solid insulation strips between the plates. The plates of some condensers are adjustable such as the *variable* air-spaced tuning condensers to be found in any broadcast receiver. A greater portion of the condensers, in a broadcast receiver for instance, are of the fixed-capacitance type.

Audio amplifiers, radio receivers and transmitters require condensers for various voltage ratings, capacitance sizes and physical dimensions to care for the many capacitance requirements in their circuits. Several commercial condensers are shown in Figs. 137*a*, 137*b*, and 137*c*.

197. The Electric Field. If two conductors are placed very near each other, as in Fig. 138, there will be an attraction of the charge in one conductor to the opposite charge in the other conductor. (Review electrical charges in section 9.) Current would flow from one conductor to the other (*from negative to positive*) were it not for the fact that the conductors are insulated from each other by the air space between them. Even though current does not actually flow across the gap, there

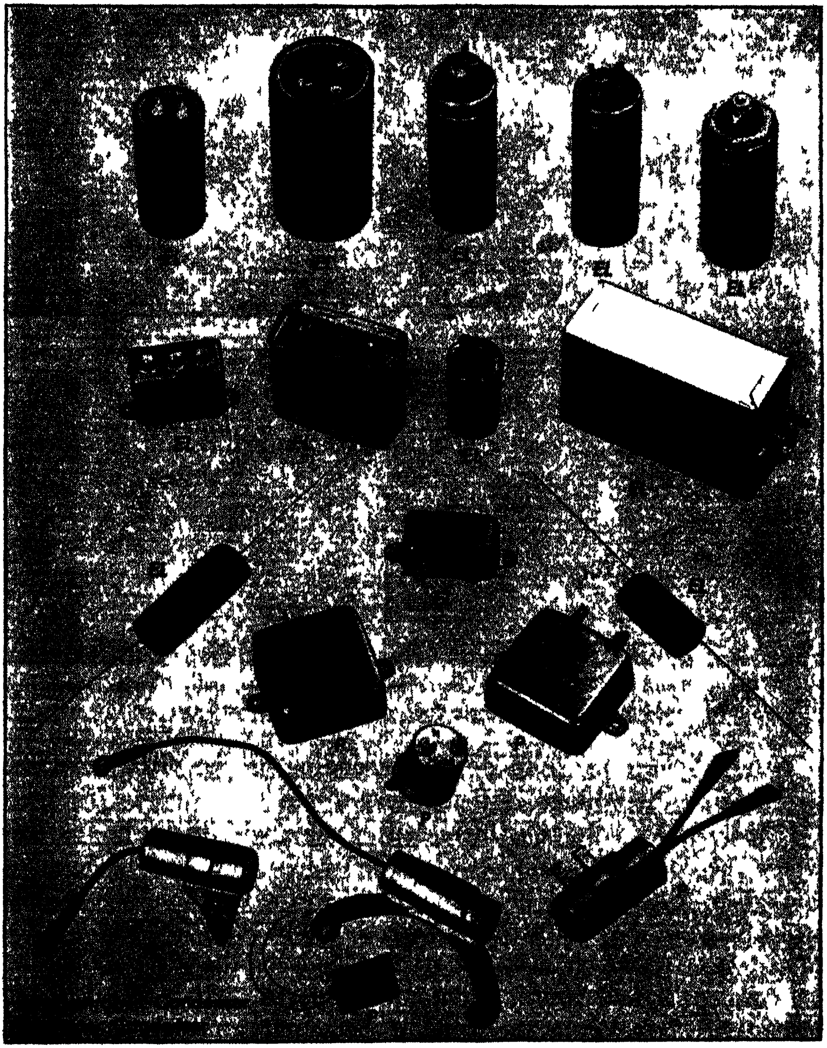


Fig. 137a. Typical paper and electrolytic type condensers. Electrolytic condensers are indicated by *EL* and paper ones by *P*. (Courtesy, P. R. Mallory Company, Inc.)

Fig. 137b. Mica type fixed capacity condensers. (Courtesy, Cornell-Dubilier Electric Corporation.)

is a strong urge for it to do so, depending on the distance of the spacing and the voltage of the battery.

If the battery voltage is increased, a value will finally be reached where the stress or strain between the two conductors will be great enough to cause an arc of current between the two conductors. The stress between the conductors is represented by *electric lines of force* of an *electric field* which is said to exist between the two conductors. The student should carefully compare the electric field to the magnetic field. When he takes up the study of the actual transmission of radio waves through space in a subsequent volume, he will find that the transmission depends upon both fields and that the waves are called *electromagnetic waves.*

The voltage necessary to cause a current to jump across any specified space from one conductor to another depends upon several fac-

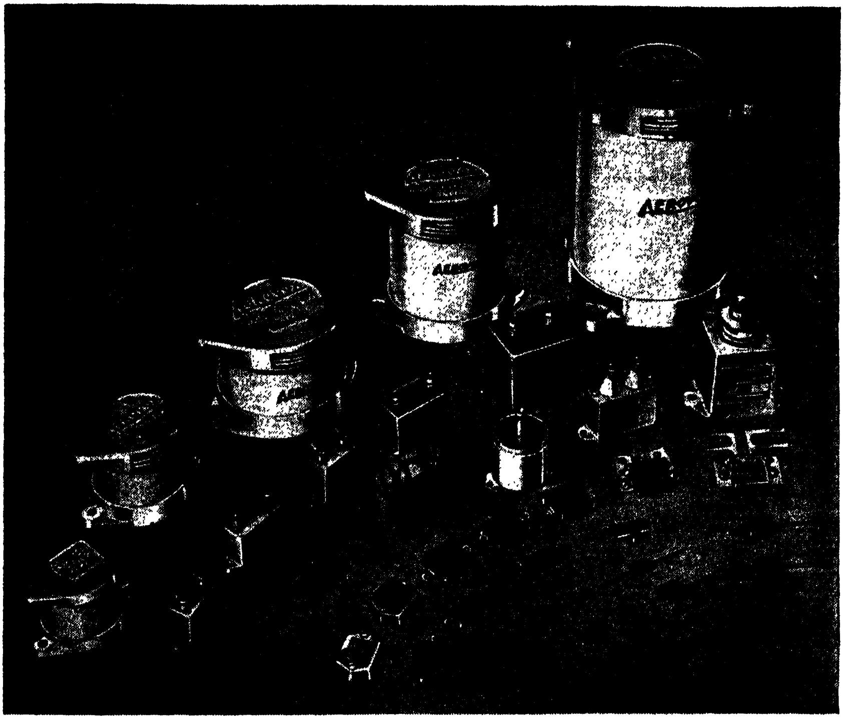


Fig. 137c. Typical mica type condensers, ranging in size from the small bakelite encased receiver and small transmitter type to the large ceramic and metal encased high voltage and high current type used in high-powered transmitters. (Courtesy, Aerovox Corporation.)

tors. If the space is air, it depends upon the density of the air, its temperature, and the kind and amount of dust or other foreign material in the air. If the space is filled with mica, paper, and so on, it depends upon the electrical properties of the material called *dielectric*

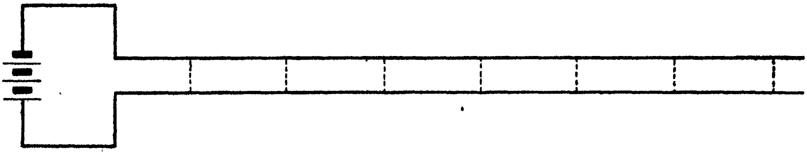


Fig. 138. An electric field exists between the two conductors because of the attraction between the charges which are of opposite polarity.

properties. For that reason, the material used between the plates of a condenser is called *the dielectric.*

198. Dielectric Constant. In the study of the magnetic field, it was found that some substances provided better paths for magnetic lines of force than others. The measure of such a property in a material was called its *permeability*.

Certain materials also provide better paths for the electric lines of force of the electric field than others. This is the dielectric property of the material, and its measure is the *dielectric constant.* Like the permeability of a material, the dielectric constant refers to air, which is arbitrarily set as 1.

The dielectric constant of a material is the ratio of the electric lines of force in the material to the electric lines of force that would be produced in air under like conditions. The dielectric constant of several materials are shown in Table IV.

TABLE IV. DIELECTRIC CONSTANTS FOR SEVERAL MATERIALS.

Material	*Dielectric Constant*
Air	1
Bakelite	3 to 5
Glass	4.5 to 10
Mica	4.5 to 8.5
Paper	2 to 3.5
Porcelain	6 to 7
Quartz	4.5
Rubber (hard)	3

199. Dielectric Materials. It will be noticed from Table IV that all the materials listed are *insulators*. It is not desirable that any current flow through the dielectric from one plate to the other plate of a condenser. Such a flow of current would represent a loss. The resistance of the dielectric to the flow of the current, if sufficiently low, would cause the dielectric to heat. Such heating of the dielectric may damage it to the extent that it may break down fully and allow the two plates of the condenser to become short-circuited.

The dielectric must not only be a good insulator, but it must also be capable of withstanding high voltages so that the thickness of the required strip may be kept to a minimum. It will be shown in section

202 that the capacitance of a condenser varies inversely as the distance between the plates.

The property of a material to withstand high voltages per unit thickness without breaking down the material is called the *dielectric strength* of the material.

200. Energy Storing Properties of a Condenser. The two plates of a condenser may be compared to two rooms separated by a solid partition as shown in Fig. 139 *a*. If the two rooms are joined externally by a long narrow corridor people may move to and fro from one room to the other. There is just enough space in the hallway for only a few people and the rooms will hold many times more. Suppose that at first there are an equal number of people in each room and a few people move from room *A* to *B*. Now room *A* has less people than before and room *B* has more people than before.

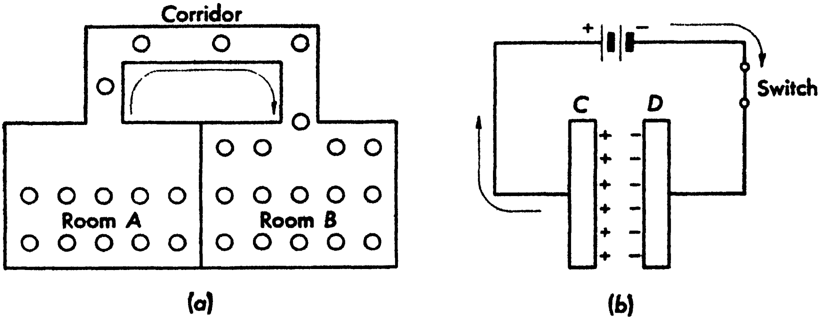


Fig. 139. The movement of electrons from plate *C* to plate *D* of the condenser in *b* may be compared to people moving from room *A* to room *B* in *a* by means of the corridor.

Compare Fig. 139 *a* to *b*. The two condenser plates *C* and *D* represent considerable conductor area. Before the switch is closed, there is an equal number of electrons in each plate. When the switch is closed, the action of the battery is to rob plate *C* of electrons and give plate *D* an excess of electrons.

From the definitions of positive and negative charges:

1. Plate *C* has lost some of its electrons and now has less than it had before and is said to be *positively charged.*
2. Plate *D* has gained electrons, having more now than before, and it is said to be *negatively charged.*

Naturally, the number of people that rooms *A* and *B* will hold depends on their capacity, and the storing property of the plates of a condenser depends on its capacitance.

201. Unit of Capacitance. The unit of measure of capacitance is the *farad.*

A condenser has a capacitance of 1 farad when 1 volt applied across the terminals causes it to take a charge of 1 coulomb, or if a *voltage change of 1 volt per second* across its terminals causes a current of *1 ampere to flow.*

Any condenser with a capacitance of 1 farad would be of huge proportions. All condensers in present-day use are much smaller than the farad and it is more convenient to use the microfarad (1/1,000,000) or micromicrofarad (1/1,000,000,000,000) in expressing their capacitance.

202. Capacitance of a Condenser. The capacitance of a condenser depends on a number of factors.

The capacitance of a simple two-plate condenser whose plates are relatively large and closely spaced is given by the equation:

$$C = \frac{.0885KA}{10^6 \times t} \tag{94}$$

where

C = capacitance in microfarads

K = dielectric constant of dielectric

A = area of the dielectric in square centimeters

t = thickness of the dielectric in centimeters (1 inch = 2.54 centimeters)

Example:

A particular condenser used in a broadcast receiver is formed by two narrow strips of tinfoil[1] separated by a thin layer of paper .02 centimeter thick. Normally, such a condenser is rolled up, which not only gives more capacitance for a given amount of tinfoil but also makes it very compact. In this instance, the condenser was unrolled and found to have a capacitance of .0015 microfarad. The dielectric constant of the paper is 3 and the condenser plates have an equal amount of tinfoil in each. Determine the number of square centimeters of tinfoil used.

---

[1] Aluminum foil and other material are also used.

Solution:

$$A = \frac{10^6 \times tC}{.0885\,K} = \frac{10^6 \times .02 \times .0015}{.0885 \times 3}$$

$$= \frac{30}{.2655} = 113.1 \text{ square centimeters, area of one plate}$$

$$2 \times 113.1 = 226.2 \text{ square centimeters, total area of tinfoil in condenser}$$

203. Condensers Connected in Series-parallel. Two condensers connected in series have a total capacitance which is less than that of either condenser. This can best be understood by inspecting equation 94 for determining the capacitance of a condenser. The capacitance of a condenser varies inversely as the thickness of the dielectric. Two or more condensers, connected in series, increase the thickness of the total dielectric in the circuit, thereby reducing the capacitance.

For condensers connected in series:

$$\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \tag{95}$$

The polarity of the charge on several condensers connected in series is shown in Fig. 140.

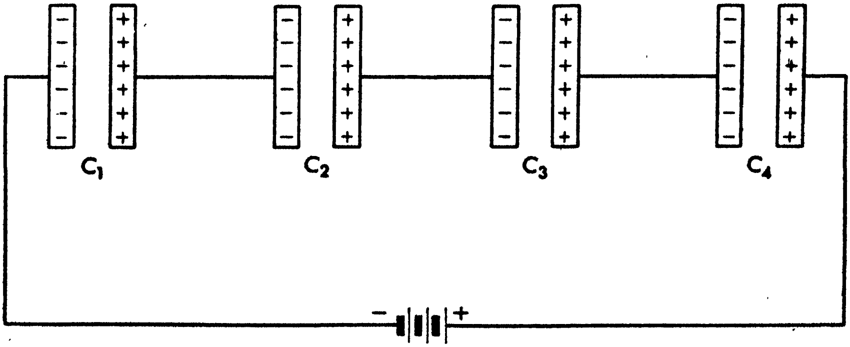


Fig. 140. **Diagram showing the polarity of the charges on the plates of several condensers connected in series.**

Added thickness of the dielectric increases the maximum allowable voltage that can be applied to the condenser plates. If the thickness of the dielectric is doubled, the maximum allowable or break-down voltage is doubled. Two condensers, identical as to capacitance and voltage ratings, may be connected in series in order *to double* the voltage ratings of the combination of the two condensers as compared to one condenser. For instance, two identical condensers, rated at a maximum a-c voltage of 1,000 volts, may be connected in series and placed across an a-c voltage of 2,000 volts. This does not overload either condenser as the voltage drop across each condenser is 1,000 volts.

Connecting condensers in series does not always increase the overall allowable operating voltage in direct accordance with the number of condensers used. The distribution of the voltage drop (voltage drop across each condenser) is determined by the reactance of each condenser and its losses. The decrease in total capacitance, as a result of connecting several condensers in series, usually offsets the advantage of increased operating voltage gained by connecting condensers in series. The usual practice, when connecting several condensers in series, is to place a small-wattage, high-ohmage resistor across each condenser to stabilize the voltage drop across each condenser.

The total capacitance of several condensers connected in parallel is determined by adding the capacitances of the condensers:

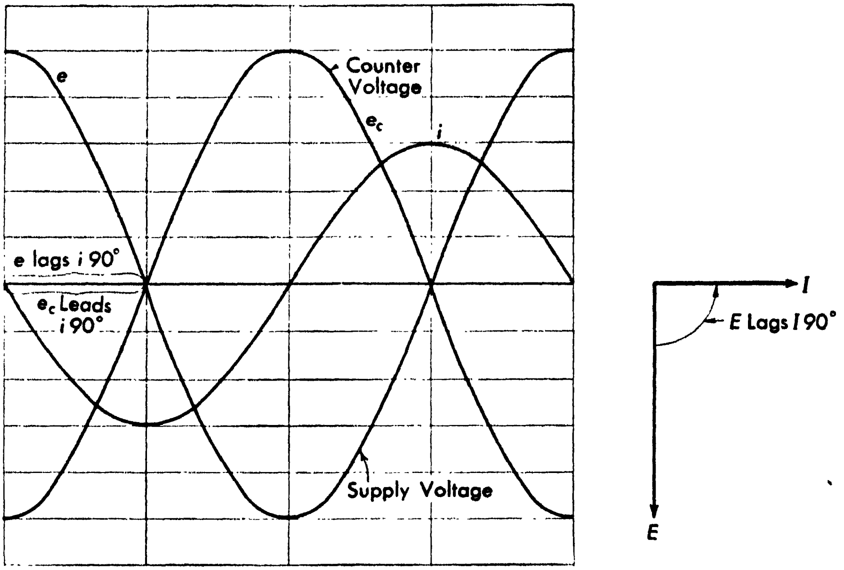
$$C_{total} = C_1 + C_2 + C_3 + \dots \tag{96}$$

204. Rate of Change of Voltage. When an alternating-current voltage is placed across the terminals of a condenser, the current is alternating and the charge of the condenser is continually changing.

The rate of change of a sine wave was discussed in section 192. The value of the current in the circuit to the condenser is determined by the *rate of change of the voltage* across the condenser.

205. Phase Relation of *E* and *I* for a Condenser. The current to a condenser will be a *maximum* when the rate of change of the voltage is a maximum and occurs *when the voltage is zero* for an a-c voltage (when the sine wave of the voltage crosses the axis from minus to plus or from plus to minus). Likewise, the current will be zero when the rate of change of the voltage across the condenser is zero, which occurs when the voltage is a maximum. Obviously, the phase angle between the current and voltage is 90°.

The current reaches its *maximum positive value* at the instant the voltage is passing through its *zero* value in a *positive direction.* From this, it is self-evident that the *voltage across the condenser lags the condenser current 90°*. The voltage drop or counter voltage which acts in opposition (180° out of phase) to the supply voltage *leads* the condenser current 90°. These waves are shown in Fig. 141.



(a) (b)

Fig. 141. Phase relation between supply voltage, current and counter voltage of a condenser.

206. Capacitive Reactance. The condenser current increases as the frequency of the changing voltage increases. It will be noticed that this is exactly *opposite* to the behavior of inductance.

The capacitive reactance, like inductance, opposes current flow and is measured in ohms.

$$X_c = \frac{1}{2\pi f C} \tag{97}$$

where

X_c = capacitive reactance measured in ohms
2π = radians in one revolution of the radius vector
f = frequency in cycles
C = capacitance in farads

From Ohm's law:

(98) $$E = IX_c$$

Exercise 56:

1. Determine the capacitance of an r-f by-pass condenser using a mica separator .02 centimeter thick and having a dielectric constant of 5 if the area of one plate is 2 square centimeters.

2. How much additional plate material would be required for the above condenser to maintain the above capacitance if two mica separators were used? Three separators?

3. Determine the resultant capacitance of the following condensers connected in series: .002 microfarad; .01 microfarad; .005 microfarad.

4. What is the capacitive reactance of the condenser combination of problem 3 at 1,300 kilocycles?

207. Power in a Condenser. The action of power in a condenser is similar to that of an inductance in that it takes power from the source

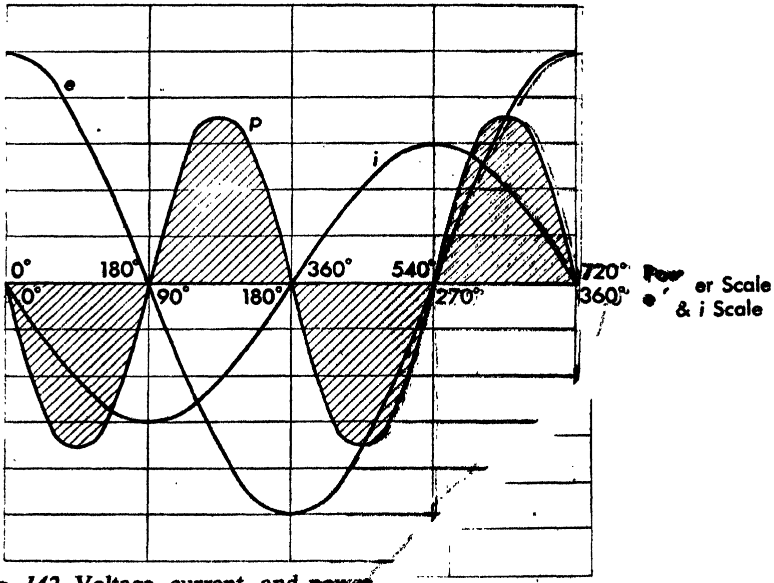


Fig. 142. Voltage, current, and power waves for a condenser showing that the power used by a pure capacitance is zero.

on one portion of the cycle and returns it to the source on the next portion of the cycle. This is shown in Fig. 142. Like the power wave for an inductance, the positive loops equal the negative loops for a power wave of a condenser, and the net power used by a condenser is zero (neglecting losses).

The student should compare the power wave of a condenser shown in Fig. 142 to the power wave of an inductance shown in Fig. 136. It will be noted that the two waves are 180° out of phase.

208. Condenser Properties. The power factor (cos θ) of a perfect condenser is zero since, from Fig. 141, θ is 90° and cos 90° = 0. No condenser is perfect, but many closely approach perfection and have only a very slight power factor. High-quality condensers used in the radio-frequency circuits of modern commercial transmitters from about 100 watts up usually have the power factor stamped on the condenser or it is available from the manufacturer's printed literature. It is well to consider the power factor in comparing condensers since it represents the loss of a condenser. Losses of a condenser increase as the power factor increases.

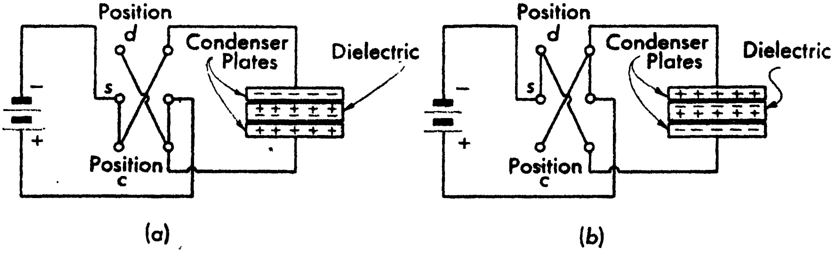


Fig. 143. Stress on the dielectric of a condenser due to electric charges of opposite polarity on the plates.

Condenser losses are principally due to molecular friction in the dielectric. Figure 143 shows the stress in the dielectric of a charged condenser. If the switch *S* is rapidly operated to and fro from positions *c* and *d*, the change in polarity of the dielectric causes molecular friction which heats the dielectric. The value of the charge and the rapidity at which the charge changes determine the loss due to molecular friction. Some condensers that are entirely satisfactory at the 60-cycle power frequency are unusable at relatively low radio frequencies, while other condensers may be satisfactory at low radio frequencies but have prohibitive losses at higher radio frequencies.

Current flowing through the dielectric due to its inability to be a perfect insulator, resistance of the plates, and corona discharges are some of the other factors contributing to condenser losses.

209. Variable Condensers. Tuning circuits of receivers and transmitters require a convenient means of changing the value of either the inductances or capacitances in the circuits. Economy and mechanical simplicity favor a variable capacitance device called a *variable condenser*. Several half-circular or similarly shaped plates are spaced on a rotatable shaft so that they will pass in between similarly shaped and spaced stationary plates that are insulated from the rotary plates.

The rotary assembly is usually mounted by ball bearings or other bearings directly on the condenser frame, making it necessary to insulate the stationary plate assembly (called *stator*) from the frame by means of insulation strips. The dielectric between the plates is air with no molecular friction losses as encountered in solid dielectrics such as Bakelite, hard rubber, paper, or mica, used in *fixed* capacitance condensers.

The low dielectric constant of air compared to the dielectric constants of other dielectrics and the relatively wide separation of the plates required for air limits the maximum practical size of such a condenser to about .0005 microfarad.

210. Mica Condensers. Mica is used extensively in high-quality condensers, where size permits, because of its excellent insulating properties and low losses. Only the better grades of mica are suitable for use in condensers. The expensiveness of mica and the inability to roll mica-insulated condensers into compact sizes limit its use to the construction of only the small capacitance size condensers.

211. Paper Condensers. The losses of paper as a dielectric for condensers are greater than air or mica, but the ease of working with paper in producing condensers by mass-production methods, its inexpensiveness, and the ability to manufacture paper with extremely small thicknesses all lend to its wide use as a dielectric for condensers.

The ability to manufacture paper that is extremely thin makes it possible to have condensers up to several microfarads with a volume of only a few cubic inches. All paper dielectric condensers are rolled up, which not only increases the capacitance for a given amount of plate area but also makes the condenser very compact. It is better to use several layers of thin paper rather than one thickness of paper that has the same thickness as that of the combined layers of thin paper. The reason for this is that any pinhole, impurity, or other fault encountered with a single layer of paper might seriously lower the maximum allowable operating voltage or ruin the condenser entirely. In the instance of several layers of paper, it is not probable that fault points in any one layer would be adjacent to fault points in any of the other layers.

The paper dielectric of a paper condenser is usually impregnated with wax or other insulating compound and the finished condenser dipped into the compound to make it moistureproof before it is sealed into a paper or metal case.

212. Electrolytic Condensers. It was shown in the study of the paper condenser that one of the chief advantages of paper as a dielectric was that it could be manufactured in extremely thin sheets, making it possible to place the two plates of a condenser close together. From equation 94, the capacitance of a condenser increases as the thickness of the dielectric decreases.

The electrolytic condenser has a dielectric that is much thinner than is possible with paper. This makes it possible to have a condenser with still greater capacitance per unit volume.

An electrolytic condenser consists of a metal plate (usually aluminum) coated with a thin oxide *insulating film* immersed in a suitable solution (electrolyte) which constitutes the *other plate of the condenser*. Such a condenser is called a *wet electrolytic condenser*. It is possible to dispense with the liquid electrolyte and use a damp paste solution as the electrolyte. Such a condenser is called a *dry electrolytic condenser*.

The electrolytic condensers described above are usable only on d-c or pulsating d-c voltages, and the *positive* terminal of the voltage must be connected to the oxide-coated plate. The negative terminal of the condenser is formed by the container (usually aluminum) which encases the electrolyte and the oxide-coated plate (positive terminal). The polarity of an electrolytic condenser is always indicated on the condenser case. This polarity must always be observed when connecting such a condenser into a circuit as the oxide film offers little opposition to current in the opposite direction.

The capacitance of an electrolytic condenser may be increased by etching or otherwise roughening the surface of the oxide coated plate to increase its surface area.

It is possible to use electrolytic condensers in a-c circuits provided *two* are used and connected in series with like polarities connected together.

The thinness of the oxide film provides an extremely high value of capacitance per unit volume, compared to other condensers, but it also limits the maximum operating voltage to about 600 volts. The losses of electrolytic condensers are relatively high and change with use, age, and other factors. Despite these and other disadvantages, the compactness and economy of the electrolytic condenser have caused it to be widely used in the radio field for by-pass purposes and in power-supply filter circuits.

REVIEW QUESTIONS

1. What is an electric condenser?

2. What is an electric field?

3. Discuss the dielectric constant of materials.

4. Are dielectrics classed as conductors or insulators?

5. Discuss the energy storing property of condensers.

6. Give and define the unit of capacitance.

7. How does the thickness of the dielectric of a condenser affect its capacitance?

8. Why is the value of the charging current in a condenser determined by the rate of change of the voltage across the condenser?

9. What is the phase relation between voltage and current for a condenser? Why?

10. Discuss capacitive reactance.

11. What are: (a) Variable condensers? (b) Mica condensers? (c) Paper condensers?

12. Why are several thin layers of paper insulation used in paper condensers instead of one layer whose thickness is the same as the combined thicknesses of the several thin layers?

13. Give the essential parts and the function of each in an electrolytic condenser.

14. Why is it possible to make some electrolytic condensers whose capacitance is greater and whose physical size is smaller than paper or mica condensers rated at the same voltage?

15. The characteristics of electrolytic condensers limit their manufacture for working voltages up to what maximum value? Why?

ANSWERS

Exercise 56, page 244:

1. .00004425 microfarad or 44.25 micromicrofarad
2. Twice as much; three times as much.
3. .00125 microfarad
4. 97.9 ohms

CHAPTER XIV *Impedance*

We are indeed fortunate that nature smiled on us so graciously and gave us inductance and capacitance which are such excellent servants that work for us tirelessly and yet take no power for themselves. What a catastrophe it would be if the coils of the power transformer on our street took all the power furnished them and passed none along to us!

Resistance, inductance, and capacitance have been covered in the order given and all found to offer an opposition (measured in ohms) to the flow of current in an a-c circuit. The opposition of all three or any two acting in an a-c circuit cannot be added directly. The combined action of all or any two is termed *impedance*, represented the by letter Z.

213. Determination of Impedance. It is not so easy to determine the total impedance of a circuit as it is to determine its total resistance. The reason for this is that resistance, inductive reactance, and capacitive reactance *do not act together* in opposing the current flowing in the circuit and cannot be added directly in either the series or parallel circuit.

(99) The *resistance* and *reactance* values in a *series a-c circuit* can only be added *vectorially* in order to determine the total circuit *impedance.*

(100) The total *impedance* of a *parallel circuit* must be determined *indirectly* from Ohm's law.

(101) $$Z = \frac{E}{I}$$

where

Z = total circuit impedance

E = supply voltage

I = total current

The total impedance of a *series* circuit may also be determined from equation 101.

214. Alternating-current Voltage and Current in Impedance Circuits. Not only are impedance circuits more difficult than resistance circuits, but it is more difficult to add the voltage drops in a series a-c circuit or the currents in a parallel a-c circuit when inductance and capacitance, as well as resistance, are present. Alternating-current circuits containing *series-parallel combinations* of resistance, inductance, and capacitance are further involved:

(102) The *voltage* drops across resistances, and reactances of a *series a-c circuit* can only be added *vectorially.*

(103) The *current* in the resistive and reactive branches of a *parallel a-c circuit* can only be added *vectorially.*

215. Phase of E-I for Series, Parallel Circuits. The phase relationship between the supply voltage and current in a *series* a-c circuit is:

$$\theta_{E-I} = \tan^{-1}\frac{X}{R} \tag{104}$$

where

θ_{E-I} = phase angle
X = resultant reactance (algebraic sum)
R = total resistance

The phase angle between the supply voltage and total current in a *parallel* a-c circuit is:

$$\theta_{E-I} = \tan^{-1}\frac{I_X}{I_R} \tag{105}$$

where

θ_{E-I} = phase angle
I_X = reactance component of the total current
I_R = resistance component of the total current

216. Resistance and Inductance in Series. It has been shown that the phase relation between the voltage across a resistor and the current through the resistor is zero, and the phase relation between the voltage across and the current through an inductance is 90° (current *lagging* voltage).

When a resistor and an inductance are connected in series, the phase relation across each remains *unchanged* but the phase relation between the *supply voltage* and the *circuit current* is affected by the combined action of both the resistor and inductance. This is shown in the circuit of Fig. 144 *a*. The vectorial addition of the voltage drop across the resistance and the voltage drop across the inductance is shown in Fig. 144 *b*. It is obvious that the resistance voltage E_R and the inductance voltage E_L form the sides of a right triangle and the hypotenuse represents the total voltage E. From equation 46:

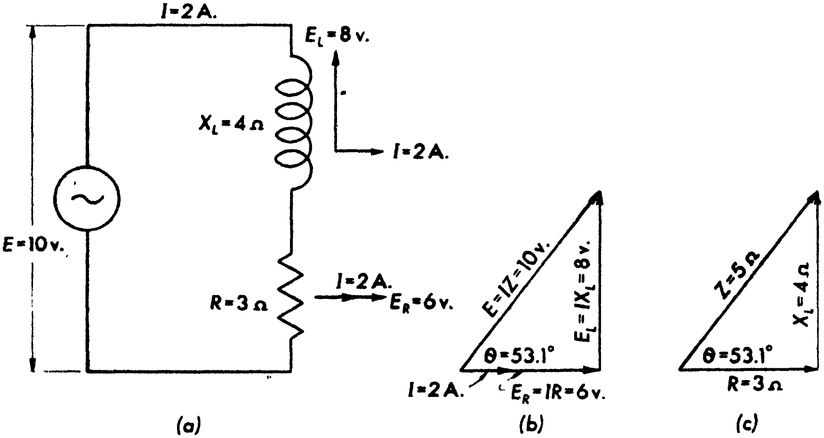


Fig. 144. Showing: *a.* Inductance and resistance connected in series. *b.* Vectorial addition of voltages. *c.* Vectorial addition of resistance and inductive reactance.

(106) $$E = \sqrt{E_R^2 + E_L^2}$$

(107) $$E^2 = E_R^2 + E_L^2$$

Substituting in equation 106:

$$E = \sqrt{(6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10 \text{ volts}$$

From Ohm's law:

$$E = IZ;\ E_R = IR;\ \text{and } E_L = IX_L$$

Substituting in equation 107:

$$(IZ)^2 = (IR)^2 + (IX_L)^2$$

or

$$Z^2 = \frac{\cancel{I^2}}{\cancel{I^2}}(R^2 + X_L^2)$$

and

$$Z^2 = R^2 + X_L^2$$

or

(108) $Z = \sqrt{R^2 + X_L^2}$

The vectorial addition of resistance and inductive reactance for the series circuit of Fig. 144 *a* is shown in *c* and is:

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ ohms}$$

This value may be checked by solving for Z from Ohm's law.

$$Z = \frac{E}{I} = \frac{10}{2} = 5 \text{ ohms}$$

$\theta = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{4}{3} = 53.1°$ phase angle between supply voltage E and circuit current I (*I lagging E*).

The power wave for the circuit of Fig. 144 *a* is shown in Fig. 145. Here, as in Figs. 136 and 142, the positive loops represent power *taken from* the supply source, and the negative loops represent power *returned to* the supply source. The *net* sum (algebraic sum) is the power used by the circuit. From equation 77, the power in an a-c circuit is: $P = EI \cos \theta$. The power in this instance is: $P = 10 \times 2 \times .6 = 12$ watts. The power used by the various elements of a series a-c circuit is determined by the phase relation of the current and voltage of the element. The phase angle for the inductance is 90° and cos 90° = 0. Therefore, the inductance does not use any power. The phase angle for the resistance is 0° and cos 0° = 1. Therefore, the power in the resistance is: $P = 2 \times 6 = 12$ watts.

217. Resistance and Capacitance in Series. An a-c circuit containing resistance and capacitance connected in series is shown in Fig. 146 *a*. It has been shown that the phase relation between the voltage across a condenser and its charging current is 90°, as is the case with a pure inductance except that the current *leads* the voltage for a capacitance and *lags* the voltage for an inductance.

From equation 46:

(109) $E = \sqrt{E_R^2 + E_C^2}$

(110) $Z = \sqrt{R^2 + X_C^2}$

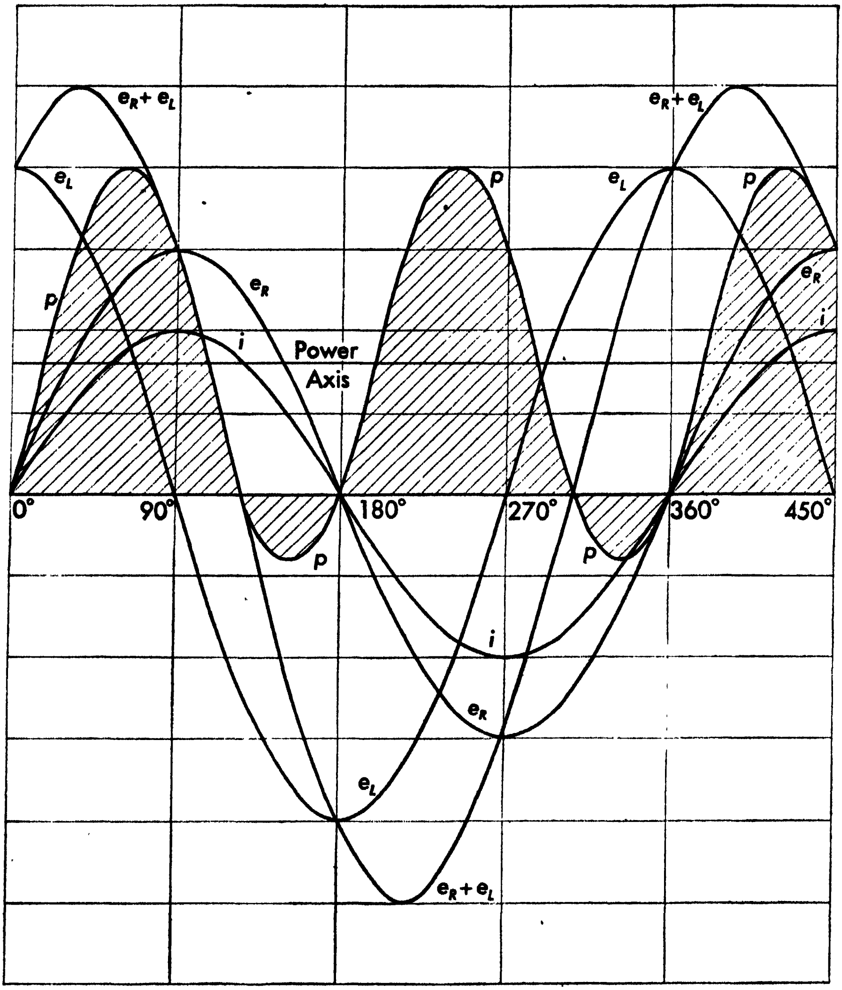


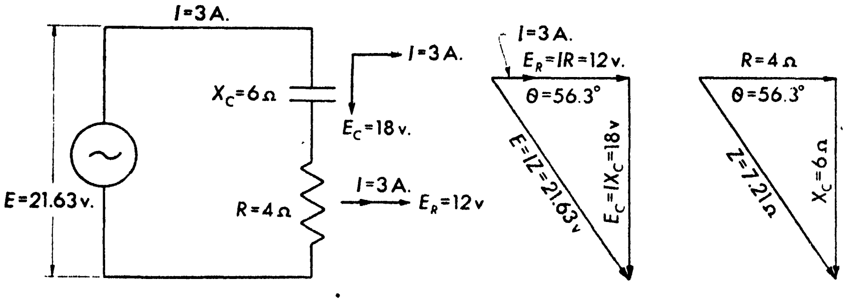
Fig. 145. **Showing how a resistance connected in series with an inductance causes the positive loops of the power wave to be greater than the negative loops.**

Substituting in equation 109:

$$E = \sqrt{(12)^2 + (18)^2} = \sqrt{144 + 324}$$
$$= \sqrt{468} = 21.63 \text{ volts}$$

Substituting in equation 110:

$$Z = \sqrt{(4)^2 + (6)^2} = \sqrt{16 + 36}$$
$$= \sqrt{52} = 7.21 \text{ ohms}$$



(a) (b) (c)

Fig. 146. a. Capacitance and resistance connected in series. *b.* Vectorial addition of voltages. *c.* Vectorial addition of resistance and capacitive reactance.

From Ohm's law:

$$Z = \frac{E}{I} = \frac{21.63}{3} = 7.21 \text{ ohms}$$

$$\theta = \tan^{-1} \frac{X_c}{R} = \tan^{-1} \frac{6}{4} = 56.3^\circ \text{ phase angle between supply}$$

voltage E and circuit current I (*I leads E*).

218. Resistance, Inductance, and Capacitance in Series. The student should compare sections 216 and 217 and note the similarity between Figs. 144 and 146.

A series a-c circuit containing resistance, inductance, and capacitance is shown in Fig. 147 *a*. The vector diagram of voltages and current is shown in *b* and the vector diagram of impedance is shown in *c*. The phase angle between the inductive voltage E_L and the capacitive voltage E_C is 180°, as shown by the vector diagram of Fig. 147 *b*. These two voltages have a canceling effect upon each other since they are of opposite polarity. The equation for the vectorial addition of E_L, E_C, and E_R is:

$$(111) \qquad E = \sqrt{E_R{}^2 + (E_L - E_C)^2}$$

Substituting the proper values from the circuit of Fig. 147 in equation 111, we obtain:

$$E = \sqrt{(4)^2 + (10 - 6)^2} = \sqrt{32} = 5.66 \text{ volts}$$

The equation for the vectorial addition of R, X_L, and X_C is:

$$(112) \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Substituting the proper values from Fig. 147 in equation 112, we obtain:

$$Z = \sqrt{(2)^2 + (5 - 3)^2} = \sqrt{8} = 2.83 \text{ ohms}$$

and from Ohm's law:

$$Z = \frac{E}{I} = \frac{5.66}{2} = 2.83 \text{ ohms}$$

$$\theta = \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \frac{2}{2} = 45° \text{ phase angle between the}$$

supply voltage E and the circuit current I (*I lagging E*).

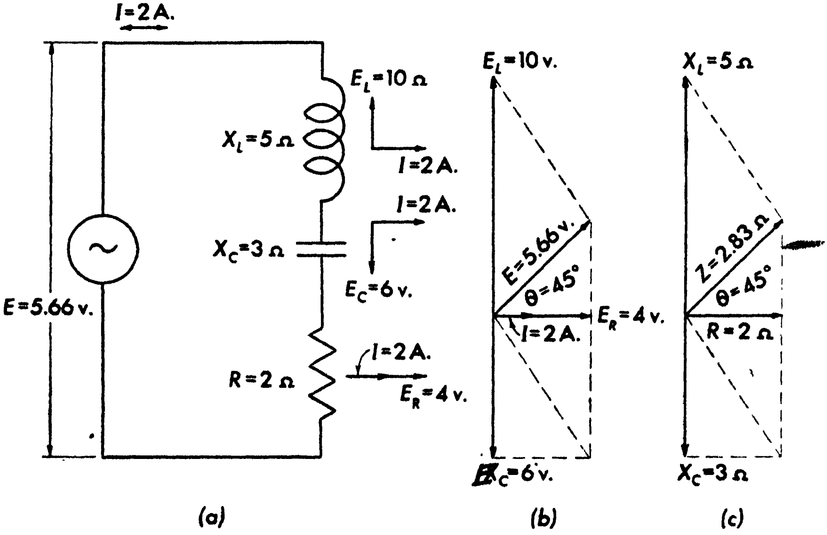


Fig. 147. a. Resistance, inductance and capacitance connected in series. *b.* Vector diagram showing voltage and current phase relations. *c.* Vector diagram of impedance.

219. Series Resonance. Coils and condensers are extremely important items in the make-up of radio transmitters and receivers, but the reactance (equations 92 and 97) of even a relatively small inductance or capacitor is quite substantial at frequencies encountered in radio. Since the action of capacitance is opposite to that of inductance, it is convenient to make the resultant reactance of a circuit zero by having both capacitance and inductance present and their values such that $X_L = X_C$ at the operating frequency. Such a circuit is said to be *resonant* and the frequency, f_r, required for X_L to equal X_C is called the *resonant frequency*.

$$X_L = X_C$$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{113}$$

The only opposition offered to the flow of current in a series resonant circuit, *at the resonant frequency*, is the resistance (if present) of the circuit. The inductive reactance does not equal the capacitive reactance of the circuit at any frequency other than the resonant frequency. At a frequency higher than the resonant frequency, X_L is greater and X_C is less and the circuit is said to be *inductive*. At a frequency lower than the resonant frequency, X_L is less and X_C is greater and the circuit is said to be *capacitive*.

The series resonant circuit provides an excellent means of accepting a desired frequency and rejecting all others. A practical example of this is the coil and condenser in series with the receiving antenna of Fig. 148. The variable condenser is adjusted so that its capacitive reactance is equal to the inductive reactance due to the inductance of both the coil and the antenna wire. The resonant frequency may be *lowered* by *increasing* the capacitance and *increased* by *lowering* the capacitance of the condenser.

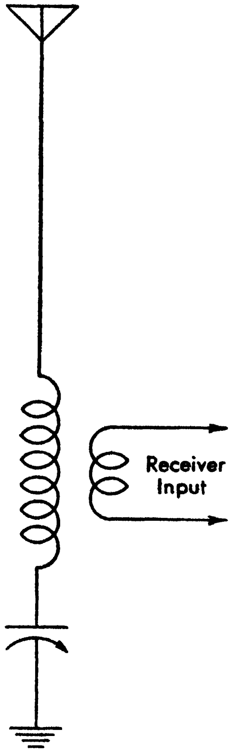
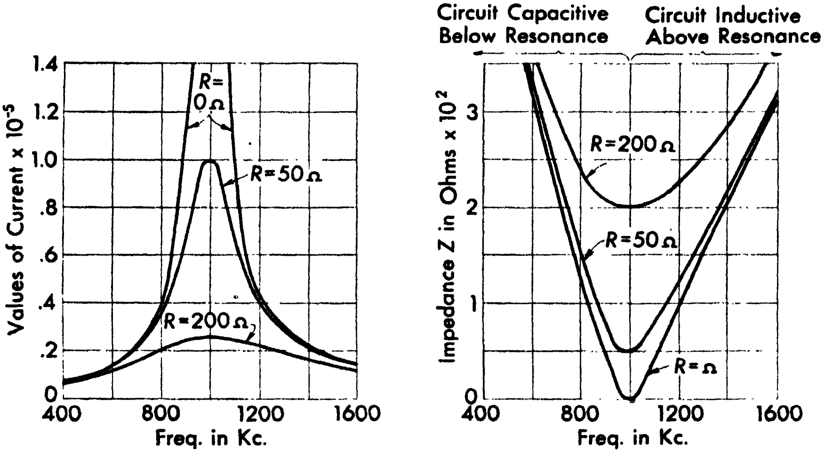


Fig. 148. A series resonant antenna circuit which offers no reactive opposition to the resonant frequency but opposes other frequencies both above and below the resonant frequency.

220. Effect of Resistance on Series Resonance. At any frequency, either above or below the resonant frequency of a series circuit, the current is not only opposed by the resistance of the circuit but by reactance as well. The total opposition (Z) at any frequency is determined by the vectorial sum of the reactances and the resistance.

Suppose that the antenna length, coil, and condenser of Fig. 148

are selected such that the circuit will tune to resonance at 1,000 kilocycles with the variable condenser set at half capacitance. If there were no resistance in the circuit, there would be no opposition to the flow of current. In actual practice, we find considerable resistance present in such a circuit. The effect of resistance in a series a-c circuit is shown in Fig. 149. The effect of resistance on the flow of current is shown in *a* and the effect of resistance on the impedance of the circuit is shown in *b*. Figure 149 shows that resistance in a series a-c circuit tends to flatten both the current curve and the impedance curve.



(a) (b)

Fig. 149. Effect of resistance on a series resonant circuit such as that of Fig. 148. *a*. Variations of current with frequency. *b*. Variation of impedance with frequency.

The student must fully realize the significance of the curves of Fig. 149. As long as the circuit is tuned to resonance at 1,000 kilocycles, it will offer less opposition to the flow of current at that frequency than to a current at any other. In fact, the opposition to currents at frequencies other than 1,000 kilocycles increases rapidly as we tune away from the resonant frequency on either side of resonance. As the amount of resistance in the circuit is increased, the above condition is not so pronounced. A point may be reached where the resistance in the circuit is so high that there is little difference in the current at the resonant frequency and currents at other frequencies to either side of resonance. This is shown in Fig. 149 *a* for $R = 200$ ohms. The significance of this fact is that our ability to tune in a particular station on a certain frequency and reject all other stations on adjacent frequencies diminishes as the resistance of the circuit increases. The ability of the tuning circuits of a receiver to do this is termed the *selectivity* of the receiver.

It must be kept in mind that the point or frequency of resonance may be changed by changing either the amount of inductance or amount of capacitance. In practice, it is usually more convenient to construct a condenser whose capacitance is continuously variable from its minimum to its maximum value than it is to construct an inductance whose inductance is continuously variable. The tuning of most receivers is accomplished by the use of variable condensers. The number of variable condensers is determined by the number of circuits that must be tuned. These circuits may not always be series resonant circuits. In fact, they are usually parallel resonant (section 224) circuits. The circuits are so designed that all the variable tuning condensers may be ganged together to a single tuning control that has the frequency range calibration directly on the face of the receiver tuning scale.

Exercise 57:

1. A 30-ohm resistor is connected in series with a coil of 25 millihenries inductance. Determine the impedance and phase angle between voltage and current at 800 kilocycles. What is the power factor?
2. Draw the vector diagram for impedance in problem 1.
3. Draw the vector diagram for current and voltage in problem 1.
4. A .002-microfarad condenser, .001-microfarad condenser, and a 100-ohm resistor are connected in series. What are the impedance and phase angle at 1,150 kilocycles?
5. Prepare a set of waves to scale for problem 1 similar to those of Fig. 145 if the supply voltage is 260 volts.
6. Prepare a set of waves to scale for problem 4 similar to those for Fig. 145 if the supply voltage is 460 volts.
7. A condenser of 6 ohms reactance is connected in series with a coil with 10 ohms reactance and a resistor of 8 ohms resistance in an r-f circuit operating at 400 kilocycles. Determine: (*a*) impedance at 400 kilocycles, (*b*) capacitance of condenser, (*c*) inductance of coil, (*d*) at what frequency the coil and condenser will resonate, (*e*) the impedance at the new frequency.
8. What r-f voltage in the antenna of Fig. 148 was required when the antenna resistance was considered to be 50 ohms? 200 ohms?

221. Resistance and Inductance in Parallel. In a series a-c circuit, the current through each element is the same, the supply voltage is across the external ends of the circuit, and the voltage drop across each element is determined by the resistance or reactance of the element. The opposite is true for a parallel a-c circuit. The supply *voltage* is impressed *across* the terminals of *all elements*, and the *current* through each element is *determined by the resistance or reactance of the element.*

As pointed out in section 213, the impedance of a parallel a-c circuit containing reactance cannot be determined vectorially as in the instance of a series a-c circuit containing reactance. However, the currents in a parallel circuit are added vectorially (section 214).

If no voltage or current is given, any voltage may be assumed in order to solve for the impedance of any parallel circuit.

Consider the circuit shown in *a* of Fig. 150. The current in each branch of the circuit is determined by Ohm's law:

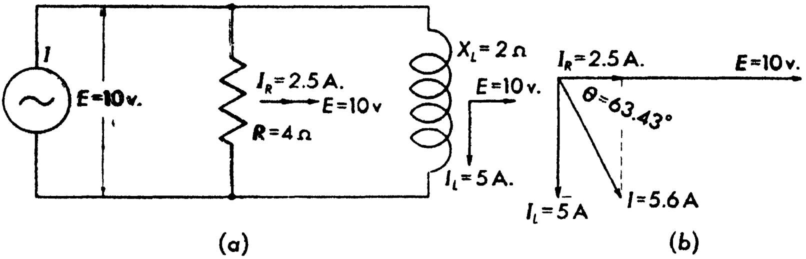


Fig. 150. Showing: *a.* Inductance and resistance connected in parallel. *b.* Vectorial addition of the currents of *a.*

(114) $$I_R = \frac{E}{R}$$

(115) $$I_L = \frac{E}{X_L}$$

and the total current is determined from the expression:

(116) $$I = \sqrt{I_R^2 + I_L^2}$$

where

I = total current

I_R = current through the resistance

I_L = current through the inductance

$$I_R = \frac{10}{4} = 2.5 \text{ amperes}$$

$$I_L = \frac{10}{2} = 5 \text{ amperes}$$

$$I = \sqrt{(2.5)^2 + (5)^2} = 5.6 \text{ amperes}$$

$$Z = \frac{E}{I} = \frac{10}{5.6} = 1.786 \text{ ohms impedance of the parallel circuit.}$$

From equation 105, the phase angle between the total current and the supply voltage is:

$$\theta = \tan^{-1}\frac{5}{2.5} = \tan^{-1} 2 = 63.43^\circ \text{ that } I \text{ lags } E.$$

222. Resistance and Capacitance in Parallel. The solution of a parallel a-c circuit containing resistance and capacitance is solved in the same general way that was outlined in the previous section for the solution of a parallel circuit containing resistance and inductance.

Figure 151 *a* shows a resistor and condenser connected in parallel. The current in each branch of the circuit is determined from Ohm's law. I_R is determined from equation 114 and I_C from equation 117.

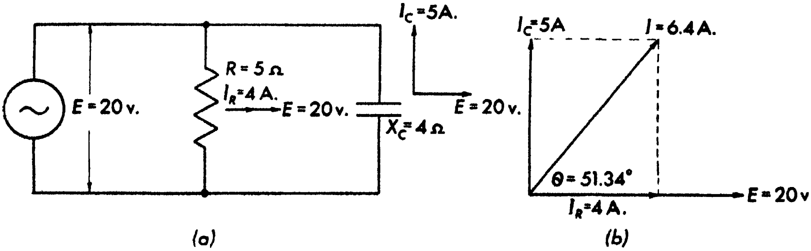


Fig. 151. Showing: *a.* A resistor and a condenser connected in parallel. *b.* Vector diagram of the currents of *a.*

$$I_C = \frac{E}{X_C} \tag{117}$$

The total current I is determined from equation 118.

$$I = \sqrt{I_R^2 + I_C^2} \tag{118}$$

where

I = total current

I_C = current flow to the condenser

I_R = current flow through the resistor

$$I_R = \frac{20}{5} = 4 \text{ amperes}$$

$$I_C = \frac{20}{4} = 5 \text{ amperes}$$

$$I = \sqrt{(4)^2 + (5)^2} = 6.4 \text{ amperes}$$

$$Z = \frac{E}{I} = \frac{20}{6.4} = 3.12 \text{ ohms impedance of the parallel circuit.}$$

From equation 105, the phase angle between the total current and the supply voltage is:

$$\theta = \tan^{-1}\frac{5}{4} = \tan^{-1} 1.25 = 51.34° \text{ that } I \textit{ leads } E.$$

223. Resistance, Inductance, and Capacitance in Parallel. A resistor, an inductance, and a capacitor are shown connected in parallel in Fig. 152 *a*. The current in each branch of the circuit is determined from Ohm's law.

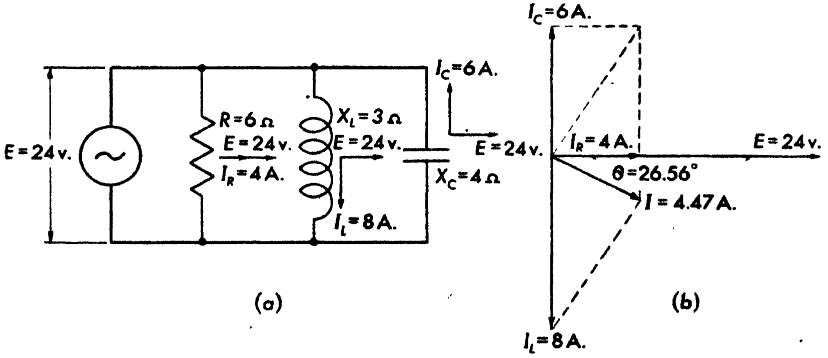


Fig. 152. a. An a-c circuit containing a resistor, an inductance, and a capacitor connected in parallel. *b.* Vector diagram of the currents.

$I_R = \frac{E}{R} = \frac{24}{6} = 4$ amperes of current flowing through the resistor

$I_L = \frac{E}{X_L} = \frac{24}{3} = 8$ amperes of current flowing through the inductance

$I_C = \frac{E}{X_C} = \frac{24}{4} = 6$ amperes charging current of the condenser.

The total current is determined vectorially as indicated in Fig. 152 *b* or from:

$$I = \sqrt{I_R^2 + (I_L - I_C)^2} \tag{119}$$

Substituting in equation 119:

$I = \sqrt{(4)^2 + (8 - 6)^2} = 4.47$ amperes, total circuit current.

Since the *capacitive reactance* is *greater* than the *inductive reactance*, the *capacitive current* is *less* than the *inductive current* and the *resultant current* is *inductive* and *lags* the *supply voltage*. The circuit is also considered to be inductive and acts as an inductance. If the capacitive reactance had been *less* than the inductive reactance, the capacitive current would have been *greater* than the inductive current, and the resultant current would have been capacitive and *leading* the supply voltage. The circuit in this instance would be capacitive and act as a capacitance.

The phase angle between the resultant inductive current and the supply voltage of Fig. 152 is:

$\theta = \tan^{-1} \frac{8 - 6}{4} = 26.56°$ that *I lags E*.

$Z = \frac{E}{I} = \frac{24}{4.47} = 5.37$ ohms impedance of the circuit.

224. Parallel Resonance. In the study of series resonance, it was shown that the inductive reactance of the circuit equaled the capacitive reactance of the circuit at the resonant frequency. This is also true for *parallel resonance* if the inductive current is equal to the capacitive current at the resonant frequency. Since the reactive currents are opposite (180° out of phase), they cancel each other, and the only current taken by a parallel circuit is due to the *resistance* of the circuit.

If there is no resistance in a parallel resonant circuit, the current from the supply source is zero and the impedance of the circuit is infinite as shown from Ohm's law:

$$Z = \frac{E}{I} = \frac{E}{0} = \text{infinity}$$

It is never possible to have a coil that is free from resistance. The actual current flowing through a coil is determined by considering its inductive reactance and its resistance as a *series circuit.* The current is determined from the *impedance* of this series circuit. Obviously, the *reactive current* due to the inductance of the coil is *less* than it would be without coil resistance. Likewise, if the value of inductance for a parallel resonant circuit is determined by equating X_L to X_C (equation 113), the value will be true only if the coil is considered to *be free of resistance.* The coil resistance added (vectorially) to the coil reactance will *limit* the *reactive component* of the coil current to a value *less* than that of the condenser,[1] which it must equal for the parallel circuit to be resonant. The inductance of the coil must be *reduced* for the coil current to be increased sufficiently beyond that of the condenser to allow the *inductive component* of the *coil current* to increase to a value exactly equaling the condenser current.

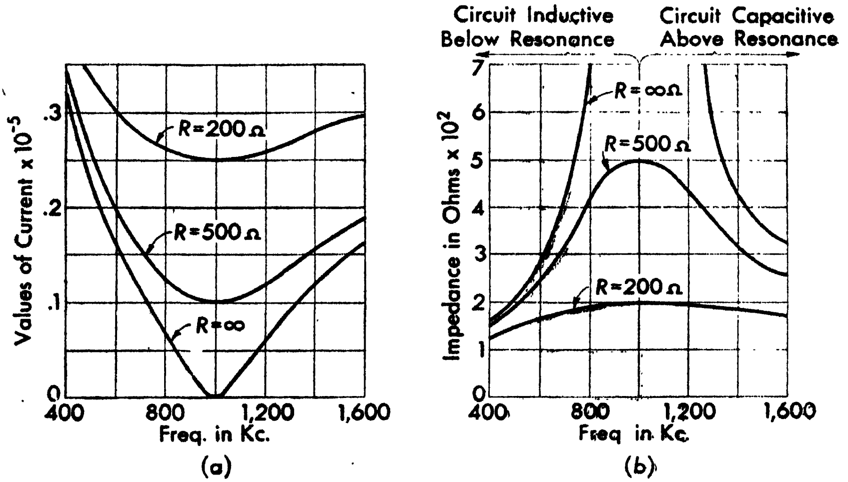
225. Effect of Resistance on Parallel Resonance. In a series resonant circuit, it is desirable that the resistance of the circuit be as low as possible since series circuits are used as *acceptor* circuits. A series circuit accepts the desired signal (resonant frequency) and opposes all others as shown in Fig. 149.

It is also desirable that the resistance in a parallel resonant circuit be kept to a minimum since parallel circuits are used as *rejector* circuits. In such circuits the current is low and the voltage is high. Such a circuit is used to great advantage in the input circuit of vacuum tubes in receivers operating as class A amplifiers. Such tubes are *voltage-operated* devices, and it is desirable that the received radio-signal voltage in the tube input circuit be as high as possible.

The effect of resistance in a parallel circuit is shown in Fig. 153. These curves should be compared with those of Fig. 149 showing the effect of resistance in a series circuit. The data used to plot the curves of Fig. 153 is shown in compiled form in Fig. 154.

---

[1] Assuming the condenser to be perfect since, in practice, it is usually possible to select a condenser whose losses are negligible.



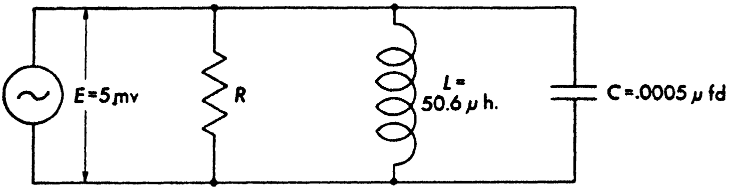
(a) (b)

Fig. 153. Effect of resistance on a parallel resonant circuit such as that of Fig. 152. *a.* Variation of current with frequency. *b.* Variation of impedance with frequency.

226. Series, Parallel A-c Circuits. Earlier in the text, in the study of resistors, series circuits were first considered, then parallel circuits, and finally, series-parallel circuits. The circuits containing both series and parallel combinations of resistors were solved by methods introduced in the solutions of series circuits and parallel circuits. Likewise, the solution of a-c circuits containing both series and parallel elements of resistance, inductive reactance, and capacitive reactance may be solved by the methods introduced in previous sections of this chapter.

227. Impedance Matching. In power circuits the frequency is so low (60 cyles) that inductive reactance values are relatively low in comparison to the inductive reactance values encountered at radio frequencies. Even so, power companies often find it advantageous to add capacitive reactance in order to cancel the effect of inductance. This allows a greater transfer of power to be made from the power company to the consumer.

Nevertheless, the process of plugging a receptical into an a-c wall-plug outlet is far more simple than is the process of obtaining a transfer of radio power from one circuit to another. The circuit which



$X_L = 2\pi f L$ $\qquad$ $X_C = \frac{1}{2\pi f C}$

$I_R = \frac{E}{R}$ $\qquad$ $I_L = \frac{E}{X_L}$ $\qquad$ $I_C = \frac{E}{X_C}$ $\qquad$ $I = \sqrt{I_R^2 + (I_L - I_C)^2}$

KC	OHMS				AMPERES x 10^{-5}			
f	X_L	X_C	R	Z	I_L	I_C	I_R	I
400	127	795	∞	151	.394	.063	0	.331
600	191	530	∞	297	.262	.094	0	.168
800	254	398	∞	704	.197	.126	0	.071
1,000	318	318	∞	∞	.157	.157	0	0
1,200	382	265	∞	862	.131	.189	0	.058
1,400	445	227	∞	423	.112	.220	0	.118
1,600	509	199	∞	327	.098	.251	0	.153
400	127	795	200	120	.394	.063	.250	.415
600	191	530	200	166	.262	.094	.250	.301
800	254	398	200	192	.197	.126	.250	.260
1,000	318	318	200	200	.157	.157	.250	.250
1,200	382	265	200	195	.131	.189	.250	.256
1,400	445	227	200	180	.112	.220	.250	.277
1,600	509	199	200	171	.098	.251	.250	.293
400	127	795	500	145	.394	.063	.100	.345
600	191	530	500	255	.262	.094	.100	.196
800	254	398	500	407	.197	.126	.100	.123
1,000	318	318	500	500	.157	.157	.100	.100
1,200	382	265	500	431	.131	.189	.100	.116
1,400	445	227	500	323	.112	.220	.100	.155
1,600	509	199	500	273	.098	.251	.100	.183

Fig. 154. Compiled data used in plotting the curves of Fig. 153.

receives the power is called the *load.* A maximum transfer of power is obtained from one circuit to another when the impedances of the two circuits are *matched.* Two impedances are matched to each other when their resistance values are equal and their reactive components are equal and opposite.

Impedance matching, in a practical sense, means the conditions under which the maximum *power* is transferred from one circuit to another. The transformer is used extensively to couple one circuit to another. The output of an audio-frequency amplifier is usually the secondary winding of an output transformer. The output impedance of the transformer may be any impedance, but a value of 500 ohms is very common. If the amplifier is coupled to the input (primary winding) transformer of another amplifier, it is desirable that the input impedance of the input transformer also be 500 ohms in order to match the input of the second amplifier to the output of the first for a maximum transfer of power.

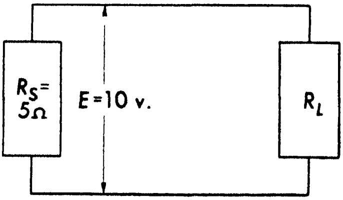


Fig. 155. Showing the power source resistance R_S to be in series with the load resistance R_L and the total circuit resistance $R = R_S + R_L$. A generator with an internal resistance R_S will transfer the maximum power to the load resistor R_L when $R_S = R_L$.

The maximum transfer of power, occurring when the load impedance is matched to the source impedance, is demonstrated in the circuit of Fig. 155. A generator with an internal resistance R_S is connected to a load resistor R_L. Current flowing through resistor R_L must also flow through R_S and the total circuit resistance R is:

$$R = R_S + R_L \tag{120}$$

$$I = \frac{E}{R_S + R_L} \tag{121}$$

$$P = I^2R \tag{122}$$

$$P_L = I^2R_L \tag{123}$$

where

R = total circuit resistance
R_S = resistance of power source
R_L = resistance of load
I = current
P = total power
P_L = power received by load resistor

The power in the load resistor of Fig. 155 for various values of load resistance is shown by the following data:

E	I	R	R_S	R_L	P	P_S	P_L
10	1.662	6	5	1	16.62	13.85	2.77
10	1.428	7	5	2	14.28	10.19	4.09
10	1.250	8	5	3	12.50	7.81	4.69
10	1.112	9	5	4	11.12	6.18	4.94
10	**1**	**10**	**5**	**5**	**10**	**5**	**5**
10	.908	11	5	6	9.08	4.13	4.95
10	.833	12	5	7	8.33	3.47	4.86
10	.768	13	5	8	7.68	2.95	4.73

An inspection of the foregoing data shows that considerable power is lost when we fail to match the load impedance to the source impedance.

It can be shown in the impedance matching of transformer windings that the turns ratio is equal to the square root of the impedance ratio:

$$\frac{N_P}{N_S} = \sqrt{\frac{Z_P}{Z_S}} \tag{124}$$

where

N_P = turns on the primary
N_S = turns on the secondary
Z_P = primary impedance
Z_S = secondary impedance

Exercise 58:

1. A 10-ohm resistor is connected in parallel with a coil with 10 ohms of reactance. Determine: (*a*) the impedance; (*b*) phase angle between the

supply voltage and total current; (*c*) draw circuit and vectors as was done in Fig. 150, if the supply voltage is 10 volts alternating current.

2. Repeat *a* and *b* of problem 1 for a line voltage of 15 volts and also for 25 volts.

3. Consider problem 7 of Exercise 57: Suppose that the condenser, resistor, and coil are now connected in parallel. (*a*) What will be the resonant frequency? (*b*) Suppose that the resistor is connected in series with the coil and this circuit connected in parallel with the condenser. Will the circuit resonate, above, below, or at the same frequency as for *b?*

4. (*a*) Prepare a set of data similar to that shown and for the frequencies and circuit of Fig. 154 for $R = 100$ ohms. (*b*) Use this data to plot graphs similar to those of Fig. 153 and compare.

5. The output transformer of an amplifier must couple from the 9,000-ohm plate impedance of the last tube to a 500-ohm line. What turns ratio will be required for the windings?

6. The output winding of the transformer of problem 5 has 200 turns and a voltage of 30 volts. It is desired to use a matching transformer to couple the amplifier to a loudspeaker with a voice coil of 5 ohms impedance. Determine: (*a*) turns on the secondary; (*b*) current in the voice coil; (*c*) power used by the loudspeaker.

REVIEW QUESTIONS

1. What is impedance?
2. How is the impedance of a series circuit determined?
3. Can the impedance of a parallel circuit be determined vectorially?
4. How are the voltages in a series circuit containing resistance and reactance added?
5. How are the currents in a parallel circuit containing resistance and reactance added?
6. How is the E-I phase of: (a) A series circuit determined? (b) A parallel circuit determined?
7. What is meant by: (a) Series resonance; (b) Parallel resonance?
8. How does resistance affect the impedance of: (a) A series circuit; (b) A parallel circuit?
9. Why is a series circuit capacitive below resonance and inductive above resonance while the opposite is true for a parallel circuit?
10. Why may any voltage be assumed for the solution of a parallel circuit when no voltage or currents are given?
11. Why is the impedance of a parallel resonant circuit infinite if there is no resistor in the circuit? (Meaning that actually the circuit resistance in such a case is infinite.)
12. What is meant by impedance matching?
13. Why is it advantageous to match impedances when connecting one circuit to another such as connecting the output of a receiver to a speaker?

14. How does the turns ratio of a transformer compare with the impedance ratio of the transformer?

ANSWERS

Exercise 57, page 259:

1. 129.2 ohms impedance; 76.6° phase angle of E leading; power factor = .2317.

4. 230 ohms impedance; 64.4° phase angle of E lagging I.

7. *a.* 8.94 ohms impedance
  b. .0662 microfarad
  c. 3.97 microhenries
  d. 310 kilocycles resonant frequency
  e. 8 ohms impedance

8. $\frac{1}{2}$ millivolt in each instance.

Exercise 58, page 268:

1. *a.* 7.07 ohms impedance
  b. 45° I lagging E

2. For line voltage = 15 volts
  a. 7.07 ohms impedance
  b. 45° I lagging E
  For line voltage = 25 volts
  a. 7.07 ohms impedance
  b. 45° I lagging E

3. *a.* 400 kilocycles
  b. Resonate at a frequency less than 400 kilocycles

5. 4.24 to 1

6. *a.* 20 turns
  b. .6 ampere
  c. 1.8 watts

APPENDIX

Greek Alphabet

Name	*Pronunciation*	*Capital*	*Lower Case*
Alpha	al′fa	Α	α
Beta	bay′ta	Β	β
Gamma	gam′a	Γ	γ
Delta	del′ta	Δ	δ
Epsilon	ep′si-lahn	Ε	ϵ
Zeta	zay′ta	Ζ	ζ
Eta	ay′ta	Η	η
Theta	thay′ta	Θ	θ
Iota	eye-o′ta	Ι	ι
Kappa	kap′pa	Κ	κ
Lambda	lam′da	Λ	λ
Mu	myou	Μ	μ
Nu	nyou	Ν	ν
Xi	zie	Ξ	ξ
Omicron	o′mi-krahn	Ο	ο
Pi	pie	Π	π
Rho	ro	Ρ	ρ
Sigma	sig′ma	Σ	σ
Tau	tow (as in how)	Τ	τ
Upsilon	up′si-lahn	ϒ	υ
Phi	fie	Φ	ϕ
Chi	kie	Χ	χ
Psi	sie	Ψ	ψ
Omega	o-may′ga	Ω	ω

LOGARITHMIC AND TRIGONOMETRIC TABLES

TABLE V

COMMON LOGARITHMS OF NUMBERS

FROM

1 TO 10 000

TO

FIVE DECIMAL PLACES

1 — 100

N	Log	N	Log	N	Log	N	Log	N	Log
0	———	20	1.30 103	40	1.60 206	60	1.77 815	80	1.90 309
1	0.00 000	21	1.32 222	41	1.61 278	61	1.78 533	81	1.90 849
2	0.30 103	22	1.34 242	42	1.62 325	62	1.79 239	82	1.91 381
3	0.47 712	23	1.36 173	43	1.63 347	63	1.79 934	83	1.91 908
4	0.60 206	24	1.38 021	44	1.64 345	64	1.80 618	84	1.92 428
5	0.69 897	25	1.39 794	45	1.65 321	65	1.81 291	85	1.92 942
6	0.77 815	26	1.41 497	46	1.66 276	66	1.81 954	86	1.93 450
7	0.84 510	27	1.43 136	47	1.67 210	67	1.82 607	87	1.93 952
8	0.90 309	28	1.44 716	48	1.68 124	68	1.83 251	88	1.94 448
9	0.95 424	29	1.46 240	49	1.69 020	69	1.83 885	89	1.94 939
10	1.00 000	30	1.47 712	50	1.69 897	70	1.84 510	90	1.95 424
11	1.04 139	31	1.49 136	51	1.70 757	71	1.85 126	91	1.95 904
12	1.07 918	32	1.50 515	52	1.71 600	72	1.85 733	92	1.96 379
13	1.11 394	33	1.51 851	53	1.72 428	73	1.86 332	93	1.96 848
14	1.14 613	34	1.53 148	54	1.73 239	74	1.86 923	94	1.97 313
15	1.17 609	35	1.54 407	55	1.74 036	75	1.87 506	95	1.97 772
16	1.20 412	36	1.55 630	56	1.74 819	76	1.88 081	96	1.98 227
17	1.23 045	37	1.56 820	57	1.75 587	77	1.88 649	97	1.98 677
18	1.25 527	38	1.57 978	58	1.76 343	78	1.89 209	98	1.99 123
19	1.27 875	39	1.59 106	59	1.77 085	79	1.89 763	99	1.99 564
N	Log	N	Log	N	Log	N	Log	N	Log

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	44	43	42
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	29	**28**
1	2.9	2.8
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	**26**
1	2.7	2.6
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25	**24**
1	2.5	2.4
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

	23	**22**
1	2.3	2.2
2	4.6	4.4
3	6.9	6.6
4	9.2	8.8
5	11.5	11.0
6	13.8	13.2
7	16.1	15.4
8	18.4	17.6
9	20.7	19.8

	21
1	2.1
2	4.2
3	6.3
4	8.4
5	10.5
6	12.6
7	14.7
8	16.8
9	18.9

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

log 2
=.30102 99957

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20	19
1	2.0	1.9
2	4.0	3.8
3	6.0	5.7
4	8.0	7.6
5	10.0	9.5
6	12.0	11.4
7	14.0	13.3
8	16.0	15.2
9	18.0	17.1

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
250	39 794	811	829	846	863	881	898	915	933	950
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123
252	40 140	157	175	192	209	226	243	261	278	295
253	312	329	346	364	381	398	415	432	449	466
254	483	500	518	535	552	569	586	603	620	637
255	654	671	688	705	722	739	756	773	790	807
256	824	841	858	875	892	909	926	943	960	976
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145
258	41 162	179	196	212	229	246	263	280	296	313
259	330	347	363	380	397	414	430	447	464	481
260	497	514	531	547	564	581	597	614	631	647
261	664	681	697	714	731	747	764	780	797	814
262	830	847	863	880	896	913	929	946	963	979
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144
264	42 160	177	193	210	226	243	259	275	292	308
265	325	341	357	374	390	406	423	439	455	472
266	488	504	521	537	553	570	586	602	619	635
267	651	667	684	700	716	732	749	765	781	797
268	813	830	846	862	878	894	911	927	943	959
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120
270	43 136	152	169	185	201	217	233	249	265	281
271	297	313	329	345	361	377	393	409	425	441
272	457	473	489	505	521	537	553	569	584	600
273	616	632	648	664	680	696	712	727	743	759
274	775	791	807	823	838	854	870	886	902	917
275	933	949	965	981	996	*012	*028	*044	*059	*075
276	44 091	107	122	138	154	170	185	201	217	232
277	248	264	279	295	311	326	342	358	373	389
278	404	420	436	451	467	483	498	514	529	545
279	560	576	592	607	623	638	654	669	685	700
280	716	731	747	762	778	793	809	824	840	855
281	871	886	902	917	932	948	963	979	994	*010
282	45 025	040	056	071	086	102	117	133	148	163
283	179	194	209	225	240	255	271	286	301	317
284	332	347	362	378	393	408	423	439	454	469
285	484	500	515	530	545	561	576	591	606	621
286	637	652	667	682	697	712	728	743	758	773
287	788	803	818	834	849	864	879	894	909	924
288	939	954	969	984	*000	*015	*030	*045	*060	*075
289	46 090	105	120	135	150	165	180	195	210	225
290	240	255	270	285	300	315	330	345	359	374
291	389	404	419	434	449	464	479	494	509	523
292	538	553	568	583	598	613	627	642	657	672
293	687	702	716	731	746	761	776	790	805	820
294	835	850	864	879	894	909	923	938	953	967
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114
296	47 129	144	159	173	188	202	217	232	246	261
297	276	290	305	319	334	349	363	378	392	407
298	422	436	451	465	480	494	509	524	538	553
299	567	582	596	611	625	640	654	669	683	698
300	712	727	741	756	770	784	799	813	828	842
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	18	17
1	1.8	1.7
2	3.6	3.4
3	5.4	5.1
4	7.2	6.8
5	9.0	8.5
6	10.8	10.2
7	12.6	11.9
8	14.4	13.6
9	16.2	15.3

M
$= \log_{10} e$
$= \log_{10} 2.718\cdots$
$= .43429\ 44819$

	16	15
1	1.6	1.5
2	3.2	3.0
3	4.8	4.5
4	6.4	6.0
5	8.0	7.5
6	9.6	9.0
7	11.2	10.5
8	12.8	12.0
9	14.4	13.5

	14
1	1.4
2	2.8
3	4.2
4	5.6
5	7.0
6	8.4
7	9.8
8	11.2
9	12.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	
306	572	586	601	615	629	643	657	671	686	700	
307	714	728	742	756	770	785	799	813	827	841	
308	855	869	883	897	911	926	940	954	968	982	
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49 136	150	164	178	192	206	220	234	248	262	
311	276	290	304	318	332	346	360	374	388	402	
312	415	429	443	457	471	485	499	513	527	541	
313	554	568	582	596	610	624	638	651	665	679	
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50 106	120	133	147	161	174	188	202	215	229	
318	243	256	270	284	297	311	325	338	352	365	
319	379	393	406	420	433	447	461	474	488	501	
320	515	529	542	556	569	583	596	610	623	637	
321	651	664	678	691	705	718	732	745	759	772	
322	786	799	813	826	840	853	866	880	893	907	
323	920	934	947	961	974	987	*001	*014	*028	*041	
324	51 055	068	081	095	108	121	135	148	162	175	
325	188	202	215	228	242	255	268	282	295	308	
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	
328	587	601	614	627	640	654	667	680	693	706	
329	720	733	746	759	772	786	799	812	825	838	
330	851	865	878	891	904	917	930	943	957	970	
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52 114	127	140	153	166	179	192	205	218	231	
333	244	257	270	284	297	310	323	336	349	362	
334	375	388	401	414	427	440	453	466	479	492	
335	504	517	530	543	556	569	582	595	608	621	
336	634	647	660	673	686	699	711	724	737	750	
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	
339	53 020	033	046	058	071	084	097	110	122	135	
340	148	161	173	186	199	212	224	237	250	263	
341	275	288	301	314	326	339	352	364	377	390	
342	403	415	428	441	453	466	479	491	504	517	
343	529	542	555	567	580	593	605	618	631	643	
344	656	668	681	694	706	719	732	744	757	769	
345	782	794	807	820	832	845	857	870	882	895	
346	908	920	933	945	958	970	983	995	*008	*020	
347	54 033	045	058	070	083	095	108	120	133	145	
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

log 3
=.47712 12547
log π
=.49714 98727

	15	14
1	1.5	1.4
2	3.0	2.8
3	4.5	4.2
4	6.0	5.6
5	7.5	7.0
6	9.0	8.4
7	10.5	9.8
8	12.0	11.2
9	13.5	12.6

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

N.	0	1	2	3	4	5	6	7	8	9
350	54 407	419	432	444	456	469	481	494	506	518
351	531	543	555	568	580	593	605	617	630	642
352	654	667	679	691	704	716	728	741	753	765
353	777	790	802	814	827	839	851	864	876	888
354	900	913	925	937	949	962	974	986	998	*011
355	55 023	035	047	060	072	084	096	108	121	133
356	145	157	169	182	194	206	218	230	242	255
357	267	279	291	303	315	328	340	352	364	376
358	388	400	413	425	437	449	461	473	485	497
359	509	522	534	546	558	570	582	594	606	618
360	630	642	654	666	678	691	703	715	727	739
361	751	763	775	787	799	811	823	835	847	859
362	871	883	895	907	919	931	943	955	967	979
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098
364	56 110	122	134	146	158	170	182	194	205	217
365	229	241	253	265	277	289	301	312	324	336
366	348	360	372	384	396	407	419	431	443	455
367	467	478	490	502	514	526	538	549	561	573
368	585	597	608	620	632	644	656	667	679	691
369	703	714	726	738	750	761	773	785	797	808
370	820	832	844	855	867	879	891	902	914	926
371	937	949	961	972	984	996	*008	*019	*031	*043
372	57 054	066	078	089	101	113	124	136	148	159
373	171	183	194	206	217	229	241	252	264	276
374	287	299	310	322	334	345	357	368	380	392
375	403	415	426	438	449	461	473	484	496	507
376	519	530	542	553	565	576	588	600	611	623
377	634	646	657	669	680	692	703	715	726	738
378	749	761	772	784	795	807	818	830	841	852
379	864	875	887	898	910	921	933	944	955	967
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081
381	58 092	104	115	127	138	149	161	172	184	195
382	206	218	229	240	252	263	274	286	297	309
383	320	331	343	354	365	377	388	399	410	422
384	433	444	456	467	478	490	501	512	524	535
385	546	557	569	580	591	602	614	625	636	647
386	659	670	681	692	704	715	726	737	749	760
387	771	782	794	805	816	827	838	850	861	872
388	883	894	906	917	928	939	950	961	973	984
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095
390	59 106	118	129	140	151	162	173	184	195	207
391	218	229	240	251	262	273	284	295	306	318
392	329	340	351	362	373	384	395	406	417	428
393	439	450	461	472	483	494	506	517	528	539
394	550	561	572	583	594	605	616	627	638	649
395	660	671	682	693	704	715	726	737	748	759
396	770	780	791	802	813	824	835	846	857	868
397	879	890	901	912	923	934	945	956	966	977
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086
399	60 097	108	119	130	141	152	163	173	184	195
400	206	217	228	239	249	260	271	282	293	304
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	13	12
1	1.3	1.2
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9
400	60 206	217	228	239	249	260	271	282	293	304
401	314	325	336	347	358	369	379	390	401	412
402	423	433	444	455	466	477	487	498	509	520
403	531	541	552	563	574	584	595	606	617	627
404	638	649	660	670	681	692	703	713	724	735
405	746	756	767	778	788	799	810	821	831	842
406	853	863	874	885	895	906	917	927	938	949
407	959	970	981	991	*002	*013	*023	*034	*045	*055
408	61 066	077	087	098	109	119	130	140	151	162
409	172	183	194	204	215	225	236	247	257	268
410	278	289	300	310	321	331	342	352	363	374
411	384	395	405	416	426	437	448	458	469	479
412	490	500	511	521	532	542	553	563	574	584
413	595	606	616	627	637	648	658	669	679	690
414	700	711	721	731	742	752	763	773	784	794
415	805	815	826	836	847	857	868	878	888	899
416	909	920	930	941	951	962	972	982	993	*003
417	62 014	024	034	045	055	066	076	086	097	107
418	118	128	138	149	159	170	180	190	201	211
419	221	232	242	252	263	273	284	294	304	315
420	325	335	346	356	366	377	387	397	408	418
421	428	439	449	459	469	480	490	500	511	521
422	531	542	552	562	572	583	593	603	613	624
423	634	644	655	665	675	685	696	706	716	726
424	737	747	757	767	778	788	798	808	818	829
425	839	849	859	870	880	890	900	910	921	931
426	941	951	961	972	982	992	*002	*012	*022	*033
427	63 043	053	063	073	083	094	104	114	124	134
428	144	155	165	175	185	195	205	215	225	236
429	246	256	266	276	286	296	306	317	327	337
430	347	357	367	377	387	397	407	417	428	438
431	448	458	468	478	488	498	508	518	528	538
432	548	558	568	579	589	599	609	619	629	639
433	649	659	669	679	689	699	709	719	729	739
434	749	759	769	779	789	799	809	819	829	839
435	849	859	869	879	889	899	909	919	929	939
436	949	959	969	979	988	998	*008	*018	*028	*038
437	64 048	058	068	078	088	098	108	118	128	137
438	147	157	167	177	187	197	207	217	227	237
439	246	256	266	276	286	296	306	316	326	335
440	345	355	365	375	385	395	404	414	424	434
441	444	454	464	473	483	493	503	513	523	532
442	542	552	562	572	582	591	601	611	621	631
443	640	650	660	670	680	689	699	709	719	729
444	738	748	758	768	777	787	797	807	816	826
445	836	846	856	865	875	885	895	904	914	924
446	933	943	953	963	972	982	992	*002	*011	*021
447	65 031	040	050	060	070	079	089	099	108	118
448	128	137	147	157	167	176	186	196	205	215
449	225	234	244	254	263	273	283	292	302	312
450	321	331	341	350	360	369	379	389	398	408
N.	0	1	2	3	4	5	6	7	8	9

Prop. Pts.

	11	10
1	1.1	1.0
2	2.2	2.0
3	3.3	3.0
4	4.4	4.0
5	5.5	5.0
6	6.6	6.0
7	7.7	7.0
8	8.8	8.0
9	9.9	9.0

log M
=log [log e]
=9.63778 431
−10

	9
1	0.9
2	1.8
3	2.7
4	3.6
5	4.5
6	5.4
7	6.3
8	7.2
9	8.1

Prop. Pts.

450 — Logarithms of Numbers — 500

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66 087	096	106	115	124	134	143	153	162	172	
459	181	191	200	210	219	229	238	247	257	266	
460	276	285	295	304	314	323	332	342	351	361	
461	370	380	389	398	408	417	427	436	445	455	
462	464	474	483	492	502	511	521	530	539	549	
463	558	567	577	586	596	605	614	624	633	642	
464	652	661	671	680	689	699	708	717	727	736	
465	745	755	764	773	783	792	801	811	820	829	
466	839	848	857	867	876	885	894	904	913	922	
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	
473	486	495	504	514	523	532	541	550	560	569	
474	578	587	596	605	614	624	633	642	651	660	
475	669	679	688	697	706	715	724	733	742	752	
476	761	770	779	788	797	806	815	825	834	843	
477	852	861	870	879	888	897	906	916	925	934	
478	943	952	961	970	979	988	997	*006	*015	*024	
479	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	
488	842	851	860	869	878	886	895	904	913	922	
489	931	940	949	958	966	975	984	993	*002	*011	
490	69 020	028	037	046	055	064	073	082	090	099	
491	108	117	126	135	144	152	161	170	179	188	
492	197	205	214	223	232	241	249	258	267	276	
493	285	294	302	311	320	329	338	346	355	364	
494	373	381	390	399	408	417	425	434	443	452	
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	10	9
1	1.0	0.9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

	8
1	0.8
2	1.6
3	2.4
4	3.2
5	4.0
6	4.8
7	5.6
8	6.4
9	7.2

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	log 5
505	329	338	346	355	364	372	381	389	398	406	=.69897 00043
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	
510	757	766	774	783	791	800	808	817	825	834	
511	842	851	859	868	876	885	893	902	910	919	
512	927	935	944	952	961	969	978	986	995	*003	
513	71 012	020	029	037	046	054	063	071	079	088	
514	096	105	113	122	130	139	147	155	164	172	
515	181	189	198	206	214	223	231	240	248	257	
516	265	273	282	290	299	307	315	324	332	341	
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	
525	72 016	024	032	041	049	057	066	074	082	090	
526	099	107	115	123	132	140	148	156	165	173	
527	181	189	198	206	214	222	230	239	247	255	
528	263	272	280	288	296	304	313	321	329	337	
529	346	354	362	370	378	387	395	403	411	419	
530	428	436	444	452	460	469	477	485	493	501	
531	509	518	526	534	542	550	558	567	575	583	
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73 078	086	094	102	111	119	127	135	143	151	
539	159	167	175	183	191	199	207	215	223	231	
540	239	247	255	263	272	280	288	296	304	312	
541	320	328	336	344	352	360	368	376	384	392	
542	400	408	416	424	432	440	448	456	464	472	
543	480	488	496	504	512	520	528	536	544	552	
544	560	568	576	584	592	600	608	616	624	632	
545	640	648	656	664	672	679	687	695	703	711	
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	9	8
1	0.9	0.8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	7
1	0.7
2	1.4
3	2.1
4	2.8
5	3.5
6	4.2
7	4.9
8	5.6
9	6.3

550 — Logarithms of Numbers — 600

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	
564	128	136	143	151	159	166	174	182	189	197	
565	205	213	220	228	236	243	251	259	266	274	
566	282	289	297	305	312	320	328	335	343	351	
567	358	366	374	381	389	397	404	412	420	427	
568	435	442	450	458	465	473	481	488	496	504	
569	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	
585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
588	938	945	953	960	967	975	982	989	997	*004	
589	77 012	019	026	034	041	048	056	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	
608	390	398	405	412	419	426	433	440	447	455	
609	462	469	476	483	490	497	504	512	519	526	
610	533	540	547	554	561	569	576	583	590	597	
611	604	611	618	625	633	640	647	654	661	668	
612	675	682	689	696	704	711	718	725	732	739	
613	746	753	760	767	774	781	789	796	803	810	
614	817	824	831	838	845	852	859	866	873	880	
615	888	895	902	909	916	923	930	937	944	951	
616	958	965	972	979	986	993	*000	*007	*014	*021	
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	
623	449	456	463	470	477	484	491	498	505	511	
624	518	525	532	539	546	553	560	567	574	581	
625	588	595	602	609	616	623	630	637	644	650	
626	657	664	671	678	685	692	699	706	713	720	
627	727	734	741	748	754	761	768	775	782	789	
628	796	803	810	817	824	831	837	844	851	858	
629	865	872	879	886	893	900	906	913	920	927	
630	934	941	948	955	962	969	975	982	989	996	
631	80 003	010	017	024	030	037	044	051	058	065	
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	
638	482	489	496	502	509	516	523	530	536	543	
639	550	557	564	570	577	584	591	598	604	611	
640	618	625	632	638	645	652	659	665	672	679	
641	686	693	699	706	713	720	726	733	740	747	
642	754	760	767	774	781	787	794	801	808	814	
643	821	828	835	841	848	855	862	868	875	882	
644	889	895	902	909	916	922	929	936	943	949	
645	956	963	969	976	983	990	996	*003	*010	*017	
646	81 023	030	037	043	050	057	064	070	077	084	
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	8	7
1	0.8	0.7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

	6
1	0.6
2	1.2
3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

650 — Logarithms of Numbers — 700

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	
662	086	092	099	105	112	119	125	132	138	145	
663	151	158	164	171	178	184	191	197	204	210	
664	217	223	230	236	243	249	256	263	269	276	
665	282	289	295	302	308	315	321	328	334	341	
666	347	354	360	367	373	380	387	393	400	406	
667	413	419	426	432	439	445	452	458	465	471	
668	478	484	491	497	504	510	517	523	530	536	
669	543	549	556	562	569	575	582	588	595	601	
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	
685	569	575	582	588	594	601	607	613	620	626	
686	632	639	645	651	658	664	670	677	683	689	
687	696	702	708	715	721	727	734	740	746	753	
688	759	765	771	778	784	790	797	803	809	816	
689	822	828	835	841	847	853	860	866	872	879	
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

N.	0	1	2	3	4	5	6	7	8	9
700	84 510	516	522	528	535	541	547	553	559	566
701	572	578	584	590	597	603	609	615	621	628
702	634	640	646	652	658	665	671	677	683	689
703	696	702	708	714	720	726	733	739	745	751
704	757	763	770	776	782	788	794	800	807	813
705	819	825	831	837	844	850	856	862	868	874
706	880	887	893	899	905	911	917	924	930	936
707	942	948	954	960	967	973	979	985	991	997
708	85 003	009	016	022	028	034	040	046	052	058
709	065	071	077	083	089	095	101	107	114	120
710	126	132	138	144	150	156	163	169	175	181
711	187	193	199	205	211	217	224	230	236	242
712	248	254	260	266	272	278	285	291	297	303
713	309	315	321	327	333	339	345	352	358	364
714	370	376	382	388	394	400	406	412	418	425
715	431	437	443	449	455	461	467	473	479	485
716	491	497	503	509	516	522	528	534	540	546
717	552	558	564	570	576	582	588	594	600	606
718	612	618	625	631	637	643	649	655	661	667
719	673	679	685	691	697	703	709	715	721	727
720	733	739	745	751	757	763	769	775	781	788
721	794	800	806	812	818	824	830	836	842	848
722	854	860	866	872	878	884	890	896	902	908
723	914	920	926	932	938	944	950	956	962	968
724	974	980	986	992	998	*004	*010	*016	*022	*028
725	86 034	040	046	052	058	064	070	076	082	088
726	094	100	106	112	118	124	130	136	141	147
727	153	159	165	171	177	183	189	195	201	207
728	213	219	225	231	237	243	249	255	261	267
729	273	279	285	291	297	303	308	314	320	326
730	332	338	344	350	356	362	368	374	380	386
731	392	398	404	410	415	421	427	433	439	445
732	451	457	463	469	475	481	487	493	499	504
733	510	516	522	528	534	540	546	552	558	564
734	570	576	581	587	593	599	605	611	617	623
735	629	635	641	646	652	658	664	670	676	682
736	688	694	700	705	711	717	723	729	735	741
737	747	753	759	764	770	776	782	788	794	800
738	806	812	817	823	829	835	841	847	853	859
739	864	870	876	882	888	894	900	906	911	917
740	923	929	935	941	947	953	958	964	970	976
741	982	988	994	999	*005	*011	*017	*023	*029	*035
742	87 040	046	052	058	064	070	075	081	087	093
743	099	105	111	116	122	128	134	140	146	151
744	157	163	169	175	181	186	192	198	204	210
745	216	221	227	233	239	245	251	256	262	268
746	274	280	286	291	297	303	309	315	320	326
747	332	338	344	349	355	361	367	373	379	384
748	390	396	402	408	413	419	425	431	437	442
749	448	454	460	466	471	477	483	489	495	500
750	506	512	518	523	529	535	541	547	552	558
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

log 7
=.84509 80400

	7	6
1	0.7	0.6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

	5
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

Prop. Pts.

750 — Logarithms of Numbers — 800

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	
765	366	372	377	383	389	395	400	406	412	417	
766	423	429	434	440	446	451	457	463	468	474	
767	480	485	491	497	502	508	513	519	525	530	
768	536	542	547	553	559	564	570	576	581	587	
769	593	598	604	610	615	621	627	632	638	643	
770	649	655	660	666	672	677	683	689	694	700	
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	
783	376	382	387	393	398	404	409	415	421	426	
784	432	437	443	448	454	459	465	470	476	481	
785	487	492	498	504	509	515	520	526	531	537	
786	542	548	553	559	564	570	575	581	586	592	
787	597	603	609	614	620	625	631	636	642	647	
788	653	658	664	669	675	680	686	691	697	702	
789	708	713	719	724	730	735	741	746	752	757	
790	763	768	774	779	785	790	796	801	807	812	
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
809	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
811	902	907	913	918	924	929	934	940	945	950	
812	956	961	966	972	977	982	988	993	998	*004	
813	91 009	014	020	025	030	036	041	046	052	057	
814	062	068	073	078	084	089	094	100	105	110	
815	116	121	126	132	137	142	148	153	158	164	
816	169	174	180	185	190	196	201	206	212	217	
817	222	228	233	238	243	249	254	259	265	270	
818	275	281	286	291	297	302	307	312	318	323	
819	328	334	339	344	350	355	360	365	371	376	
820	381	387	392	397	403	408	413	418	424	429	
821	434	440	445	450	455	461	466	471	477	482	
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	
832	92 012	018	023	028	033	038	044	049	054	059	
833	065	070	075	080	085	091	096	101	106	111	
834	117	122	127	132	137	143	148	153	158	163	
835	169	174	179	184	189	195	200	205	210	215	
836	221	226	231	236	241	247	252	257	262	267	
837	273	278	283	288	293	298	304	309	314	319	
838	324	330	335	340	345	350	355	361	366	371	
839	376	381	387	392	397	402	407	412	418	423	
840	428	433	438	443	449	454	459	464	469	474	
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Prop. Pts.**

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

N.	0	1	2	3	4	5	6	7	8	9
850	92 942	947	952	957	962	967	973	978	983	988
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039
852	93 044	049	054	059	064	069	075	080	085	090
853	095	100	105	110	115	120	125	131	136	141
854	146	151	156	161	166	171	176	181	186	192
855	197	202	207	212	217	222	227	232	237	242
856	247	252	258	263	268	273	278	283	288	293
857	298	303	308	313	318	323	328	334	339	344
858	349	354	359	364	369	374	379	384	389	394
859	399	404	409	414	420	425	430	435	440	445
860	450	455	460	465	470	475	480	485	490	495
861	500	505	510	515	520	526	531	536	541	546
862	551	556	561	566	571	576	581	586	591	596
863	601	606	611	616	621	626	631	636	641	646
864	651	656	661	666	671	676	682	687	692	697
865	702	707	712	717	722	727	732	737	742	747
866	752	757	762	767	772	777	782	787	792	797
867	802	807	812	817	822	827	832	837	842	847
868	852	857	862	867	872	877	882	887	892	897
869	902	907	912	917	922	927	932	937	942	947
870	952	957	962	967	972	977	982	987	992	997
871	94 002	007	012	017	022	027	032	037	042	047
872	052	057	062	067	072	077	082	086	091	096
873	101	106	111	116	121	126	131	136	141	146
874	151	156	161	166	171	176	181	186	191	196
875	201	206	211	216	221	226	231	236	240	245
876	250	255	260	265	270	275	280	285	290	295
877	300	305	310	315	320	325	330	335	340	345
878	349	354	359	364	369	374	379	384	389	394
879	399	404	409	414	419	424	429	433	438	443
880	448	453	458	463	468	473	478	483	488	493
881	498	503	507	512	517	522	527	532	537	542
882	547	552	557	562	567	571	576	581	586	591
883	596	601	606	611	616	621	626	630	635	640
884	645	650	655	660	665	670	675	680	685	689
885	694	699	704	709	714	719	724	729	734	738
886	743	748	753	758	763	768	773	778	783	787
887	792	797	802	807	812	817	822	827	832	836
888	841	846	851	856	861	866	871	876	880	885
889	890	895	900	905	910	915	919	924	929	934
890	939	944	949	954	959	963	968	973	978	983
891	988	993	998	*002	*007	*012	*017	*022	*027	*032
892	95 036	041	046	051	056	061	066	071	075	080
893	085	090	095	100	105	109	114	119	124	129
894	134	139	143	148	153	158	163	168	173	177
895	182	187	192	197	202	207	211	216	221	226
896	231	236	240	245	250	255	260	265	270	274
897	279	284	289	294	299	303	308	313	318	323
898	328	332	337	342	347	352	357	361	366	371
899	376	381	386	390	395	400	405	410	415	419
900	424	429	434	439	444	448	453	458	463	468
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

Prop. Pts.

	6	5
1	0.6	0.5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

	4
1	0.4
2	0.8
3	1.2
4	1.6
5	2.0
6	2.4
7	2.8
8	3.2
9	3.6

Prop. Pts.

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
911	952	957	961	966	971	976	980	985	990	995	
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96 047	052	057	061	066	071	076	080	085	090	
914	095	099	104	109	114	118	123	128	133	137	
915	142	147	152	156	161	166	171	175	180	185	
916	190	194	199	204	209	213	218	223	227	232	
917	237	242	246	251	256	261	265	270	275	280	
918	284	289	294	298	303	308	313	317	322	327	
919	332	336	341	346	350	355	360	365	369	374	
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	
932	942	946	951	956	960	965	970	974	979	984	
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97 035	039	044	049	053	058	063	067	072	077	
935	081	086	090	095	100	104	109	114	118	123	
936	128	132	137	142	146	151	155	160	165	169	
937	174	179	183	188	192	197	202	206	211	216	
938	220	225	230	234	239	243	248	253	257	262	
939	267	271	276	280	285	290	294	299	304	308	
940	313	317	322	327	331	336	340	345	350	354	
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	
962	318	322	327	331	336	340	345	349	354	358	
963	363	367	372	376	381	385	390	394	399	403	
964	408	412	417	421	426	430	435	439	444	448	
965	453	457	462	466	471	475	480	484	489	493	
966	498	502	507	511	516	520	525	529	534	538	
967	543	547	552	556	561	565	570	574	579	583	
968	588	592	597	601	605	610	614	619	623	628	
969	632	637	641	646	650	655	659	664	668	673	
970	677	682	686	691	695	700	704	709	713	717	
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
981	167	171	176	180	185	189	193	198	202	207	
982	211	216	220	224	229	233	238	242	247	251	
983	255	260	264	269	273	277	282	286	291	295	
984	300	304	308	313	317	322	326	330	335	339	
985	344	348	352	357	361	366	370	374	379	383	
986	388	392	396	401	405	410	414	419	423	427	
987	432	436	441	445	449	454	458	463	467	471	
988	476	480	484	489	493	498	502	506	511	515	
989	520	524	528	533	537	542	546	550	555	559	
990	564	568	572	577	581	585	590	594	599	603	
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	0	1	2	3	4	5	6	7	8	9	Prop. Pts.

	5	4
1	0.5	0.4
2	1.0	0.8
3	1.5	1.2
4	2.0	1.6
5	2.5	2.0
6	3.0	2.4
7	3.5	2.8
8	4.0	3.2
9	4.5	3.6

Degs.	0	.5	1	1.5	2	2.5	3	3.5	4	4.5
sin	0.0000	0.0087	0.0175	0.0262	0.0349	0.0436	0.0523	0.0610	0.0698	0.0785
cos	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9986	0.9981	0.9976	0.9969
tan	0.0000	0.0087	0.0175	0.0262	0.0349	0.0437	0.0524	0.0612	0.0699	0.0787

Degs.	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5
sin	0.0872	0.0958	0.1045	0.1132	0.1219	0.1305	0.1392	0.1478	0.1564	0.1650
cos	0.9962	0.9954	0.9945	0.9936	0.9925	0.9914	0.9903	0.9890	0.9877	0.9863
tan	0.0875	0.0963	0.1051	0.1139	0.1228	0.1317	0.1405	0.1495	0.1584	0.1673

Degs.	10	10.5	11	11.5	12	12.5	13	13.5	14	14.5
sin	0.1736	0.1822	0.1908	0.1994	0.2079	0.2164	0.2250	0.2334	0.2419	0.2504
cos	0.9848	0.9833	0.9816	0.9799	0.9781	0.9763	0.9744	0.9724	0.9703	0.9681
tan	0.1763	0.1853	0.1944	0.2035	0.2126	0.2217	0.2309	0.2401	0.2493	0.2586

Degs.	15	15.5	16	16.5	17	17.5	18	18.5	19	19.5
sin	0.2588	0.2672	0.2756	0.2840	0.2924	0.3007	0.3090	0.3173	0.3256	0.3338
cos	0.9659	0.9636	0.9613	0.9588	0.9563	0.9537	0.9511	0.9483	0.9455	0.9426
tan	0.2679	0.2773	0.2867	0.2962	0.3057	0.3153	0.3249	0.3346	0.3443	0.3541

Degs.	20	20.5	21	21.5	22	22.5	23	23.5	24	24.5
sin	0.3420	0.3502	0.3584	0.3665	0.3746	0.3827	0.3907	0.3987	0.4067	0.4147
cos	0.9397	0.9367	0.9336	0.9304	0.9272	0.9239	0.9205	0.9171	0.9135	0.9100
tan	0.3640	0.3739	0.3839	0.3939	0.4040	0.4142	0.4245	0.4348	0.4452	0.4557

Degs.	25	25.5	26	26.5	27	27.5	28	28.5	29	29.5
sin	0.4226	0.4305	0.4384	0.4462	0.4540	0.4617	0.4695	0.4772	0.4848	0.4924
cos	0.9063	0.9026	0.8988	0.8949	0.8910	0.8870	0.8829	0.8788	0.8746	0.8704
tan	0.4663	0.4770	0.4877	0.4986	0.5095	0.5206	0.5317	0.5430	0.5543	0.5658

Degs.	30	30.5	31	31.5	32	32.5	33	33.5	34	34.5
sin	0.5000	0.5075	0.5150	0.5225	0.5299	0.5373	0.5446	0.5519	0.5592	0.5664
cos	0.8660	0.8616	0.8572	0.8526	0.8480	0.8434	0.8387	0.8339	0.8290	0.8241
tan	0.5774	0.5890	0.6009	0.6128	0.6249	0.6371	0.6494	0.6619	0.6745	0.6873

Degs.	35	35.5	36	36.5	37	37.5	38	38.5	39	39.5
sin	0.5736	0.5807	0.5878	0.5948	0.6018	0.6088	0.6157	0.6225	0.6293	0.6361
cos	0.8192	0.8141	0.8090	0.8039	0.7986	0.7934	0.7880	0.7826	0.7771	0.7716
tan	0.7002	0.7133	0.7265	0.7400	0.7536	0.7673	0.7813	0.7954	0.8098	0.8243

Degs.	40	40.5	41	41.5	42	42.5	43	43.5	44	44.5
sin	0.6428	0.6494	0.6561	0.6626	0.6691	0.6756	0.6820	0.6884	0.6947	0.7009
cos	0.7660	0.7604	0.7547	0.7490	0.7431	0.7373	0.7314	0.7254	0.7193	0.7133
tan	0.8391	0.8541	0.8693	0.8847	0.9004	0.9163	0.9325	0.9490	0.9657	0.9827

Degs.	45	45.5	46	46.5	47	47.5	48	48.5	49	49.5
sin	0.7071	0.7133	0.7193	0.7254	0.7314	0.7373	0.7431	0.7490	0.7547	0.7604
cos	0.7071	0.7009	0.6947	0.6884	0.6820	0.6756	0.6691	0.6626	0.6561	0.6494
tan	1.0000	1.0176	1.0355	1.0538	1.0724	1.0913	1.1106	1.1303	1.1504	1.1708

Degs.	50	50.5	51	51.5	52	52.5	53	53.5	54	54.5
sin	0.7660	0.7716	0.7771	0.7826	0.7880	0.7934	0.7986	0.8039	0.8090	0.8141
cos	0.6428	0.6361	0.6293	0.6225	0.6157	0.6088	0.6018	0.5948	0.5878	0.5807
tan	1.1918	1.2131	1.2349	1.2572	1.2799	1.3032	1.3270	1.3514	1.3764	1.4019

Degs.	55	55.5	56	56.5	57	57.5	58	58.5	59	59.5
sin	0.8192	0.8241	0.8290	0.8339	0.8387	0.8434	0.8480	0.8526	0.8572	0.8616
cos	0.5736	0.5664	0.5592	0.5519	0.5446	0.5373	0.5299	0.5225	0.5150	0.5075
tan	1.4281	1.4550	1.4826	1.5108	1.5399	1.5697	1.6003	1.6319	1.6643	1.6977

Degs.	60	60.5	61	61.5	62	62.5	63	63.5	64	64.5
sin	0.8660	0.8704	0.8746	0.8788	0.8829	0.8870	0.8910	0.8949	0.8988	0.9026
cos	0.5000	0.4924	0.4848	0.4772	0.4695	0.4617	0.4540	0.4462	0.4384	0.4305
tan	1.7321	1.7675	1.8040	1.8418	1.8807	1.9210	1.9626	2.0057	2.0503	2.0965

Degs.	65	65.5	66	66.5	67	67.5	68	68.5	69	69.5
sin	0.9063	0.9100	0.9135	0.9171	0.9205	0.9239	0.9272	0.9304	0.9336	0.9367
cos	0.4226	0.4147	0.4067	0.3987	0.3907	0.3827	0.3746	0.3665	0.3584	0.3502
tan	2.1445	2.1943	2.2460	2.2998	2.3559	2.4142	2.4751	2.5386	2.6051	2.6746

Degs.	70	70.5	71	71.5	72	72.5	73	73.5	74	74.5
sin	0.9397	0.9426	0.9455	0.9483	0.9511	0.9537	0.9563	0.9588	0.9613	0.9636
cos	0.3420	0.3338	0.3256	0.3173	0.3090	0.3007	0.2924	0.2840	0.2756	0.2672
tan	2.7475	2.8239	2.9042	2.9887	3.0777	3.1716	3.2709	3.3759	3.4874	3.6059

Degs.	75	75.5	76	76.5	77	77.5	78	78.5	79	79.5
sin	0.9659	0.9681	0.9703	0.9724	0.9744	0.9763	0.9781	0.9799	0.9816	0.9833
cos	0.2588	0.2504	0.2419	0.2334	0.2250	0.2164	0.2079	0.1994	0.1908	0.1822
tan	3.7321	3.8667	4.0108	4.1653	4.3315	4.5107	4.7046	4.9152	5.1446	5.3955

Degs.	80	80.5	81	81.5	82	82.5	83	83.5	84	84.5
sin	0.9848	0.9863	0.9877	0.9890	0.9903	0.9914	0.9925	0.9936	0.9945	0.9954
cos	0.1736	0.1650	0.1564	0.1478	0.1392	0.1305	0.1219	0.1132	0.1045	0.0958
tan	5.6713	5.9758	6.3138	6.6912	7.1154	7.5958	8.1443	8.7769	9.5144	10.385

Degs.	85	85.5	86	86.5	87	87.5	88	88.5	89	89.5
sin	0.9962	0.9969	0.9976	0.9981	0.9986	0.9990	0.9994	0.9997	0.9998	0.9999
cos	0.0872	0.0785	0.0698	0.0610	0.0523	0.0436	0.0349	0.0262	0.0175	0.0087
tan	11.43	12.71	14.30	16.35	19.08	22.90	28.64	38.19	57.29	114.6

Degs.	90	90.5	91	91.5	92	92.5	93	93.5	94	94.5
sin	1	0.9999	0.9998	0.9997	0.9994	0.9990	0.9986	0.9981	0.9976	0.9969
cos	0	−.0087	−.0175	−.0262	−.0349	−.0436	−.0523	−.0610	−.0698	−.0785
tan	∞	−114.6	−57.29	−38.19	−28.64	−22.90	−19.08	−16.35	−14.30	−12.71

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
1.00	1.0000	1.00000	3.16228	1.00000	1.00000	2.15443	4.64159	1.00000
1.01	1.0201	1.00499	3.17805	1.03030	1.00332	2.16159	4.65701	.990099
1.02	1.0404	1.00995	3.19374	1.06121	1.00662	2.16870	4.67233	.980392
1.03	1.0609	1.01489	3.20936	1.09273	1.00990	2.17577	4.68755	.970874
1.04	1.0816	1.01980	3.22490	1.12486	1.01316	2.18279	4.70267	.961538
1.05	1.1025	1.02470	3.24037	1.15762	1.01640	2.18976	4.71769	.952381
1.06	1.1236	1.02956	3.25576	1.19102	1.01961	2.19669	4.73262	.943396
1.07	1.1449	1.03441	3.27109	1.22504	1.02281	2.20358	4.74746	.934579
1.08	1.1664	1.03923	3.28634	1.25971	1.02599	2.21042	4.76220	.925926
1.09	1.1881	1.04403	3.30151	1.29503	1.02914	2.21722	4.77686	.917431
1.10	1.2100	1.04881	3.31662	1.33100	1.03228	2.22398	4.79142	.909091
1.11	1.2321	1.05357	3.33167	1.36763	1.03540	2.23070	4.80590	.900901
1.12	1.2544	1.05830	3.34664	1.40493	1.03850	2.23738	4.82028	.892857
1.13	1.2769	1.06301	3.36155	1.44290	1.04158	2.24402	4.83459	.884956
1.14	1.2996	1.06771	3.37639	1.48154	1.04464	2.25062	4.84881	.877193
1.15	1.3225	1.07238	3.39116	1.52088	1.04769	2.25718	4.86294	.869565
1.16	1.3456	1.07703	3.40588	1.56090	1.05072	2.26370	4.87700	.862069
1.17	1.3689	1.08167	3.42053	1.60161	1.05373	2.27019	4.89097	.854701
1.18	1.3924	1.08628	3.43511	1.64303	1.05672	2.27664	4.90487	.847458
1.19	1.4161	1.09087	3.44964	1.68516	1.05970	2.28305	4.91868	.840336
1.20	1.4400	1.09545	3.46410	1.72800	1.06266	2.28943	4.93242	.833333
1.21	1.4641	1.10000	3.47851	1.77156	1.06560	2.29577	4.94609	.826446
1.22	1.4884	1.10454	3.49285	1.81585	1.06853	2.30208	4.95968	.819672
1.23	1.5129	1.10905	3.50714	1.86087	1.07144	2.30835	4.97319	.813008
1.24	1.5376	1.11355	3.52136	1.90662	1.07434	2.31459	4.98663	.806452
1.25	1.5625	1.11803	3.53553	1.95312	1.07722	2.32079	5.00000	.800000
1.26	1.5876	1.12250	3.54965	2.00038	1.08008	2.32697	5.01330	.793651
1.27	1.6129	1.12694	3.56371	2.04838	1.08293	2.33311	5.02653	.787402
1.28	1.6384	1.13137	3.57771	2.09715	1.08577	2.33921	5.03968	.781250
1.29	1.6641	1.13578	3.59166	2.14669	1.08859	2.34529	5.05277	.775194
1.30	1.6900	1.14018	3.60555	2.19700	1.09139	2.35133	5.06580	.769231
1.31	1.7161	1.14455	3.61939	2.24809	1.09418	2.35735	5.07875	.763359
1.32	1.7424	1.14891	3.63318	2.29997	1.09696	2.36333	5.09164	.757576
1.33	1.7689	1.15326	3.64692	2.35264	1.09972	2.36928	5.10447	.751880
1.34	1.7956	1.15758	3.66060	2.40610	1.10247	2.37521	5.11723	.746269
1.35	1.8225	1.16190	3.67423	2.46038	1.10521	2.38110	5.12993	.740741
1.36	1.8496	1.16619	3.68782	2.51546	1.10793	2.38697	5.14256	.735294
1.37	1.8769	1.17047	3.70135	2.57135	1.11064	2.39280	5.15514	.729927
1.38	1.9044	1.17473	3.71484	2.62807	1.11334	2.39861	5.16765	.724638
1.39	1.9321	1.17898	3.72827	2.68562	1.11602	2.40439	5.18010	.719424
1.40	1.9600	1.18322	3.74166	2.74400	1.11869	2.41014	5.19249	.714286
1.41	1.9881	1.18743	3.75500	2.80322	1.12135	2.41587	5.20483	.709220
1.42	2.0164	1.19164	3.76829	2.86329	1.12399	2.42156	5.21710	.704225
1.43	2.0449	1.19583	3.78153	2.92421	1.12662	2.42724	5.22932	.699301
1.44	2.0736	1.20000	3.79473	2.98598	1.12924	2.43288	5.24148	.694444
1.45	2.1025	1.20416	3.80789	3.04862	1.13185	2.43850	5.25359	.689655
1.46	2.1316	1.20830	3.82099	3.11214	1.13445	2.44409	5.26564	.684932
1.47	2.1609	1.21244	3.83406	3.17652	1.13703	2.44966	5.27763	.680272
1.48	2.1904	1.21655	3.84708	3.24179	1.13960	2.45520	5.28957	.675676
1.49	2.2201	1.22066	3.86005	3.30795	1.14216	2.46072	5.30146	.671141
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
1.50	2.2500	1.22474	3.87298	3.37500	1.14471	2.46621	5.31329	.666667
1.51	2.2801	1.22882	3.88587	3.44295	1.14725	2.47168	5.32507	.662252
1.52	2.3104	1.23288	3.89872	3.51181	1.14978	2.47712	5.33680	.657895
1.53	2.3409	1.23693	3.91152	3.58158	1.15230	2.48255	5.34848	.653595
1.54	2.3716	1.24097	3.92428	3.65226	1.15480	2.48794	5.36011	.649351
1.55	2.4025	1.24499	3.93700	3.72388	1.15729	2.49332	5.37169	.645161
1.56	2.4336	1.24900	3.94968	3.79642	1.15978	2.49867	5.38321	.641026
1.57	2.4649	1.25300	3.96232	3.86989	1.16225	2.50399	5.39469	.636943
1.58	2.4964	1.25698	3.97492	3.94431	1.16471	2.50930	5.40612	.632911
1.59	2.5281	1.26095	3.98748	4.01968	1.16717	2.51458	5.41750	.628931
1.60	2.5600	1.26491	4.00000	4.09600	1.16961	2.51984	5.42884	.625000
1.61	2.5921	1.26886	4.01248	4.17328	1.17204	2.52508	5.44012	.621118
1.62	2.6244	1.27279	4.02492	4.25153	1.17446	2.53030	5.45136	.617284
1.63	2.6569	1.27671	4.03733	4.33075	1.17687	2.53549	5.46256	.613497
1.64	2.6896	1.28062	4.04969	4.41094	1.17927	2.54067	5.47370	.609756
1.65	2.7225	1.28452	4.06202	4.49212	1.18167	2.54582	5.48481	.606061
1.66	2.7556	1.28841	4.07431	4.57430	1.18405	2.55095	5.49586	.602410
1.67	2.7889	1.29228	4.08656	4.65746	1.18642	2.55607	5.50688	.598802
1.68	2.8224	1.29615	4.09878	4.74163	1.18878	2.56116	5.51785	.595238
1.69	2.8561	1.30000	4.11096	4.82681	1.19114	2.56623	5.52877	.591716
1.70	2.8900	1.30384	4.12311	4.91300	1.19348	2.57128	5.53966	.588235
1.71	2.9241	1.30767	4.13521	5.00021	1.19582	2.57631	5.55050	.584795
1.72	2.9584	1.31149	4.14729	5.08845	1.19815	2.58133	5.56130	.581395
1.73	2.9929	1.31529	4.15933	5.17772	1.20046	2.58632	5.57205	.578035
1.74	3.0276	1.31909	4.17133	5.26802	1.20277	2.59129	5.58277	.574713
1.75	3.0625	1.32288	4.18330	5.35938	1.20507	2.59625	5.59344	.571429
1.76	3.0976	1.32665	4.19524	5.45178	1.20736	2.60118	5.60408	.568182
1.77	3.1329	1.33041	4.20714	5.54523	1.20964	2.60610	5.61467	.564972
1.78	3.1684	1.33417	4.21900	5.63975	1.21192	2.61100	5.62523	.561798
1.79	3.2041	1.33791	4.23084	5.73534	1.21418	2.61588	5.63574	.558659
1.80	3.2400	1.34164	4.24264	5.83200	1.21644	2.62074	5.64622	.555556
1.81	3.2761	1.34536	4.25441	5.92974	1.21869	2.62559	5.65665	.552486
1.82	3.3124	1.34907	4.26615	6.02857	1.22093	2.63041	5.66705	.549451
1.83	3.3489	1.35277	4.27785	6.12849	1.22316	2.63522	5.67741	.546448
1.84	3.3856	1.35647	4.28952	6.22950	1.22539	2.64001	5.68773	.543478
1.85	3.4225	1.36015	4.30116	6.33162	1.22760	2.64479	5.69802	.540541
1.86	3.4596	1.36382	4.31277	6.43486	1.22981	2.64954	5.70827	.537634
1.87	3.4969	1.36748	4.32435	6.53920	1.23201	2.65428	5.71848	.534759
1.88	3.5344	1.37113	4.33590	6.64467	1.23420	2.65901	5.72865	.531915
1.89	3.5721	1.37477	4.34741	6.75127	1.23639	2.66371	5.73879	.529101
1.90	3.6100	1.37840	4.35890	6.85900	1.23856	2.66840	5.74890	.526316
1.91	3.6481	1.38203	4.37035	6.96787	1.24073	2.67307	5.75897	.523560
1.92	3.6864	1.38564	4.38178	7.07789	1.24289	2.67773	5.76900	.520833
1.93	3.7249	1.38924	4.39318	7.18906	1.24505	2.68237	5.77900	.518135
1.94	3.7636	1.39284	4.40454	7.30138	1.24719	2.68700	5.78896	.515464
1.95	3.8025	1.39642	4.41588	7.41488	1.24933	2.69161	5.79889	.512821
1.96	3.8416	1.40000	4.42719	7.52954	1.25146	2.69620	5.80879	.510204
1.97	3.8809	1.40357	4.43847	7.64537	1.25359	2.70078	5.81865	.507614
1.98	3.9204	1.40712	4.44972	7.76239	1.25571	2.70534	5.82848	.505051
1.99	3.9601	1.41067	4.46094	7.88060	1.25782	2.70989	5.83827	.502513
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
2.00	4.0000	1.41421	4.47214	8.00000	1.25992	2.71442	5.84804	.500000
2.01	4.0401	1.41774	4.48330	8.12060	1.26202	2.71893	5.85777	.497512
2.02	4.0804	1.42127	4.49444	8.24241	1.26411	2.72344	5.86746	.495050
2.03	4.1209	1.42478	4.50555	8.36543	1.26619	2.72792	5.87713	.492611
2.04	4.1616	1.42829	4.51664	8.48966	1.26827	2.73239	5.88677	.490196
2.05	4.2025	1.43178	4.52769	8.61512	1.27033	2.73685	5.89637	.487805
2.06	4.2436	1.43527	4.53872	8.74182	1.27240	2.74129	5.90594	.485437
2.07	4.2849	1.43875	4.54973	8.86974	1.27445	2.74572	5.91548	.483092
2.08	4.3264	1.44222	4.56070	8.99891	1.27650	2.75014	5.92499	.480769
2.09	4.3681	1.44568	4.57165	9.12933	1.27854	2.75454	5.93447	.478469
2.10	4.4100	1.44914	4.58258	9.26100	1.28058	2.75892	5.94392	.476190
2.11	4.4521	1.45258	4.59347	9.39393	1.28261	2.76330	5.95334	.473934
2.12	4.4944	1.45602	4.60435	9.52813	1.28463	2.76766	5.96273	.471698
2.13	4.5369	1.45945	4.61519	9.66360	1.28665	2.77200	5.97209	.469434
2.14	4.5796	1.46287	4.62601	9.80034	1.28866	2.77633	5.98142	.467290
2.15	4.6225	1.46629	4.63681	9.93838	1.29066	2.78065	5.99073	.465116
2.16	4.6656	1.46969	4.64758	10.0777	1.29266	2.78495	6.00000	.462963
2.17	4.7089	1.47309	4.65833	10.2183	1.29465	2.78924	6.00925	.460829
2.18	4.7524	1.47648	4.66905	10.3602	1.29664	2.79352	6.01846	.458716
2.19	4.7961	1.47986	4.67974	10.5035	1.29862	2.79779	6.02765	.456621
2.20	4.8400	1.48324	4.69042	10.6480	1.30059	2.80204	6.03681	.454545
2.21	4.8841	1.48661	4.70106	10.7939	1.30256	2.80628	6.04594	.452489
2.22	4.9284	1.48997	4.71169	10.9410	1.30452	2.81050	6.05505	.450450
2.23	4.9729	1.49332	4.72229	11.0896	1.30648	2.81472	6.06413	.448430
2.24	5.0176	1.49666	4.73286	11.2394	1.30843	2.81892	6.07318	.446429
2.25	5.0625	1.50000	4.74342	11.3906	1.31037	2.82311	6.08220	.444444
2.26	5.1076	1.50333	4.75395	11.5432	1.31231	2.82728	6.09120	.442478
2.27	5.1529	1.50665	4.76445	11.6971	1.31424	2.83145	6.10017	.440529
2.28	5.1984	1.50997	4.77493	11.8524	1.31617	2.83560	6.10911	.438596
2.29	5.2441	1.51327	4.78539	12.0090	1.31809	2.83974	6.11803	.436681
2.30	5.2900	1.51658	4.79583	12.1670	1.32001	2.84387	6.12693	.434783
2.31	5.3361	1.51987	4.80625	12.3264	1.32192	2.84798	6.13579	.432900
2.32	5.3824	1.52315	4.81664	12.4872	1.32382	2.85209	6.14463	.431034
2.33	5.4289	1.52643	4.82701	12.6493	1.32572	2.85618	6.15345	.429185
2.34	5.4756	1.52971	4.83735	12.8129	1.32761	2.86026	6.16224	.427350
2.35	5.5225	1.53297	4.84768	12.9779	1.32950	2.86433	6.17101	.425532
2.36	5.5696	1.53623	4.85798	13.1443	1.33139	2.86838	6.17975	.423729
2.37	5.6169	1.53948	4.86826	13.3121	1.33326	2.87243	6.18846	.421941
2.38	5.6644	1.54272	4.87852	13.4813	1.33514	2.87646	6.19715	.420168
2.39	5.7121	1.54596	4.88876	13.6519	1.33700	2.88049	6.20582	.418410
2.40	5.7600	1.54919	4.89898	13.8240	1.33887	2.88450	6.21447	.416667
2.41	5.8081	1.55242	4.90918	13.9975	1.34072	2.88850	6.22308	.414938
2.42	5.8564	1.55563	4.91935	14.1725	1.34257	2.89249	6.23168	.413223
2.43	5.9049	1.55885	4.92950	14.3489	1.34442	2.89647	6.24025	.411523
2.44	5.9536	1.56205	4.93964	14.5268	1.34626	2.90044	6.24880	.409836
2.45	6.0025	1.56525	4.94975	14.7061	1.34810	2.90439	6.25732	.408163
2.46	6.0516	1.56844	4.95984	14.8869	1.34993	2.90834	6.26583	.406504
2.47	6.1009	1.57162	4.96991	15.0692	1.35176	2.91227	6.27431	.404858
2.48	6.1504	1.57480	4.97996	15.2530	1.35358	2.91620	6.28276	.403226
2.49	6.2001	1.57797	4.98999	15.4382	1.35540	2.92011	6.29119	.401606
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
2.50	6.2500	1.58114	5.00000	15.6250	1.35721	2.92402	6.29961	.400000
2.51	6.3001	1.58430	5.00999	15.8133	1.35902	2.92791	6.30799	.398406
2.52	6.3504	1.58745	5.01996	16.0030	1.36082	2.93179	6.31636	.396825
2.53	6.4009	1.59060	5.02991	16.1943	1.36262	2.93567	6.32470	.395257
2.54	6.4516	1.59374	5.03984	16.3871	1.36441	2.93953	6.33303	.393701
2.55	6.5025	1.59687	5.04975	16.5814	1.36620	2.94338	6.34133	.392157
2.56	6.5536	1.60000	5.05964	16.7772	1.36798	2.94723	6.34960	.390625
2.57	6.6049	1.60312	5.06952	16.9746	1.36976	2.95106	6.35786	.389105
2.58	6.6564	1.60624	5.07937	17.1735	1.37153	2.95488	6.36610	.387597
2.59	6.7081	1.60935	5.08920	17.3740	1.37330	2.95869	6.37431	.386100
2.60	6.7600	1.61245	5.09902	17.5760	1.37507	2.96250	6.38250	.384615
2.61	6.8121	1.61555	5.10882	17.7796	1.37683	2.96629	6.39068	.383142
2.62	6.8644	1.61864	5.11859	17.9847	1.37859	2.97007	6.39883	.381679
2.63	6.9169	1.62173	5.12835	18.1914	1.38034	2.97385	6.40696	.380228
2.64	6.9696	1.62481	5.13809	18.3997	1.38208	2.97761	6.41507	.378788
2.65	7.0225	1.62788	5.14782	18.6096	1.38383	2.98137	6.42316	.377358
2.66	7.0756	1.63095	5.15752	18.8211	1.38557	2.98511	6.43123	.375940
2.67	7.1289	1.63401	5.16720	19.0342	1.38730	2.98885	6.43928	.374532
2.68	7.1824	1.63707	5.17687	19.2488	1.38903	2.99257	6.44731	.373134
2.69	7.2361	1.64012	5.18652	19.4651	1.39076	2.99629	6.45531	.371747
2.70	7.2900	1.64317	5.19615	19.6830	1.39248	3.00000	6.46330	.370370
2.71	7.3441	1.64621	5.20577	19.9025	1.39419	3.00370	6.47127	.369004
2.72	7.3984	1.64924	5.21536	20.1236	1.39591	3.00739	6.47922	.367647
2.73	7.4529	1.65227	5.22494	20.3464	1.39761	3.01107	6.48715	.366300
2.74	7.5076	1.65529	5.23450	20.5708	1.39932	3.01474	6.49507	.364964
2.75	7.5625	1.65831	5.24404	20.7969	1.40102	3.01841	6.50296	.363636
2.76	7.6176	1.66132	5.25357	21.0246	1.40272	3.02206	6.51083	.362319
2.77	7.6729	1.66433	5.26308	21.2539	1.40441	3.02570	6.51868	.361011
2.78	7.7284	1.66733	5.27257	21.4850	1.40610	3.02934	6.52652	.359712
2.79	7.7841	1.67033	5.28205	21.7176	1.40778	3.03297	6.53434	.358423
2.80	7.8400	1.67332	5.29150	21.9520	1.40946	3.03659	6.54213	.357143
2.81	7.8961	1.67631	5.30094	22.1880	1.41114	3.04020	6.54991	.355872
2.82	7.9524	1.67929	5.31037	22.4258	1.41281	3.04380	6.55767	.354610
2.83	8.0089	1.68226	5.31977	22.6652	1.41448	3.04740	6.56541	.353357
2.84	8.0656	1.68523	5.32917	22.9063	1.41614	3.05098	6.57314	.352113
2.85	8.1225	1.68819	5.33854	23.1491	1.41780	3.05456	6.58084	.350877
2.86	8.1796	1.69115	5.34790	23.3937	1.41946	3.05813	6.58853	.349650
2.87	8.2369	1.69411	5.35724	23.6399	1.42111	3.06169	6.59620	.348432
2.88	8.2944	1.69706	5.36656	23.8879	1.42276	3.06524	6.60385	.347222
2.89	8.3521	1.70000	5.37587	24.1376	1.42440	3.06878	6.61149	.346021
2.90	8.4100	1.70294	5.38516	24.3890	1.42604	3.07232	6.61911	.344828
2.91	8.4681	1.70587	5.39444	24.6422	1.42768	3.07584	6.62671	.343643
2.92	8.5264	1.70880	5.40370	24.8971	1.42931	3.07936	6.63429	.342466
2.93	8.5849	1.71172	5.41295	25.1538	1.43094	3.08287	6.64185	.341297
2.94	8.6436	1.71464	5.42218	25.4122	1.43257	3.08638	6.64940	.340136
2.95	8.7025	1.71756	5.43139	25.6724	1.43419	3.08987	6.65693	.338983
2.96	8.7616	1.72047	5.44059	25.9343	1.43581	3.09336	6.66444	.337838
2.97	8.8209	1.72337	5.44977	26.1981	1.43743	3.09684	6.67194	.336700
2.98	8.8804	1.72627	5.45894	26.4636	1.43904	3.10031	6.67942	.335570
2.99	8.9401	1.72916	5.46809	26.7309	1.44065	3.10378	6.68688	.334448
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
3.00	9.0000	1.73205	5.47723	27.0000	1.44225	3.10723	6.69433	.333333
3.01	9.0601	1.73494	5.48635	27.2709	1.44385	3.11068	6.70176	.332226
3.02	9.1204	1.73781	5.49545	27.5436	1.44545	3.11412	6.70917	.331126
3.03	9.1809	1.74069	5.50454	27.8181	1.44704	3.11756	6.71657	.330033
3.04	9.2416	1.74356	5.51362	28.0945	1.44863	3.12098	6.72395	.328947
3.05	9.3025	1.74642	5.52268	28.3726	1.45022	3.12440	6.73132	.327869
3.06	9.3636	1.74929	5.53173	28.6526	1.45180	3.12781	6.73866	.326797
3.07	9.4249	1.75214	5.54076	28.9344	1.45338	3.13121	6.74600	.325733
3.08	9.4864	1.75499	5.54977	29.2181	1.45496	3.13461	6.75331	.324675
3.09	9.5481	1.75784	5.55878	29.5036	1.45653	3.13800	6.76061	.323625
3.10	9.6100	1.76068	5.56776	29.7910	1.45810	3.14138	6.76790	.322581
3.11	9.6721	1.76352	5.57674	30.0802	1.45967	3.14475	6.77517	.321543
3.12	9.7344	1.76635	5.58570	30.3713	1.46123	3.14812	6.78242	.320513
3.13	9.7969	1.76918	5.59464	30.6643	1.46279	3.15148	6.78966	.319489
3.14	9.8596	1.77200	5.60357	30.9591	1.46434	3.15483	6.79688	.318471
3.15	9.9225	1.77482	5.61249	31.2559	1.46590	3.15818	6.80409	.317460
3.16	9.9856	1.77764	5.62139	31.5545	1.46745	3.16152	6.81128	.316456
3.17	10.0489	1.78045	5.63028	31.8550	1.46899	3.16485	6.81846	.315457
3.18	10.1124	1.78326	5.63915	32.1574	1.47054	3.16817	6.82562	.314465
3.19	10.1761	1.78606	5.64801	32.4618	1.47208	3.17149	6.83277	.313480
3.20	10.2400	1.78885	5.65685	32.7680	1.47361	3.17480	6.83990	.312500
3.21	10.3041	1.79165	5.66569	33.0762	1.47515	3.17811	6.84702	.311526
3.22	10.3684	1.79444	5.67450	33.3862	1.47668	3.18140	6.85412	.310559
3.23	10.4329	1.79722	5.68331	33.6983	1.47820	3.18469	6.86121	.309598
3.24	10.4976	1.80000	5.69210	34.0122	1.47973	3.18798	6.86829	.308642
3.25	10.5625	1.80278	5.70088	34.3281	1.48125	3.19125	6.87534	.307692
3.26	10.6276	1.80555	5.70964	34.6460	1.48277	3.19452	6.88239	.306748
3.27	10.6929	1.80831	5.71839	34.9658	1.48428	3.19778	6.88942	.305810
3.28	10.7584	1.81108	5.72713	35.2876	1.48579	3.20104	6.89643	.304878
3.29	10.8241	1.81384	5.73585	35.6113	1.48730	3.20429	6.90344	.303951
3.30	10.8900	1.81659	5.74456	35.9370	1.48881	3.20753	6.91042	.303030
3.31	10.9561	1.81934	5.75326	36.2647	1.49031	3.21077	6.91740	.302115
3.32	11.0224	1.82209	5.76194	36.5944	1.49181	3.21400	6.92436	.301205
3.33	11.0889	1.82483	5.77062	36.9260	1.49330	3.21722	6.93130	.300300
3.34	11.1556	1.82757	5.77927	37.2597	1.49480	3.22044	6.93823	.299401
3.35	11.2225	1.83030	5.78792	37.5954	1.49629	3.22365	6.94515	.298507
3.36	11.2896	1.83303	5.79655	37.9331	1.49777	3.22686	6.95205	.297619
3.37	11.3569	1.83576	5.80517	38.2728	1.49926	3.23006	6.95894	.296736
3.38	11.4244	1.83848	5.81378	38.6145	1.50074	3.23325	6.96582	.295858
3.39	11.4921	1.84120	5.82237	38.9582	1.50222	3.23643	6.97268	.294985
3.40	11.5600	1.84391	5.83095	39.3040	1.50369	3.23961	6.97953	.294118
3.41	11.6281	1.84662	5.83952	39.6518	1.50517	3.24278	6.98637	.293255
3.42	11.6964	1.84932	5.84808	40.0017	1.50664	3.24595	6.99319	.292398
3.43	11.7649	1.85203	5.85662	40.3536	1.50810	3.24911	7.00000	.291545
3.44	11.8336	1.85472	5.86515	40.7076	1.50957	3.25227	7.00680	.290698
3.45	11.9025	1.85742	5.87367	41.0636	1.51103	3.25542	7.01358	.289855
3.46	11.9716	1.86011	5.88218	41.4217	1.51249	3.25856	7.02035	.289017
3.47	12.0409	1.86279	5.89067	41.7819	1.51394	3.26169	7.02711	.288184
3.48	12.1104	1.86548	5.89915	42.1442	1.51540	3.26482	7.03385	.287356
3.49	12.1801	1.86815	5.90762	42.5085	1.51685	3.26795	7.04058	.286533
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$
3.50	12.2500	1.87083	5.91608	42.8750	1.51829	3.27107	7.04730	.285714
3.51	12.3201	1.87350	5.92453	43.2436	1.51974	3.27418	7.05400	.284900
3.52	12.3904	1.87617	5.93296	43.6142	1.52118	3.27729	7.06070	.284091
3.53	12.4609	1.87883	5.94138	43.9870	1.52262	3.28039	7.06738	.283286
3.54	12.5316	1.88149	5.94979	44.3619	1.52406	3.28348	7.07404	.282486
3.55	12.6025	1.88414	5.95819	44.7389	1.52549	3.28657	7.08070	.281690
3.56	12.6736	1.88680	5.96657	45.1180	1.52692	3.28965	7.08734	.280899
3.57	12.7449	1.88944	5.97495	45.4993	1.52835	3.29273	7.09397	.280112
3.58	12.8164	1.89209	5.98331	45.8827	1.52978	3.29580	7.10059	.279330
3.59	12.8881	1.89473	5.99166	46.2683	1.53120	3.29887	7.10719	.278552
3.60	12.9600	1.89737	6.00000	46.6560	1.53262	3.30193	7.11379	.277778
3.61	13.0321	1.90000	6.00833	47.0459	1.53404	3.30498	7.12037	.277008
3.62	13.1044	1.90263	6.01664	47.4379	1.53545	3.30803	7.12694	.276243
3.63	13.1769	1.90526	6.02495	47.8321	1.53686	3.31107	7.13349	.275482
3.64	13.2496	1.90788	6.03324	48.2285	1.53827	3.31411	7.14004	.274725
3.65	13.3225	1.91050	6.04152	48.6271	1.53968	3.31714	7.14657	.273973
3.66	13.3956	1.91311	6.04979	49.0279	1.54109	3.32017	7.15309	.273224
3.67	13.4689	1.91572	6.05805	49.4309	1.54249	3.32319	7.15960	.272480
3.68	13.5424	1.91833	6.06630	49.8360	1.54389	3.32621	7.16610	.271739
3.69	13.6161	1.92094	6.07454	50.2434	1.54529	3.32922	7.17258	.271003
3.70	13.6900	1.92354	6.08276	50.6530	1.54668	3.33222	7.17905	.270270
3.71	13.7641	1.92614	6.09098	51.0648	1.54807	3.33522	7.18552	.269542
3.72	13.8384	1.92873	6.09918	51.4788	1.54946	3.33822	7.19197	.268817
3.73	13.9129	1.93132	6.10737	51.8951	1.55085	3.34120	7.19840	.268097
3.74	13.9876	1.93391	6.11555	52.3136	1.55223	3.34419	7.20483	.267380
3.75	14.0625	1.93649	6.12372	52.7344	1.55362	3.34716	7.21125	.266667
3.76	14.1376	1.93907	6.13188	53.1574	1.55500	3.35014	7.21765	.265957
3.77	14.2129	1.94165	6.14003	53.5826	1.55637	3.35310	7.22405	.265252
3.78	14.2884	1.94422	6.14817	54.0102	1.55775	3.35607	7.23043	.264550
3.79	14.3641	1.94679	6.15630	54.4399	1.55912	3.35902	7.23680	.263852
3.80	14.4400	1.94936	6.16441	54.8720	1.56049	3.36198	7.24316	.263158
3.81	14.5161	1.95192	6.17252	55.3063	1.56186	3.36492	7.24950	.262467
3.82	14.5924	1.95448	6.18061	55.7430	1.56322	3.36786	7.25584	.261780
3.83	14.6689	1.95704	6.18870	56.1819	1.56459	3.37080	7.26217	.261097
3.84	14.7456	1.95959	6.19677	56.6231	1.56595	3.37373	7.26848	.260417
3.85	14.8225	1.96214	6.20484	57.0666	1.56731	3.37666	7.27479	.259740
3.86	14.8996	1.96469	6.21289	57.5125	1.56866	3.37958	7.28108	.259067
3.87	14.9769	1.96723	6.22093	57.9606	1.57001	3.38249	7.28736	.258398
3.88	15.0544	1.96977	6.22896	58.4111	1.57137	3.38540	7.29363	.257732
3.89	15.1321	1.97231	6.23699	58.8639	1.57271	3.38831	7.29989	.257069
3.90	15.2100	1.97484	6.24500	59.3190	1.57406	3.39121	7.30614	.256410
3.91	15.2881	1.97737	6.25300	59.7765	1.57541	3.39411	7.31238	.255754
3.92	15.3664	1.97990	6.26099	60.2363	1.57675	3.39700	7.31861	.255102
3.93	15.4449	1.98242	6.26897	60.6985	1.57809	3.39988	7.32483	.254453
3.94	15.5236	1.98494	6.27694	61.1630	1.57942	3.40277	7.33104	.253807
3.95	15.6025	1.98746	6.28490	61.6299	1.58076	3.40564	7.33723	.253165
3.96	15.6816	1.98997	6.29285	62.0991	1.58209	3.40851	7.34342	.252525
3.97	15.7609	1.99249	6.30079	62.5708	1.58342	3.41138	7.34960	.251889
3.98	15.8404	1.99499	6.30872	63.0448	1.58475	3.41424	7.35576	.251256
3.99	15.9201	1.99750	6.31664	63.5212	1.58608	3.41710	7.36192	.250627
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
n	n^2	$\sqrt{n}$	$\sqrt{10\,n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10\,n}$	$\sqrt[3]{100\,n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.00	16.0000	2.00000	6.32456	64.0000	1.58740	3.41995	7.36806	.250000
4.01	16.0801	2.00250	6.33246	64.4812	1.58872	3.42280	7.37420	.249377
4.02	16.1604	2.00499	6.34035	64.9648	1.59004	3.42564	7.38032	.248756
4.03	16.2409	2.00749	6.34823	65.4508	1.59136	3.42848	7.38644	.248139
4.04	16.3216	2.00998	6.35610	65.9393	1.59267	3.43131	7.39254	.247525
4.05	16.4025	2.01246	6.36396	66.4301	1.59399	3.43414	7.39864	.246914
4.06	16.4836	2.01494	6.37181	66.9234	1.59530	3.43697	7.40472	.246305
4.07	16.5649	2.01742	6.37966	67.4191	1.59661	3.43979	7.41080	.245700
4.08	16.6464	2.01990	6.38749	67.9173	1.59791	3.44260	7.41686	.245098
4.09	16.7281	2.02237	6.39531	68.4179	1.59922	3.44541	7.42291	.244499
4.10	16.8100	2.02485	6.40312	68.9210	1.60052	3.44822	7.42896	.243902
4.11	16.8921	2.02731	6.41093	69.4265	1.60182	3.45102	7.43499	.243309
4.12	16.9744	2.02978	6.41872	69.9345	1.60312	3.45382	7.44102	.242718
4.13	17.0569	2.03224	6.42651	70.4450	1.60441	3.45661	7.44703	.242131
4.14	17.1396	2.03470	6.43428	70.9579	1.60571	3.45939	7.45304	.241546
4.15	17.2225	2.03715	6.44205	71.4734	1.60700	3.46218	7.45904	.240964
4.16	17.3056	2.03961	6.44981	71.9913	1.60829	3.46496	7.46502	.240385
4.17	17.3889	2.04206	6.45755	72.5117	1.60958	3.46773	7.47100	.239808
4.18	17.4724	2.04450	6.46529	73.0346	1.61086	3.47050	7.47697	.239234
4.19	17.5561	2.04695	6.47302	73.5601	1.61215	3.47327	7.48292	.238663
4.20	17.6400	2.04939	6.48074	74.0880	1.61343	3.47603	7.48887	.238095
4.21	17.7241	2.05183	6.48845	74.6185	1.61471	3.47878	7.49481	.237530
4.22	17.8084	2.05426	6.49615	75.1514	1.61599	3.48154	7.50074	.236967
4.23	17.8929	2.05670	6.50384	75.6870	1.61726	3.48428	7.50666	.236407
4.24	17.9776	2.05913	6.51153	76.2250	1.61853	3.48703	7.51257	.235849
4.25	18.0625	2.06155	6.51920	76.7656	1.61981	3.48977	7.51847	.235294
4.26	18.1476	2.06398	6.52687	77.3088	1.62108	3.49250	7.52437	.234742
4.27	18.2329	2.06640	6.53452	77.8545	1.62234	3.49523	7.53025	.234192
4.28	18.3184	2.06882	6.54217	78.4028	1.62361	3.49796	7.53612	.233645
4.29	18.4041	2.07123	6.54981	78.9536	1.62487	3.50068	7.54199	.233100
4.30	18.4900	2.07364	6.55744	79.5070	1.62613	3.50340	7.54784	.232558
4.31	18.5761	2.07605	6.56506	80.0630	1.62739	3.50611	7.55369	.232019
4.32	18.6624	2.07846	6.57267	80.6216	1.62865	3.50882	7.55953	.231481
4.33	18.7489	2.08087	6.58027	81.1827	1.62991	3.51153	7.56535	.230947
4.34	18.8356	2.08327	6.58787	81.7465	1.63116	3.51423	7.57117	.230415
4.35	18.9225	2.08567	6.59545	82.3129	1.63241	3.51692	7.57698	.229885
4.36	19.0096	2.08806	6.60303	82.8819	1.63366	3.51962	7.58279	.229358
4.37	19.0969	2.09045	6.61060	83.4535	1.63491	3.52231	7.58858	.228833
4.38	19.1844	2.09284	6.61816	84.0277	1.63619	3.52499	7.59436	.228311
4.39	19.2721	2.09523	6.62571	84.6045	1.63740	3.52767	7.60014	.227790
4.40	19.3600	2.09762	6.63325	85.1840	1.63864	3.53035	7.60590	.227273
4.41	19.4481	2.10000	6.64078	85.7661	1.63988	3.53302	7.61166	.226757
4.42	19.5364	2.10238	6.64831	86.3509	1.64112	3.53569	7.61741	.226244
4.43	19.6249	2.10476	6.65582	86.9383	1.64236	3.53835	7.62315	.225734
4.44	19.7136	2.10713	6.66333	87.5284	1.64359	3.54101	7.62888	.225225
4.45	19.8025	2.10950	6.67083	88.1211	1.64483	3.54367	7.63461	.224719
4.46	19.8916	2.11187	6.67832	88.7165	1.64606	3.54632	7.64032	.224215
4.47	19.9809	2.11424	6.68581	89.3146	1.64729	3.54897	7.64603	.223714
4.48	20.0704	2.11660	6.69328	89.9154	1.64851	3.55162	7.65172	.223214
4.49	20.1601	2.11896	6.70075	90.5188	1.64974	3.55426	7.65741	.222717
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
4.50	20.2500	2.12132	6.70820	91.1250	1.65096	3.55689	7.66309	.222222
4.51	20.3401	2.12368	6.71565	91.7339	1.65219	3.55953	7.66877	.221729
4.52	20.4304	2.12603	6.72309	92.3454	1.65341	3.56215	7.67443	.221239
4.53	20.5209	2.12838	6.73053	92.9597	1.65462	3.56478	7.68009	.220751
4.54	20.6116	2.13073	6.73795	93.5767	1.65584	3.56740	7.68573	.220264
4.55	20.7025	2.13307	6.74537	94.1964	1.65706	3.57002	7.69137	.219780
4.56	20.7936	2.13542	6.75278	94.8188	1.65827	3.57263	7.69700	.219298
4.57	20.8849	2.13776	6.76018	95.4440	1.65948	3.57524	7.70262	.218818
4.58	20.9764	2.14009	6.76757	96.0719	1.66069	3.57785	7.70824	.218341
4.59	21.0681	2.14243	6.77495	96.7026	1.66190	3.58045	7.71384	.217865
4.60	21.1600	2.14476	6.78233	97.3360	1.66310	3.58305	7.71944	.217391
4.61	21.2521	2.14709	6.78970	97.9722	1.66431	3.58564	7.72503	.216920
4.62	21.3444	2.14942	6.79706	98.6111	1.66551	3.58823	7.73061	.216450
4.63	21.4369	2.15174	6.80441	99.2528	1.66671	3.59082	7.73619	.215983
4.64	21.5296	2.15407	6.81175	99.8973	1.66791	3.59340	7.74175	.215517
4.65	21.6225	2.15639	6.81909	100.645	1.66911	3.59598	7.74731	.215054
4.66	21.7156	2.15870	6.82642	101.195	1.67030	3.59856	7.75286	.214592
4.67	21.8089	2.16102	6.83374	101.848	1.67150	3.60113	7.75840	.214133
4.68	21.9024	2.16333	6.84105	102.503	1.67269	3.60370	7.76394	.213675
4.69	21.9961	2.16564	6.84836	103.162	1.67388	3.60626	7.76946	.213220
4.70	22.0900	2.16795	6.85565	103.823	1.67507	3.60883	7.77498	.212766
4.71	22.1841	2.17025	6.86294	104.487	1.67626	3.61138	7.78049	.212314
4.72	22.2784	2.17256	6.87023	105.154	1.67744	3.61394	7.78599	.211864
4.73	22.3729	2.17486	6.87750	105.824	1.67863	3.61649	7.79149	.211416
4.74	22.4676	2.17715	6.88477	106.496	1.67981	3.61903	7.79697	.210970
4.75	22.5625	2.17945	6.89202	107.172	1.68099	3.62158	7.80245	.210526
4.76	22.6576	2.18174	6.89928	107.850	1.68217	3.62412	7.80793	.210084
4.77	22.7529	2.18403	6.90652	108.531	1.68334	3.62665	7.81339	.209644
4.78	22.8484	2.18632	6.91375	109.215	1.68452	3.62919	7.81885	.209205
4.79	22.9441	2.18861	6.92098	109.902	1.68569	3.63172	7.82429	.208768
4.80	23.0400	2.19089	6.92820	110.592	1.68687	3.63424	7.82974	.208333
4.81	23.1361	2.19317	6.93542	111.285	1.68804	3.63676	7.83517	.207900
4.82	23.2324	2.19545	6.94262	111.980	1.68920	3.63928	7.84059	.207469
4.83	23.3289	2.19773	6.94982	112.679	1.69037	3.64180	7.84601	.207039
4.84	23.4256	2.20000	6.95701	113.380	1.69154	3.64431	7.85142	.206612
4.85	23.5225	2.20227	6.96419	114.084	1.69270	3.64682	7.85683	.206186
4.86	23.6196	2.20454	6.97137	114.791	1.69386	3.64932	7.86222	.205761
4.87	23.7169	2.20681	6.97854	115.501	1.69503	3.65182	7.86761	.205339
4.88	23.8144	2.20907	6.98570	116.214	1.69619	3.65432	7.87299	.204918
4.89	23.9121	2.21133	6.99285	116.930	1.69734	3.65681	7.87837	.204499
4.90	24.0100	2.21359	7.00000	117.649	1.69850	3.65931	7.88374	.204082
4.91	24.1081	2.21585	7.00714	118.371	1.69965	3.66179	7.88909	.203666
4.92	24.2064	2.21811	7.01427	119.095	1.70081	3.66428	7.89445	.203252
4.93	24.3049	2.22036	7.02140	119.823	1.70196	3.66676	7.89979	.202840
4.94	24.4036	2.22261	7.02851	120.554	1.70311	3.66924	7.90513	.202429
4.95	24.5025	2.22486	7.03562	121.287	1.70426	3.67171	7.91046	.202020
4.96	24.6016	2.22711	7.04273	122.024	1.70540	3.67418	7.91578	.201613
4.97	24.7009	2.22935	7.04982	122.763	1.70655	3.67665	7.92110	.201207
4.98	24.8004	2.23159	7.05691	123.506	1.70769	3.67911	7.92641	.200803
4.99	24.9001	2.23383	7.06399	124.251	1.70884	3.68157	7.93171	.200401
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.00	25.0000	2.23607	7.07107	125.000	1.70998	3.68403	7.93701	.200000
5.01	25.1001	2.23830	7.07814	125.752	1.71112	3.68649	7.94229	.199601
5.02	25.2004	2.24054	7.08520	126.506	1.71225	3.68894	7.94757	.199203
5.03	25.3009	2.24277	7.09225	127.264	1.71339	3.69138	7.95285	.198807
5.04	25.4016	2.24499	7.09930	128.024	1.71452	3.69383	7.95811	.198413
5.05	25.5025	2.24722	7.10634	128.788	1.71566	3.69627	7.96337	.198020
5.06	25.6036	2.24944	7.11337	129.554	1.71679	3.69871	7.96863	.197628
5.07	25.7049	2.25167	7.12039	130.324	1.71792	3.70114	7.97387	.197239
5.08	25.8064	2.25389	7.12741	131.097	1.71905	3.70357	7.97911	.196850
5.09	25.9081	2.25610	7.13442	131.872	1.72017	3.70600	7.98434	.196464
5.10	26.0100	2.25832	7.14143	132.651	1.72130	3.70843	7.98957	.196078
5.11	26.1121	2.26053	7.14843	133.433	1.72242	3.71085	7.99479	.195695
5.12	26.2144	2.26274	7.15542	134.218	1.72355	3.71327	8.00000	.195312
5.13	26.3169	2.26495	7.16240	135.006	1.72467	3.71569	8.00520	.194932
5.14	26.4196	2.26716	7.16938	135.797	1.72579	3.71810	8.01040	.194553
5.15	26.5225	2.26936	7.17635	136.591	1.72691	3.72051	8.01559	.194175
5.16	26.6256	2.27156	7.18331	137.388	1.72802	3.72292	8.02078	.193798
5.17	26.7289	2.27376	7.19027	138.188	1.72914	3.72532	8.02596	.193424
5.18	26.8324	2.27596	7.19722	138.992	1.73025	3.72772	8.03113	.193050
5.19	26.9361	2.27816	7.20417	139.798	1.73137	3.73012	8.03629	.192678
5.20	27.0400	2.28035	7.21110	140.608	1.73248	3.73251	8.04145	.192308
5.21	27.1441	2.28254	7.21803	141.421	1.73359	3.73490	8.04660	.191939
5.22	27.2484	2.28473	7.22496	142.237	1.73470	3.73729	8.05175	.191571
5.23	27.3529	2.28692	7.23187	143.056	1.73580	3.73968	8.05689	.191205
5.24	27.4576	2.28910	7.23878	143.878	1.73691	3.74206	8.06202	.190840
5.25	27.5625	2.29129	7.24569	144.703	1.73801	3.74443	8.06714	.190476
5.26	27.6676	2.29347	7.25259	145.532	1.73912	3.74681	8.07226	.190114
5.27	27.7729	2.29565	7.25948	146.363	1.74022	3.74918	8.07737	.189753
5.28	27.8784	2.29783	7.26636	147.198	1.74132	3.75155	8.08248	.189394
5.29	27.9841	2.30000	7.27324	148.036	1.74242	3.75392	8.08758	.189036
5.30	28.0900	2.30217	7.28011	148.877	1.74351	3.75629	8.09267	.188679
5.31	28.1961	2.30434	7.28697	149.721	1.74461	3.75865	8.09776	.188324
5.32	28.3024	2.30651	7.29383	150.569	1.74570	3.76101	8.10284	.187970
5.33	28.4089	2.30868	7.30068	151.419	1.74680	3.76336	8.10791	.187617
5.34	28.5156	2.31084	7.30753	152.273	1.74789	3.76571	8.11298	.187266
5.35	28.6225	2.31301	7.31437	153.130	1.74898	3.76806	8.11804	.186916
5.36	28.7296	2.31517	7.32120	153.991	1.75007	3.77041	8.12310	.186567
5.37	28.8369	2.31733	7.32803	154.854	1.75116	3.77275	8.12814	.186220
5.38	28.9444	2.31948	7.33485	155.721	1.75224	3.77509	8.13319	.185874
5.39	29.0521	2.32164	7.34166	156.591	1.75333	3.77743	8.13822	.185529
5.40	29.1600	2.32379	7.34847	157.464	1.75441	3.77976	8.14325	.185185
5.41	29.2681	2.32594	7.35527	158.340	1.75549	3.78209	8.14828	.184843
5.42	29.3764	2.32809	7.36206	159.220	1.75657	3.78442	8.15329	.184502
5.43	29.4849	2.33024	7.36885	160.103	1.75765	3.78675	8.15831	.184162
5.44	29.5936	2.33238	7.37564	160.989	1.75873	3.78907	8.16331	.183824
5.45	29.7025	2.33452	7.38241	161.879	1.75981	3.79139	8.16831	.183486
5.46	29.8116	2.33666	7.38918	162.771	1.76088	3.79371	8.17330	.183150
5.47	29.9209	2.33880	7.39594	163.667	1.76196	3.79603	8.17829	.182815
5.48	30.0304	2.34094	7.40270	164.567	1.76303	3.79834	8.18327	.182482
5.49	30.1401	2.34307	7.40945	165.469	1.76410	3.80065	8.18824	.182149
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
5.50	30.2500	2.34521	7.41620	166.375	1.76517	3.80295	8.19321	.181818
5.51	30.3601	2.34734	7.42294	167.284	1.76624	3.80526	8.19818	.181488
5.52	30.4704	2.34947	7.42967	168.197	1.76731	3.80756	8.20313	.181159
5.53	30.5809	2.35160	7.43640	169.112	1.76838	3.80985	8.20808	.180832
5.54	30.6916	2.35372	7.44312	170.031	1.76944	3.81215	8.21303	.180505
5.55	30.8025	2.35584	7.44983	170.954	1.77051	3.81444	8.21797	.180180
5.56	30.9136	2.35797	7.45654	171.880	1.77157	3.81673	8.22290	.179856
5.57	31.0249	2.36008	7.46324	172.809	1.77263	3.81902	8.22783	.179533
5.58	31.1364	2.36220	7.46994	173.741	1.77369	3.82130	8.23275	.179211
5.59	31.2481	2.36432	7.47663	174.677	1.77475	3.82358	8.23766	.178891
5.60	31.3600	2.36643	7.48331	175.616	1.77581	3.82586	8.24257	.178571
5.61	31.4721	2.36854	7.48999	176.558	1.77686	3.82814	8.24747	.178253
5.62	31.5844	2.37065	7.49667	177.504	1.77792	3.83041	8.25237	.177936
5.63	31.6969	2.37276	7.50333	178.454	1.77897	3.83268	8.25726	.177620
5.64	31.8096	2.37487	7.50999	179.406	1.78003	3.83495	8.26215	.177305
5.65	31.9225	2.37697	7.51665	180.362	1.78108	3.83722	8.26703	.176991
5.66	32.0356	2.37908	7.52330	181.321	1.78213	3.83948	8.27190	.176678
5.67	32.1489	2.38118	7.52994	182.284	1.78318	3.84174	8.27677	.176367
5.68	32.2624	2.38328	7.53658	183.250	1.78422	3.84399	8.28164	.176056
5.69	32.3761	2.38537	7.54321	184.220	1.78527	3.84625	8.28649	.175747
5.70	32.4900	2.38747	7.54983	185.193	1.78632	3.84850	8.29134	.175439
5.71	32.6041	2.38956	7.55645	186.169	1.78736	3.85075	8.29619	.175131
5.72	32.7184	2.39165	7.56307	187.149	1.78840	3.85300	8.30103	.174825
5.73	32.8329	2.39374	7.56968	188.133	1.78944	3.85524	8.30587	.174520
5.74	32.9476	2.39583	7.57628	189.119	1.79048	3.85748	8.31069	.174216
5.75	33.0625	2.39792	7.58288	190.109	1.79152	3.85972	8.31552	.173913
5.76	33.1776	2.40000	7.58947	191.103	1.79256	3.86196	8.32034	.173611
5.77	33.2929	2.40208	7.59605	192.100	1.79360	3.86419	8.32515	.173310
5.78	33.4084	2.40416	7.60263	193.101	1.79463	3.86642	8.32995	.173010
5.79	33.5241	2.40624	7.60920	194.105	1.79567	3.86865	8.33476	.172712
5.80	33.6400	2.40832	7.61577	195.112	1.79670	3.87088	8.33955	.172414
5.81	33.7561	2.41039	7.62234	196.123	1.79773	3.87310	8.34434	.172117
5.82	33.8724	2.41247	7.62889	197.137	1.79876	3.87532	8.34913	.171821
5.83	33.9889	2.41454	7.63544	198.155	1.79979	3.87754	8.35390	.171527
5.84	34.1056	2.41661	7.64199	199.177	1.80082	3.87975	8.35868	.171233
5.85	34.2225	2.41868	7.64853	200.202	1.80185	3.88197	8.36345	.170940
5.86	34.3396	2.42074	7.65506	201.230	1.80288	3.88418	8.36821	.170649
5.87	34.4569	2.42281	7.66159	202.262	1.80390	3.88639	8.37297	.170358
5.88	34.5744	2.42487	7.66812	203.297	1.80492	3.88859	8.37772	.170068
5.89	34.6921	2.42693	7.67463	204.336	1.80595	3.89080	8.38247	.169779
5.90	34.8100	2.42899	7.68115	205.379	1.80697	3.89300	8.38721	.169492
5.91	34.9281	2.43105	7.68765	206.425	1.80799	3.89519	8.39194	.169205
5.92	35.0464	2.43311	7.69415	207.475	1.80901	3.89739	8.39667	.168919
5.93	35.1649	2.43516	7.70065	208.528	1.81003	3.89958	8.40140	.168634
5.94	35.2836	2.43721	7.70714	209.585	1.81104	3.90177	8.40612	.168350
5.95	35.4025	2.43926	7.71362	210.645	1.81206	3.90396	8.41083	.168067
5.96	35.5216	2.44131	7.72010	211.709	1.81307	3.90615	8.41554	.167785
5.97	35.6409	2.44336	7.72658	212.776	1.81409	3.90833	8.42025	.167504
5.98	35.7604	2.44540	7.73305	213.847	1.81510	3.91051	8.42494	.167224
5.99	35.8801	2.44745	7.73951	214.922	1.81611	3.91269	8.42964	.166945
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
6.00	36.0000	2.44949	7.74597	216.000	1.81712	3.91487	8.43433	.166667
6.01	36.1201	2.45153	7.75242	217.082	1.81813	3.91704	8.43901	.166389
6.02	36.2404	2.45357	7.75887	218.167	1.81914	3.91921	8.44369	.166113
6.03	36.3609	2.45561	7.76531	219.256	1.82014	3.92138	8.44836	.165837
6.04	36.4816	2.45764	7.77174	220.349	1.82115	3.92355	8.45303	.165563
6.05	36.6025	2.45967	7.77817	221.445	1.82215	3.92571	8.45769	.165289
6.06	36.7236	2.46171	7.78460	222.545	1.82316	3.92787	8.46235	.165017
6.07	36.8449	2.46374	7.79102	223.649	1.82416	3.93003	8.46700	.164745
6.08	36.9664	2.46577	7.79744	224.756	1.82516	3.93219	8.47165	.164474
6.09	37.0881	2.46779	7.80385	225.867	1.82616	3.93434	8.47629	.164204
6.10	37.2100	2.46982	7.81025	226.981	1.82716	3.93650	8.48093	.163934
6.11	37.3321	2.47184	7.81665	228.099	1.82816	3.93865	8.48556	.163666
6.12	37.4544	2.47386	7.82304	229.221	1.82915	3.94079	8.49018	.163399
6.13	37.5769	2.47588	7.82943	230.346	1.83015	3.94294	8.49481	.163132
6.14	37.6996	2.47790	7.83582	231.476	1.83115	3.94508	8.49942	.162866
6.15	37.8225	2.47992	7.84219	232.608	1.83214	3.94722	8.50403	.162602
6.16	37.9456	2.48193	7.84857	233.745	1.83313	3.94936	8.50864	.162338
6.17	38.0689	2.48395	7.85493	234.885	1.83412	3.95150	8.51324	.162075
6.18	38.1924	2.48596	7.86130	236.029	1.83511	3.95363	8.51784	.161812
6.19	38.3161	2.48797	7.86766	237.177	1.83610	3.95576	8.52243	.161551
6.20	38.4400	2.48998	7.87401	238.328	1.83709	3.95789	8.52702	.161290
6.21	38.5641	2.49199	7.88036	239.483	1.83808	3.96002	8.53160	.161031
6.22	38.6884	2.49399	7.88670	240.642	1.83906	3.96214	8.53618	.160772
6.23	38.8129	2.49600	7.89303	241.804	1.84005	3.96427	8.54075	.160514
6.24	38.9376	2.49800	7.89937	242.971	1.84103	3.96638	8.54532	.160256
6.25	39.0625	2.50000	7.90569	244.141	1.84202	3.96850	8.54988	.160000
6.26	39.1876	2.50200	7.91202	245.314	1.84300	3.97062	8.55444	.159744
6.27	39.3129	2.50400	7.91833	246.492	1.84398	3.97273	8.55899	.159490
6.28	39.4384	2.50599	7.92465	247.673	1.84496	3.97484	8.56354	.159236
6.29	39.5641	2.50799	7.93095	248.858	1.84594	3.97695	8.56808	.158983
6.30	39.6900	2.50998	7.93725	250.047	1.84691	3.97906	8.57262	.158730
6.31	39.8161	2.51197	7.94355	251.240	1.84789	3.98116	8.57715	.158479
6.32	39.9424	2.51396	7.94984	252.436	1.84887	3.98326	8.58168	.158228
6.33	40.0689	2.51595	7.95613	253.636	1.84984	3.98536	8.58620	.157978
6.34	40.1956	2.51794	7.96241	254.840	1.85082	3.98746	8.59072	.157729
6.35	40.3225	2.51992	7.96869	256.048	1.85179	3.98956	8.59524	.157480
6.36	40.4496	2.52190	7.97496	257.259	1.85276	3.99165	8.59975	.157233
6.37	40.5769	2.52389	7.98123	258.475	1.85373	3.99374	8.60425	.156986
6.38	40.7044	2.52587	7.98749	259.694	1.85470	3.99583	8.60875	.156740
6.39	40.8321	2.52784	7.99375	260.917	1.85567	3.99792	8.61325	.156495
6.40	40.9600	2.52982	8.00000	262.144	1.85664	4.00000	8.61774	.156250
6.41	41.0881	2.53180	8.00625	263.375	1.85760	4.00208	8.62222	.156006
6.42	41.2164	2.53377	8.01249	264.609	1.85857	4.00416	8.62671	.155763
6.43	41.3449	2.53574	8.01873	265.848	1.85953	4.00624	8.63118	.155521
6.44	41.4736	2.53772	8.02496	267.090	1.86050	4.00832	8.63566	.155280
6.45	41.6025	2.53969	8.03119	268.336	1.86146	4.01039	8.64012	.155039
6.46	41.7316	2.54165	8.03741	269.586	1.86242	4.01246	8.64459	.154799
6.47	41.8609	2.54362	8.04363	270.840	1.86338	4.01453	8.64904	.154560
6.48	41.9904	2.54558	8.04984	272.098	1.86434	4.01660	8.65350	.154321
6.49	42.1201	2.54755	8.05605	273.359	1.86530	4.01866	8.65795	.154083
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	1/n
6.50	42.2500	2.54951	8.06226	274.625	1.86626	4.02073	8.66239	.153846
6.51	42.3801	2.55147	8.06846	275.894	1.86721	4.02279	8.66683	.153610
6.52	42.5104	2.55343	8.07465	277.168	1.86817	4.02485	8.67127	.153374
6.53	42.6409	2.55539	8.08084	278.445	1.86912	4.02690	8.67570	.153139
6.54	42.7716	2.55734	8.08703	279.726	1.87008	4.02896	8.68012	.152905
6.55	42.9025	2.55930	8.09321	281.011	1.87103	4.03101	8.68455	.152672
6.56	43.0336	2.56125	8.09938	282.300	1.87198	4.03306	8.68896	.152439
6.57	43.1649	2.56320	8.10555	283.593	1.87293	4.03511	8.69338	.152207
6.58	43.2964	2.56515	8.11172	284.890	1.87388	4.03715	8.69778	.151976
6.59	43.4281	2.56710	8.11788	286.191	1.87483	4.03920	8.70219	.151745
6.60	43.5600	2.56905	8.12404	287.496	1.87578	4.04124	8.70659	.151515
6.61	43.6921	2.57099	8.13019	288.805	1.87672	4.04328	8.71098	.151286
6.62	43.8244	2.57294	8.13634	290.118	1.87767	4.04532	8.71537	.151057
6.63	43.9569	2.57488	8.14248	291.434	1.87862	4.04735	8.71976	.150830
6.64	44.0896	2.57682	8.14862	292.755	1.87956	4.04939	8.72414	.150602
6.65	44.2225	2.57876	8.15475	294.080	1.88050	4.05142	8.72852	.150376
6.66	44.3556	2.58070	8.16088	295.408	1.88144	4.05345	8.73289	.150150
6.67	44.4889	2.58263	8.16701	296.741	1.88239	4.05548	8.73726	.149925
6.68	44.6224	2.58457	8.17313	298.078	1.88333	4.05750	8.74162	.149701
6.69	44.7561	2.58650	8.17924	299.418	1.88427	4.05953	8.74598	.149477
6.70	44.8900	2.58844	8.18535	300.763	1.88520	4.06155	8.75034	.149254
6.71	45.0241	2.59037	8.19146	302.112	1.88614	4.06357	8.75469	.149031
6.72	45.1584	2.59230	8.19756	303.464	1.88708	4.06559	8.75904	.148810
6.73	45.2929	2.59422	8.20366	304.821	1.88801	4.06760	8.76338	.148588
6.74	45.4276	2.59615	8.20975	306.182	1.88895	4.06961	8.76772	.148368
6.75	45.5625	2.59808	8.21584	307.547	1.88988	4.07163	8.77205	.148148
6.76	45.6976	2.60000	8.22192	308.916	1.89081	4.07364	8.77638	.147929
6.77	45.8329	2.60192	8.22800	310.289	1.89175	4.07564	8.78071	.147710
6.78	45.9684	2.60384	8.23408	311.666	1.89268	4.07765	8.78503	.147493
6.79	46.1041	2.60576	8.24015	313.047	1.89361	4.07965	8.78935	.147275
6.80	46.2400	2.60768	8.24621	314.432	1.89454	4.08166	8.79366	.147059
6.81	46.3761	2.60960	8.25227	315.821	1.89546	4.08365	8.79797	.146843
6.82	46.5124	2.61151	8.25833	317.215	1.89639	4.08565	8.80227	.146628
6.83	46.6489	2.61343	8.26438	318.612	1.89732	4.08765	8.80657	.146413
6.84	46.7856	2.61534	8.27043	320.014	1.89824	4.08964	8.81087	.146199
6.85	46.9225	2.61725	8.27647	321.419	1.89917	4.09163	8.81516	.145985
6.86	47.0596	2.61916	8.28251	322.829	1.90009	4.09362	8.81945	.145773
6.87	47.1969	2.62107	8.28855	324.243	1.90102	4.09561	8.82373	.145560
6.88	47.3344	2.62298	8.29458	325.661	1.90194	4.09760	8.82801	.145349
6.89	47.4721	2.62488	8.30060	327.083	1.90286	4.09958	8.83228	.145138
6.90	47.6100	2.62679	8.30662	328.509	1.90378	4.10157	8.83656	.144928
6.91	47.7481	2.62869	8.31264	329.939	1.90470	4.10355	8.84082	.144718
6.92	47.8864	2.63059	8.31865	331.374	1.90562	4.10552	8.84509	.144509
6.93	48.0249	2.63249	8.32466	332.813	1.90653	4.10750	8.84934	.144300
6.94	48.1636	2.63439	8.33067	334.255	1.90745	4.10948	8.85360	.144092
6.95	48.3025	2.63629	8.33667	335.702	1.90837	4.11145	8.85785	.143885
6.96	48.4416	2.63818	8.34266	337.154	1.90928	4.11342	8.86210	.143678
6.97	48.5809	2.64008	8.34865	338.609	1.91019	4.11539	8.86634	.143472
6.98	48.7204	2.64197	8.35464	340.068	1.91111	4.11736	8.87058	.143266
6.99	48.8601	2.64386	8.36062	341.532	1.91202	4.11932	8.87481	.143062
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	1/n

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.00	49.0000	2.64575	8.36660	343.000	1.91293	4.12129	8.87904	.142857
7.01	49.1401	2.64764	8.37257	344.472	1.91384	4.12325	8.88327	.142653
7.02	49.2804	2.64953	8.37854	345.948	1.91475	4.12521	8.88749	.142450
7.03	49.4209	2.65141	8.38451	347.429	1.91566	4.12716	8.89171	.142248
7.04	49.5616	2.65330	8.39047	348.914	1.91657	4.12912	8.89592	.142045
7.05	49.7025	2.65518	8.39643	350.403	1.91747	4.13107	8.90013	.141844
7.06	49.8436	2.65707	8.40238	351.896	1.91838	4.13303	8.90434	.141643
7.07	49.9849	2.65895	8.40833	353.393	1.91929	4.13498	8.90854	.141443
7.08	50.1264	2.66083	8.41427	354.895	1.92019	4.13693	8.91274	.141243
7.09	50.2681	2.66271	8.42021	356.401	1.92109	4.13887	8.91693	.141044
7.10	50.4100	2.66458	8.42615	357.911	1.92200	4.14082	8.92112	.140845
7.11	50.5521	2.66646	8.43208	359.425	1.92290	4.14276	8.92531	.140647
7.12	50.6944	2.66833	8.43801	360.944	1.92380	4.14470	8.92949	.140449
7.13	50.8369	2.67021	8.44393	362.467	1.92470	4.14664	8.93367	.140252
7.14	50.9796	2.67208	8.44985	363.994	1.92560	4.14858	8.93784	.140056
7.15	51.1225	2.67395	8.45577	365.526	1.92650	4.15052	8.94201	.139860
7.16	51.2656	2.67582	8.46168	367.062	1.92740	4.15245	8.94618	.139665
7.17	51.4089	2.67769	8.46759	368.602	1.92829	4.15438	8.95034	.139470
7.18	51.5524	2.67955	8.47349	370.146	1.92919	4.15631	8.95450	.139276
7.19	51.6961	2.68142	8.47939	371.695	1.93008	4.15824	8.95866	.139082
7.20	51.8400	2.68328	8.48528	373.248	1.93098	4.16017	8.96281	.138889
7.21	51.9841	2.68514	8.49117	374.805	1.93187	4.16209	8.96696	.138696
7.22	52.1284	2.68701	8.49706	376.367	1.93277	4.16402	8.97110	.138504
7.23	52.2729	2.68887	8.50294	377.933	1.93366	4.16594	8.97524	.138313
7.24	52.4176	2.69072	8.50882	379.503	1.93455	4.16786	8.97938	.138122
7.25	52.5625	2.69258	8.51469	381.078	1.93544	4.16978	8.98351	.137931
7.26	52.7076	2.69444	8.52056	382.657	1.93633	4.17169	8.98764	.137741
7.27	52.8529	2.69629	8.52643	384.241	1.93722	4.17361	8.99176	.137552
7.28	52.9984	2.69815	8.53229	385.828	1.93810	4.17552	8.99588	.137363
7.29	53.1441	2.70000	8.53815	387.420	1.93899	4.17743	9.00000	.137174
7.30	53.2900	2.70185	8.54400	389.017	1.93988	4.17934	9.00411	.136986
7.31	53.4361	2.70370	8.54985	390.618	1.94076	4.18125	9.00822	.136799
7.32	53.5824	2.70555	8.55570	392.223	1.94165	4.18315	9.01233	.136612
7.33	53.7289	2.70740	8.56154	393.833	1.94253	4.18506	9.01643	.136426
7.34	53.8756	2.70924	8.56738	395.447	1.94341	4.18696	9.02053	.136240
7.35	54.0225	2.71109	8.57321	397.065	1.94430	4.18886	9.02462	.136054
7.36	54.1696	2.71293	8.57904	398.688	1.94518	4.19076	9.02871	.135870
7.37	54.3169	2.71477	8.58487	400.316	1.94606	4.19266	9.03280	.135685
7.38	54.4644	2.71662	8.59069	401.947	1.94694	4.19455	9.03689	.135501
7.39	54.6121	2.71846	8.59651	403.583	1.94782	4.19644	9.04097	.135318
7.40	54.7600	2.72029	8.60233	405.224	1.94870	4.19834	9.04504	.135135
7.41	54.9081	2.72213	8.60814	406.869	1.94957	4.20023	9.04911	.134953
7.42	55.0564	2.72397	8.61394	408.518	1.95045	4.20212	9.05318	.134771
7.43	55.2049	2.72580	8.61974	410.172	1.95132	4.20400	9.05725	.134590
7.44	55.3536	2.72764	8.62554	411.831	1.95220	4.20589	9.06131	.134409
7.45	55.5025	2.72947	8.63134	413.494	1.95307	4.20777	9.06537	.134228
7.46	55.6516	2.73130	8.63713	415.161	1.95395	4.20965	9.06942	.134048
7.47	55.8009	2.73313	8.64292	416.833	1.95482	4.21153	9.07347	.133869
7.48	55.9504	2.73496	8.64870	418.509	1.95569	4.21341	9.07752	.133690
7.49	56.1001	2.73679	8.65448	420.190	1.95656	4.21529	9.08156	.133511
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
7.50	56.2500	2.73861	8.66025	421.875	1.95743	4.21716	9.08560	.133333
7.51	56.4001	2.74044	8.66603	423.565	1.95830	4.21904	9.08964	.133156
7.52	56.5504	2.74226	8.67179	425.259	1.95917	4.22091	9.09367	.132979
7.53	56.7009	2.74408	8.67756	426.958	1.96004	4.22278	9.09770	.132802
7.54	56.8516	2.74591	8.68332	428.661	1.96091	4.22465	9.10173	.132626
7.55	57.0025	2.74773	8.68907	430.369	1.96177	4.22651	9.10575	.132450
7.56	57.1536	2.74955	8.69483	432.081	1.96264	4.22838	9.10977	.132275
7.57	57.3049	2.75136	8.70057	433.798	1.96350	4.23024	9.11378	.132100
7.58	57.4564	2.75318	8.70632	435.520	1.96437	4.23210	9.11779	.131926
7.59	57.6081	2.75500	8.71206	437.245	1.96523	4.23396	9.12180	.131752
7.60	57.7600	2.75681	8.71780	438.976	1.96610	4.23582	9.12581	.131579
7.61	57.9121	2.75862	8.72353	440.711	1.96696	4.23768	9.12981	.131406
7.62	58.0644	2.76043	8.72926	442.451	1.96782	4.23954	9.13380	.131234
7.63	58.2169	2.76225	8.73499	444.195	1.96868	4.24139	9.13780	.131062
7.64	58.3696	2.76405	8.74071	445.944	1.96954	4.24324	9.14179	.130890
7.65	58.5225	2.76586	8.74643	447.697	1.97040	4.24509	9.14577	.130719
7.66	58.6756	2.76767	8.75214	449.455	1.97126	4.24694	9.14976	.130548
7.67	58.8289	2.76948	8.75785	451.218	1.97211	4.24879	9.15374	.130378
7.68	58.9824	2.77128	8.76356	452.985	1.97297	4.25063	9.15771	.130208
7.69	59.1361	2.77308	8.76926	454.757	1.97383	4.25248	9.16169	.130039
7.70	59.2900	2.77489	8.77496	456.533	1.97468	4.25432	9.16566	.129870
7.71	59.4441	2.77669	8.78066	458.314	1.97554	4.25616	9.16962	.129702
7.72	59.5984	2.77849	8.78635	460.100	1.97639	4.25800	9.17359	.129534
7.73	59.7529	2.78029	8.79204	461.890	1.97724	4.25984	9.17754	.129366
7.74	59.9076	2.78209	8.79773	463.685	1.97809	4.26167	9.18150	.129199
7.75	60.0625	2.78388	8.80341	465.484	1.97895	4.26351	9.18545	.129032
7.76	60.2176	2.78568	8.80909	467.289	1.97980	4.26534	9.18940	.128866
7.77	60.3729	2.78747	8.81476	469.097	1.98065	4.26717	9.19335	.128700
7.78	60.5284	2.78927	8.82043	470.911	1.98150	4.26900	9.19729	.128535
7.79	60.6841	2.79106	8.82610	472.729	1.98234	4.27083	9.20123	.128370
7.80	60.8400	2.79285	8.83176	474.552	1.98319	4.27266	9.20516	.128205
7.81	60.9961	2.79464	8.83742	476.380	1.98404	4.27448	9.20910	.128041
7.82	61.1524	2.79643	8.84308	478.212	1.98489	4.27631	9.21302	.127877
7.83	61.3089	2.79821	8.84873	480.049	1.98573	4.27813	9.21695	.127714
7.84	61.4656	2.80000	8.85438	481.890	1.98658	4.27995	9.22087	.127551
7.85	61.6225	2.80179	8.86002	483.737	1.98742	4.28177	9.22479	.127389
7.86	61.7796	2.80357	8.86566	485.588	1.98826	4.28359	9.22871	.127226
7.87	61.9369	2.80535	8.87130	487.443	1.98911	4.28540	9.23262	.127065
7.88	62.0944	2.80713	8.87694	489.304	1.98995	4.28722	9.23653	.126904
7.89	62.2521	2.80891	8.88257	491.169	1.99079	4.28903	9.24043	.126743
7.90	62.4100	2.81069	8.88819	493.039	1.99163	4.29084	9.24434	.126582
7.91	62.5681	2.81247	8.89382	494.914	1.99247	4.29265	9.24823	.126422
7.92	62.7264	2.81425	8.89944	496.793	1.99331	4.29446	9.25213	.126263
7.93	62.8849	2.81603	8.90505	498.677	1.99415	4.29627	9.25602	.126103
7.94	63.0436	2.81780	8.91067	500.566	1.99499	4.29807	9.25991	.125945
7.95	63.2025	2.81957	8.91628	502.460	1.99582	4.29987	9.26380	.125786
7.96	63.3616	2.82135	8.92188	504.358	1.99666	4.30168	9.26768	.125628
7.97	63.5209	2.82312	8.92749	506.262	1.99750	4.30348	9.27156	.125471
7.98	63.6804	2.82489	8.93308	508.170	1.99833	4.30528	9.27544	.125313
7.99	63.8401	2.82666	8.93868	510.082	1.99917	4.30707	9.27931	.125156
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.00	64.0000	2.82843	8.94427	512.000	2.00000	4.30887	9.28318	.125000
8.01	64.1601	2.83019	8.94986	513.922	2.00083	4.31066	9.28704	.124844
8.02	64.3204	2.83196	8.95545	515.850	2.00167	4.31246	9.29091	.124688
8.03	64.4809	2.83373	8.96103	517.782	2.00250	4.31425	9.29477	.124533
8.04	64.6416	2.83549	8.96660	519.718	2.00333	4.31604	9.29862	.124378
8.05	64.8025	2.83725	8.97218	521.660	2.00416	4.31783	9.30248	.124224
8.06	64.9636	2.83901	8.97775	523.607	2.00499	4.31961	9.30633	.124069
8.07	65.1249	2.84077	8.98332	525.558	2.00582	4.32140	9.31018	.123916
8.08	65.2864	2.84253	8.98888	527.514	2.00664	4.32318	9.31402	.123762
8.09	65.4481	2.84429	8.99444	529.475	2.00747	4.32497	9.31786	.123609
8.10	65.6100	2.84605	9.00000	531.441	2.00830	4.32675	9.32170	.123457
8.11	65.7721	2.84781	9.00555	533.412	2.00912	4.32853	9.32553	.123305
8.12	65.9344	2.84956	9.01110	535.387	2.00995	4.33031	9.32936	.123153
8.13	66.0969	2.85132	9.01665	537.368	2.01078	4.33208	9.33319	.123001
8.14	66.2596	2.85307	9.02219	539.353	2.01160	4.33386	9.33702	.122850
8.15	66.4225	2.85482	9.02774	541.343	2.01242	4.33563	9.34084	.122699
8.16	66.5856	2.85657	9.03327	543.338	2.01325	4.33741	9.34466	.122549
8.17	66.7489	2.85832	9.03881	545.339	2.01407	4.33918	9.34847	.122399
8.18	66.9124	2.86007	9.04434	547.343	2.01489	4.34095	9.35229	.122249
8.19	67.0761	2.86182	9.04986	549.353	2.01571	4.34271	9.35610	.122100
8.20	67.2400	2.86356	9.05539	551.368	2.01653	4.34448	9.35990	.121951
8.21	67.4041	2.86531	9.06091	553.388	2.01735	4.34625	9.36370	.121803
8.22	67.5684	2.86705	9.06642	555.412	2.01817	4.34801	9.36751	.121655
8.23	67.7329	2.86880	9.07193	557.442	2.01899	4.34977	9.37130	.121507
8.24	67.8976	2.87054	9.07744	559.476	2.01980	4.35153	9.37510	.121359
8.25	68.0625	2.87228	9.08295	561.516	2.02062	4.35329	9.37889	.121212
8.26	68.2276	2.87402	9.08845	563.560	2.02144	4.35505	9.38268	.121065
8.27	68.3929	2.87576	9.09395	565.609	2.02225	4.35681	9.38646	.120919
8.28	68.5584	2.87750	9.09945	567.664	2.02307	4.35856	9.39024	.120773
8.29	68.7241	2.87924	9.10494	569.723	2.02388	4.36032	9.39402	.120627
8.30	68.8900	2.88097	9.11043	571.787	2.02469	4.36207	9.39780	.120482
8.31	69.0561	2.88271	9.11592	573.856	2.02551	4.36382	9.40157	.120337
8.32	69.2224	2.88444	9.12140	575.930	2.02632	4.36557	9.40534	.120192
8.33	69.3889	2.88617	9.12688	578.010	2.02713	4.36732	9.40911	.120048
8.34	69.5556	2.88791	9.13236	580.094	2.02794	4.36907	9.41287	.119904
8.35	69.7225	2.88964	9.13783	582.183	2.02875	4.37081	9.41663	.119760
8.36	69.8896	2.89137	9.14330	584.277	2.02956	4.37256	9.42039	.119617
8.37	70.0569	2.89310	9.14877	586.376	2.03037	4.37430	9.42414	.119474
8.38	70.2244	2.89482	9.15423	588.480	2.03118	4.37604	9.42789	.119332
8.39	70.3921	2.89655	9.15969	590.590	2.03199	4.37778	9.43164	.119190
8.40	70.5600	2.89828	9.16515	592.704	2.03279	4.37952	9.43539	.119048
8.41	70.7281	2.90000	9.17061	594.823	2.03360	4.38126	9.43913	.118906
8.42	70.8964	2.90172	9.17606	596.948	2.03440	4.38299	9.44287	.118765
8.43	71.0649	2.90345	9.18150	599.077	2.03521	4.38473	9.44661	.118624
8.44	71.2336	2.90517	9.18695	601.212	2.03601	4.38646	9.45034	.118483
8.45	71.4025	2.90689	9.19239	603.351	2.03682	4.38819	9.45407	.118343
8.46	71.5716	2.90861	9.19783	605.496	2.03762	4.38992	9.45780	.118203
8.47	71.7409	2.91033	9.20326	607.645	2.03842	4.39165	9.46152	.118064
8.48	71.9104	2.91204	9.20869	609.800	2.03923	4.39338	9.46525	.117925
8.49	72.0801	2.91376	9.21412	611.960	2.04003	4.39510	9.46897	.117786
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
8.50	72.2500	2.91548	9.21954	614.125	2.04083	4.39683	9.47268	.117647
8.51	72.4201	2.91719	9.22497	616.295	2.04163	4.39855	9.47640	.117509
8.52	72.5904	2.91890	9.23038	618.470	2.04243	4.40028	9.48011	.117371
8.53	72.7609	2.92062	9.23580	620.650	2.04323	4.40200	9.48381	.117233
8.54	72.9316	2.92233	9.24121	622.836	2.04402	4.40372	9.48752	.117096
8.55	73.1025	2.92404	9.24662	625.026	2.04482	4.40543	9.49122	.116959
8.56	73.2736	2.92575	9.25203	627.222	2.04562	4.40715	9.49492	.116822
8.57	73.4449	2.92746	9.25743	629.423	2.04641	4.40887	9.49861	.116686
8.58	73.6164	2.92916	9.26283	631.629	2.04721	4.41058	9.50231	.116550
8.59	73.7881	2.93087	9.26823	633.840	2.04801	4.41229	9.50600	.116414
8.60	73.9600	2.93258	9.27362	636.056	2.04880	4.41400	9.50969	.116279
8.61	74.1321	2.93428	9.27901	638.277	2.04959	4.41571	9.51337	.116144
8.62	74.3044	2.93598	9.28440	640.504	2.05039	4.41742	9.51705	.116009
8.63	74.4769	2.93769	9.28978	642.736	2.05118	4.41913	9.52073	.115875
8.64	74.6496	2.93939	9.29516	644.973	2.05197	4.42084	9.52441	.115741
8.65	74.8225	2.94109	9.30054	647.215	2.05276	4.42254	9.52808	.115607
8.66	74.9956	2.94279	9.30591	649.462	2.05355	4.42425	9.53175	.115473
8.67	75.1689	2.94449	9.31128	651.714	2.05434	4.42595	9.53542	.115340
8.68	75.3424	2.94618	9.31665	653.972	2.05513	4.42765	9.53908	.115207
8.69	75.5161	2.94788	9.32202	656.235	2.05592	4.42935	9.54274	.115075
8.70	75.6900	2.94958	9.32738	658.503	2.05671	4.43105	9.54640	.114943
8.71	75.8641	2.95127	9.33274	660.776	2.05750	4.43274	9.55006	.114811
8.72	76.0384	2.95296	9.33809	663.055	2.05828	4.43444	9.55371	.114679
8.73	76.2129	2.95466	9.34345	665.339	2.05907	4.43613	9.55736	.114548
8.74	76.3876	2.95635	9.34880	667.628	2.05986	4.43783	9.56101	.114416
8.75	76.5625	2.95804	9.35414	669.922	2.06064	4.43952	9.56466	.114286
8.76	76.7376	2.95973	9.35949	672.221	2.06143	4.44121	9.56830	.114155
8.77	76.9129	2.96142	9.36483	674.526	2.06221	4.44290	9.57194	.114025
8.78	77.0884	2.96311	9.37017	676.836	2.06299	4.44459	9.57557	.113895
8.79	77.2641	2.96479	9.37550	679.151	2.06378	4.44627	9.57921	.113766
8.80	77.4400	2.96648	9.38083	681.472	2.06456	4.44796	9.58284	.113636
8.81	77.6161	2.96816	9.38616	683.798	2.06534	4.44964	9.58647	.113507
8.82	77.7924	2.96985	9.39149	686.129	2.06612	4.45133	9.59009	.113379
8.83	77.9689	2.97153	9.39681	688.465	2.06690	4.45301	9.59372	.113250
8.84	78.1456	2.97321	9.40213	690.807	2.06768	4.45469	9.59734	.113122
8.85	78.3225	2.97489	9.40744	693.154	2.06846	4.45637	9.60095	.112994
8.86	78.4996	2.97658	9.41276	695.506	2.06924	4.45805	9.60457	.112867
8.87	78.6769	2.97825	9.41807	697.864	2.07002	4.45972	9.60818	.112740
8.88	78.8544	2.97993	9.42338	700.227	2.07080	4.46140	9.61179	.112613
8.89	79.0321	2.98161	9.42868	702.595	2.07157	4.46307	9.61540	.112486
8.90	79.2100	2.98329	9.43398	704.969	2.07235	4.46475	9.61900	.112360
8.91	79.3881	2.98496	9.43928	707.348	2.07313	4.46642	9.62260	.112233
8.92	79.5664	2.98664	9.44458	709.732	2.07390	4.46809	9.62620	.112108
8.93	79.7449	2.98831	9.44987	712.122	2.07468	4.46976	9.62980	.111982
8.94	79.9236	2.98998	9.45516	714.517	2.07545	4.47142	9.63339	.111857
8.95	80.1025	2.99166	9.46044	716.917	2.07622	4.47309	9.63698	.111732
8.96	80.2816	2.99333	9.46573	719.323	2.07700	4.47476	9.64057	.111607
8.97	80.4609	2.99500	9.47101	721.734	2.07777	4.47642	9.64415	.111483
8.98	80.6404	2.99666	9.47629	724.151	2.07854	4.47808	9.64774	.111359
8.99	80.8201	2.99833	9.48156	726.573	2.07931	4.47974	9.65132	.111235
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
9.00	81.0000	3.00000	9.48683	729.000	2.08008	4.48140	9.65489	.111111
9.01	81.1801	3.00167	9.49210	731.433	2.08085	4.48306	9.65847	.110988
9.02	81.3604	3.00333	9.49737	733.871	2.08162	4.48472	9.66204	.110865
9.03	81.5409	3.00500	9.50263	736.314	2.08239	4.48638	9.66561	.110742
9.04	81.7216	3.00666	9.50789	738.763	2.08316	4.48803	9.66918	.110619
9.05	81.9025	3.00832	9.51315	741.218	2.08393	4.48969	9.67274	.110497
9.06	82.0836	3.00998	9.51840	743.677	2.08470	4.49134	9.67630	.110375
9.07	82.2649	3.01164	9.52365	746.143	2.08546	4.49299	9.67986	.110254
9.08	82.4464	3.01330	9.52890	748.613	2.08623	4.49464	9.68342	.110132
9.09	82.6281	3.01496	9.53415	751.089	2.08699	4.49629	9.68697	.110011
9.10	82.8100	3.01662	9.53939	753.571	2.08776	4.49794	9.69052	.109890
9.11	82.9921	3.01828	9.54463	756.058	2.08852	4.49959	9.69407	.109769
9.12	83.1744	3.01993	9.54987	758.551	2.08929	4.50123	9.69762	.109649
9.13	83.3569	3.02159	9.55510	761.048	2.09005	4.50288	9.70116	.109529
9.14	83.5396	3.02324	9.56033	763.552	2.09081	4.50452	9.70470	.109409
9.15	83.7225	3.02490	9.56556	766.061	2.09158	4.50616	9.70824	.109290
9.16	83.9056	3.02655	9.57079	768.575	2.09234	4.50781	9.71177	.109170
9.17	84.0889	3.02820	9.57601	771.095	2.09310	4.50945	9.71531	.109051
9.18	84.2724	3.02985	9.58123	773.621	2.09386	4.51108	9.71884	.108932
9.19	84.4561	3.03150	9.58645	776.152	2.09462	4.51272	9.72236	.108814
9.20	84.6400	3.03315	9.59166	778.688	2.09538	4.51436	9.72589	.108696
9.21	84.8241	3.03480	9.59687	781.230	2.09614	4.51599	9.72941	.108578
9.22	85.0084	3.03645	9.60208	783.777	2.09690	4.51763	9.73293	.108460
9.23	85.1929	3.03809	9.60729	786.330	2.09765	4.51926	9.73645	.108342
9.24	85.3776	3.03974	9.61249	788.889	2.09841	4.52089	9.73996	.108225
9.25	85.5625	3.04138	9.61769	791.453	2.09917	4.52252	9.74348	.108108
9.26	85.7476	3.04302	9.62289	794.023	2.09992	4.52415	9.74699	.107991
9.27	85.9329	3.04467	9.62808	796.598	2.10068	4.52578	9.75049	.107875
9.28	86.1184	3.04631	9.63328	799.179	2.10144	4.52740	9.75400	.107759
9.29	86.3041	3.04795	9.63846	801.765	2.10219	4.52903	9.75750	.107643
9.30	86.4900	3.04959	9.64365	804.357	2.10294	4.53065	9.76100	.107527
9.31	86.6761	3.05123	9.64883	806.954	2.10370	4.53228	9.76450	.107411
9.32	86.8624	3.05287	9.65401	809.558	2.10445	4.53390	9.76799	.107296
9.33	87.0489	3.05450	9.65919	812.166	2.10520	4.53552	9.77148	.107181
9.34	87.2356	3.05614	9.66437	814.781	2.10595	4.53714	9.77497	.107066
9.35	87.4225	3.05778	9.66954	817.400	2.10671	4.53876	9.77846	.106952
9.36	87.6096	3.05941	9.67471	820.026	2.10746	4.54038	9.78195	.106838
9.37	87.7969	3.06105	9.67988	822.657	2.10821	4.54199	9.78543	.106724
9.38	87.9844	3.06268	9.68504	825.294	2.10896	4.54361	9.78891	.106610
9.39	88.1721	3.06431	9.69020	827.936	2.10971	4.54522	9.79239	.106496
9.40	88.3600	3.06594	9.69536	830.584	2.11045	4.54684	9.79586	.106383
9.41	88.5481	3.06757	9.70052	833.238	2.11120	4.54845	9.79933	.106270
9.42	88.7364	3.06920	9.70567	835.897	2.11195	4.55006	9.80280	.106157
9.43	88.9249	3.07083	9.71082	838.562	2.11270	4.55167	9.80627	.106045
9.44	89.1136	3.07246	9.71597	841.232	2.11344	4.55328	9.80974	.105932
9.45	89.3025	3.07409	9.72111	843.909	2.11419	4.55488	9.81320	.105820
9.46	89.4916	3.07571	9.72625	846.591	2.11494	4.55649	9.81666	.105708
9.47	89.6809	3.07734	9.73139	849.278	2.11568	4.55809	9.82012	.105597
9.48	89.8704	3.07896	9.73653	851.971	2.11642	4.55970	9.82357	.105485
9.49	90.0601	3.08058	9.74166	854.670	2.11717	4.56130	9.82703	.105374
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

Powers — Roots — Reciprocals

n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$
9.50	90.2500	3.08221	9.74679	857.375	2.11791	4.56290	9.83048	.105263
9.51	90.4401	3.08383	9.75192	860.085	2.11865	4.56450	9.83392	.105152
9.52	90.6304	3.08545	9.75705	862.801	2.11940	4.56610	9.83737	.105042
9.53	90.8209	3.08707	9.76217	865.523	2.12014	4.56770	9.84081	.104932
9.54	91.0116	3.08869	9.76729	868.251	2.12088	4.56930	9.84425	.104822
9.55	91.2025	3.09031	9.77241	870.984	2.12162	4.57089	9.84769	.104712
9.56	91.3936	3.09192	9.77753	873.723	2.12236	4.57249	9.85113	.104603
9.57	91.5849	3.09354	9.78264	876.467	2.12310	4.57408	9.85456	.104493
9.58	91.7764	3.09516	9.78775	879.218	2.12384	4.57567	9.85799	.104384
9.59	91.9681	3.09677	9.79285	881.974	2.12458	4.57727	9.86142	.104275
9.60	92.1600	3.09839	9.79796	884.736	2.12532	4.57886	9.86485	.104167
9.61	92.3521	3.10000	9.80306	887.504	2.12605	4.58045	9.86827	.104058
9.62	92.5444	3.10161	9.80816	890.277	2.12679	4.58204	9.87169	.103950
9.63	92.7369	3.10322	9.81326	893.056	2.12753	4.58362	9.87511	.103842
9.64	92.9296	3.10483	9.81835	895.841	2.12826	4.58521	9.87853	.103734
9.65	93.1225	3.10644	9.82344	898.632	2.12900	4.58679	9.88195	.103627
9.66	93.3156	3.10805	9.82853	901.429	2.12974	4.58838	9.88536	.103520
9.67	93.5089	3.10966	9.83362	904.231	2.13047	4.58996	9.88877	.103413
9.68	93.7024	3.11127	9.83870	907.039	2.13120	4.59154	9.89217	.103306
9.69	93.8961	3.11288	9.84378	909.853	2.13194	4.59312	9.89558	.103199
9.70	94.0900	3.11448	9.84886	912.673	2.13267	4.59470	9.89898	.103093
9.71	94.2841	3.11609	9.85393	915.499	2.13340	4.59628	9.90238	.102987
9.72	94.4784	3.11769	9.85901	918.330	2.13414	4.59786	9.90578	.102881
9.73	94.6729	3.11929	9.86408	921.167	2.13487	4.59943	9.90918	.102775
9.74	94.8676	3.12090	9.86914	924.010	2.13560	4.60101	9.91257	.102669
9.75	95.0625	3.12250	9.87421	926.859	2.13633	4.60258	9.91596	.102564
9.76	95.2576	3.12410	9.87927	929.714	2.13706	4.60416	9.91935	.102459
9.77	95.4529	3.12570	9.88433	932.575	2.13779	4.60573	9.92274	.102354
9.78	95.6484	3.12730	9.88939	935.441	2.13852	4.60730	9.92612	.102249
9.79	95.8441	3.12890	9.89444	938.314	2.13925	4.60887	9.92950	.102145
9.80	96.0400	3.13050	9.89949	941.192	2.13997	4.61044	9.93288	.102041
9.81	96.2361	3.13209	9.90454	944.076	2.14070	4.61200	9.93626	.101937
9.82	96.4324	3.13369	9.90959	946.966	2.14143	4.61357	9.93964	.101833
9.83	96.6289	3.13528	9.91464	949.862	2.14216	4.61514	9.94301	.101729
9.84	96.8256	3.13688	9.91968	952.764	2.14288	4.61670	9.94638	.101626
9.85	97.0225	3.13847	9.92472	955.672	2.14361	4.61826	9.94975	.101523
9.86	97.2196	3.14006	9.92975	958.585	2.14433	4.61983	9.95311	.101420
9.87	97.4169	3.14166	9.93479	961.505	2.14506	4.62139	9.95648	.101317
9.88	97.6144	3.14325	9.93982	964.430	2.14578	4.62295	9.95984	.101215
9.89	97.8121	3.14484	9.94485	967.362	2.14651	4.62451	9.96320	.101112
9.90	98.0100	3.14643	9.94987	970.299	2.14723	4.62607	9.96655	.101010
9.91	98.2081	3.14802	9.95490	973.242	2.14795	4.62762	9.96991	.100908
9.92	98.4064	3.14960	9.95992	976.191	2.14867	4.62918	9.97326	.100806
9.93	98.6049	3.15119	9.96494	979.147	2.14940	4.63073	9.97661	.100705
9.94	98.8036	3.15278	9.96995	982.108	2.15012	4.63229	9.97996	.100604
9.95	99.0025	3.15436	9.97497	985.075	2.15084	4.63384	9.98331	.100503
9.96	99.2016	3.15595	9.97998	988.048	2.15156	4.63539	9.98665	.100402
9.97	99.4009	3.15753	9.98499	991.027	2.15228	4.63694	9.98999	.100301
9.98	99.6004	3.15911	9.98999	994.012	2.15300	4.63849	9.99333	.100200
9.99	99.8001	3.16070	9.99500	997.003	2.15372	4.64004	9.99667	.100100
10.00	100.000	3.16228	10.0000	1000.00	2.15443	4.64159	10.0000	.100000
n	n^2	$\sqrt{n}$	$\sqrt{10n}$	n^3	$\sqrt[3]{n}$	$\sqrt[3]{10n}$	$\sqrt[3]{100n}$	$1/n$

Mathematical Glossary

Adjacent side. Side (other than hypotenuse) of a right triangle adjacent to the angle (other than the right angle).

Algebra. A mathematical science whereby both numbers and symbols are used and whose values may be either positive or negative.

Angle. Measured in degrees or radians. Formed by two straight lines meeting at a point.

Antilogarithm. Number corresponding to a given logarithm.

Arithmetic. Deals with numbers.

Base—Logarithm. Number selected and used with different exponent values (called logarithms) to give corresponding numbers.

Characteristic. The whole number part of a logarithm.

Circumference. Length of a line forming a circle.

Coefficient. Any factor of a product is a coefficient of the remaining factors of the product.

Cologarithm. Logarithm of the reciprocal of a number.

Cosine (Cos). Name of the ratio: Adjacent side over the hypotenuse.

Degrees. Angular measure (360° in a circle).

Denominator. Below the division line.

Diameter. Straight line dividing a circle area in half.

Equal sign. Symbol used to separate two equal quantities.

Equation. A statement that two quantities are equal.

Equation—Simultaneous. Solution of two linear equations with a set of values that satisfies both linear (straight line) equations.

Exponent. A number, letter, or symbol which, when placed at the upper right side of another number, letter, or symbol, indicates how many times the latter is to be taken as a factor.

Fraction. A portion of a whole number.

Graph. A pictorial representation of the relation between two or more quantities.

Hypotenuse. The longest side of a right triangle and is the side opposite the right angle.

Logarithm. The logarithm of a number is the exponent to which a base is raised to obtain the number.

Logarithm—Tables. Tables giving numbers and corresponding logarithms.

Mantissa. The decimal fraction part of a logarithm.

Numbers—Negative. Less than zero.

Numbers—Positive. Greater than zero.

Numerator. Above the division line.

Opposite side. Side of a right triangle opposite to the angle (other than the right angle).

Pi. Ratio of circumference of a circle to its diameter and is equal to 3.1416 . . . .

Polynomial. An algebraic expression consisting of two or more terms.

Quadrant. Division of a plane with reference to the right angle intersection of two axes which divide the plane into four parts called quadrants.

Radian. Circumference divided by the radius which gives 2π. 360° divided by 2π gives 57.3° in one radian.

Radical. Indicating that the root of a number is to be obtained.

Radius. Half the length of the diameter of a circle.

Sine (Sin). Name of the ratio: Opposite side over the hypotenuse.

Slide rule. A mechanical device with scales whereby mathematical operations may be performed by mechanically manipulating the scales.

Symbol—Mathematical. An abbreviated name representing a quantity.

Tangent (Tan). Name of the ratio: Opposite side over the adjacent side.

Triangle. Figure bound by three sides.

Triangle—Right. A triangle having 90° in one of its three angles.

Trigonometric function. Ratio of one side of a right triangle to another (six possible ratios).

Trigonometric table. Table of angles in degrees and fractional parts thereof with corresponding values of trigonometric functions.

Trigonometry. Deals with the mathematical relations of the sides and angles of triangles to each other.

Vector. A quantity which has both magnitude and direction.

Vector components. x and y axis projections.

Electrical Glossary

Alternation. One-half cycle.

Alternating current. A current whose direction of flow changes periodically.

Ammeter. Instrument used for measuring current in amperes.

Ampere. A coulomb of electric current flowing past a given point in one second.

Ampere turns. Product of amperes times number of turns.

Atom. The smallest part to which an element may be reduced and still remain an element.

Battery. Device supplying electrical power by electrochemical action.

Capacitance. Property of an electrical circuit to store electrical energy in an electric field.

Charge-Electric. An electric charge is determined by the number of electrons present. An excess of electrons denotes a negative charge while a deficiency of electrons denotes a positive charge.

Circuit-Electric. Electrical path formed by electrical conductors.

Circuit-Parallel. Circuit elements connected such that several separate paths for current are provided.

Circuit-Series. Circuit elements connected such that current flows from one element to the next.

Condenser. Capacitance device consisting of two conducting surfaces separated by an insulating material.

Conductance. Ability of a substance to conduct electricity.

Conductor. A substance which will allow electric current to flow through it readily.

Coulomb. An electrical charge of 6.3 x 10^{18} electrons.

Current. Movement of electrons past a given point.

Current-Effective. Maximum current times .707.

Cycle. An alternation in one direction with a subsequent alternation in the opposite direction.

D'Arsonval movement. Basic unit of the most common type electrical instrument for measuring electrical quantities.

Dielectric. Insulating material between the conducting plates of a condenser.

Direct current. Current flowing in one direction only.

Dry cell. A wet cell that is not nearly so wet.

Efficiency. Power output divided by the input.

Electricity. A form of energy associated with the electron.

Electromagnetism. Production of magnetism or a magnetic field by means of a current flowing in a conductor.

Electron. Smallest division of matter—a negative charge of electricity.

Farad. Unit of capacitance.

Ferromagnetic. Material possessing magnetic properties.

Field coil-Speaker. Coil of a dynamic speaker (non-permanent magnet type) supplied with direct current to provide an electromagnetic field for the speaker.

Flux density. Number of lines for a given unit cross-sectional area at right angles to the direction of the magnetic lines of force (measured in gausses).

Frequency. Number of cycles per second.

Frequency-Audio. Audible to the human ear. (Range—approximately 20-17,000 cycles per second.)

Frequency-Radio. Used for radio transmission (as high as 30,000 MC per second).

Galvanometer. A current indicating device whose scale is not calibrated in any particular units.

Generator. A rotating device which converts mechanical energy into electrical energy (d-c or a-c) or an electrical device which converts d-c energy into radio frequency a-c energy.

Headphone. A device for converting electrical impulses (which are too weak to operate a speaker satisfactorily) into audible sound.

Henry. Unit of inductance.

Hysteresis. Action of the magnetic flux density in lagging behind the magnetizing force.

Impedance. Combination of resistance and reactance or inductive and capacitive reactance.

Inductance. Property of a coil or a circuit which opposes a change of current through it.

Instrument-Electrical. A device for measuring present value of observed electrical quantity.

Insulator. A material which offers so much opposition to the flow of current that practically no current at all flows through the material.

Magnet. A magnetized piece of metal.

Magnet-Permanent. Material retaining its magnetism after the magnetizing force is removed.

Magnet-Temporary. Material retaining its magnetism only so long as the magnetizing force is present.

Magnetic field. Magnetic lines of force surrounding a magnet or, a conductor carrying current.

Magnetic lines of force. Imaginary lines considered to represent the magnetic force associated with a magnet or a conductor carrying current.

Magnetism. The property of certain substances to attract others.

Magnetism-Residual. Trace of magnetism remaining in a material after magnetizing force is removed.

Magnetizing force. Magnetomotive force per unit length of the magnetic path. Unit is in oersteds.

Magnetomotive force. Magnetic force of a magnetic circuit measured in gilberts.

Maxwells. Total number of lines of force (sometimes called magnetic flux).

Meter-Electrical. A device which registers or measures an electrical quantity with respect to time.

Milliammeter. Current measuring instrument whose scale is calibrated in milliamperes or 1/1000 ths of an ampere.

Motor generator. An electric motor and generator built on a common shaft as one unit.

Multiplier. Resistor connected in series with a voltage measuring instrument to extend its range.

Ohm. Unit of resistance; also unit of measure of reactance.

Ohmmeter. A device for measuring resistance which has an instrument scale calibrated in ohms.

Permeability. Ratio of the magnetic-conducting ability of a material as compared to air which is one.

Phase. Angle of lead or lag between waves with the same frequency.

Polarity-Electric. Electric charge of one point of a circuit with respect to another point (plus or minus).

Polarity-Magnetic. Meaning either a north or south pole.

Poles-Magnetic. The ends of a magnet. One pole is called the north pole and the other is called the south pole.

Power-Apparent. a-c voltage times a-c current, disregarding the phase angle between the two.

Power-Average. E_{eff} times I_{eff}.

Power-Electrical. Ability of an electrical circuit to do work and may be defined as the time rate at which work can be done.

Power-Maximum. $E_{max} \times I_{max}$.

Proton. Positive charge of electricity.

Reactance. Opposition to an a-c wave due to inductive or capacitive property of a circuit.

Rectifier. A device which will allow current to flow through it in one direction only.

Relay. Electro-mechanical device with contact points, some of which move and may be used to control other circuits when the relay coil is energized.

Reluctance. Opposition of a substance to the magnetic lines of force in a magnetic circuit. No name for this unit. The name oersted was formerly used but this name, by recent international agreement, is now used to name the unit of magnetizing force.

Resistance. That property of a material which gives rise to its opposition to an electrical current flowing in the material.

Resistivity. A factor used to compare the resistance property of one material to another. Copper is usually considered to have a resistivity of 1 and all other substances are compared to cooper.

Resistor. Resistance device.

Resistor-Variable. A resistor whose resistance value may be changed.

Resonance. A condition where the inductive reactance and capacitive reactance are equal and opposite at a particular frequency.

Shunt. Resistor connected in parallel with a current measuring instrument to extend its range.

Sinusoidal. Sine wave form.

Speaker. Device for converting audio frequency electrical impulses of sufficient volume into audio sound waves of suitable volume for home, auditorium, or outdoor use.

Storage battery. A rechargeable battery.

Symbols-Electrical. Designs used in radio circuit diagrams to represent radio parts, such as resistors, tubes, coils, etc.

Transformer. Device for stepping voltage or current up or down.

Vibrator. An electrically operated vibrating contact device which interrupts direct current.

Voice coil. Small coil attached to the vibrating cone of a speaker.

Volt-Ohmmeter. Trade name given a multi-purpose instrument usually provided with several ohm scales, both d-c and a-c current and voltage scales.

Voltage. Electromotive force.

Voltage drop. A difference in potential, in an electrical circuit, due to resistance.

Voltage-Effective. Maximum voltage times .707.

Voltage regulation. Percentage voltage variation of a voltage source from full load to no load.

Volume control. A variable resistance device for raising or lowering the audio voltage of a circuit.

Watt. Unit of electrical power.

Wattmeter. Meter that measures or records the watts used by an electrical device.

Wave. A continually varying quantity such as a sine function of voltage or current.

Wave-Complex. Combination of two or more waves.

Wet cell. A battery with a liquid electrolyte.

Wheatstone bridge. A laboratory device for accurately measuring electrical quantities such as resistance, inductance, capacitance, reactance, and impedance.

Wiring. Conductors connecting several electrical units together.

Work. A force in motion, acting against an opposing force.

Index

A-c rectifiers, 64
Accuracy, answers, 14
Algebra, 15
  addition, 85
  division, 87
  multiplication, 86
  subtraction, 85
Alternating current, 152
  effective value, 178
  generation of, 175
  maximum value, 178
  measurements, 206
  voltage, effective value, 178
    maximum value, 178
Alternation, 152
Ammeter, 192
Ampere, a-c, 176
  d-c, 5
Ampere turns, 135
Angles, 154
  acute, 154
Angular difference, 169
Arithmetic, 15
Atom, 4

Batteries, 60
  B, 62
  dry cells, 60, 61
  parallel connected, 63
  series connected, 62
B-H curves, 142

Capacitance, 234
  unit of, 240
Capacitive reactance, 243
Charge, electrical, 5
  in a condenser, 239
  negative, 4
  positive, 4
  unit of measure, 5
Circuit, defined, 11
  magnetic, 133, 137, 138
  wiring, 68
    form, 72
    point-to-point, 68
Circuits, advanced, 75
  series, 40
  series parallel, 42
  series parallel a-c, 265
  simple, 73
  superposition solution, 106
Circumference of circle, relation to diameter, 166
  relation to radians, 166
Coefficients, 16
Complex waves, 174
Condenser, capacitance of, 240
  defined, 234
  energy storing properties, 239
  properties, 245
Condensers, electrolytic, 247
  mica, 246
  paper, 246
  parallel and series connected, 241
  variable, 246
Conductance, 49
Conductors, 10
Core saturation, 141
Coulomb, 5
Current, 5
  alternating, 152
    effective, 178
    maximum, 178
  direct, 5
  direction of flow, 97
  division of, 35
  in resistance branches, 37, 38
  left-hand rule, 127
  measurement of, 196
  rate of change, 228
  unit of measure, 5
  vector of, 187
Cycle, 152

D'Arsonval movement, 192
  care of, 201
  phantom view of, 194
  sketch of, 193
D-c voltmeter, 198

Degrees, 154
Dielectric, 237
Dielectric constant, 238
  table of, 238
Dielectric materials, 238
Dielectric strength, 239

Eddy currents, 227
Efficiency, 57
Electric current, 5
Electric field, 234
Electrical measurements, 192
Electrical symbols, 12
Electrodynamometer type instrument, 208
  sketch of, 209
  pictures of, 210, 211
Electromagnetism, 128
Electron, 4
Elements, 4
Equal sign, 18
Equation, defined, 18
  rules of, 19, 20
Equations, simultaneous, 90
Exponents, defined, 25
  fractional, 30
  law of for division, 26
    multiplication, 25
  negative, 26
  one, 26
  zero, 26, 27

Farad, 240
Ferromagnetic materials, 112
Field, around a conductor, 126
  electric, 234
  magnetic, 111
Flux density, 141
Flux, leakage, 227
Force, electric lines of, 237
  magnetic lines of, 114
  magnetizing, 135
  magnetomotive, 133
Fractional exponents, 30
Fractions, 29
Frequency, 153

Galvanometer, 201
Generators, 63
Gilbert, 133
Greek alphabet, 272

Headphones, 130
Henry, 220
Hipernick, 113
Hysteresis, 146
  losses, 147

Impedance, 250
  determination of, 250
  capacitance, inductance, and resistance in parallel, 262
  capacitance, inductance, and resistance in series, 255
  capacitance and resistance in parallel, 261
  capacitance and resistance in series, 253
  inductance and resistance in parallel, 259
  inductance and resistance in series, 251
Impedance matching, 265–268
Inductance, defined, 219
  mutual, 222
  pictures of, 221, 222
  self, 219
  unit of, 220
Inductances connected in series-parallel, 227
Inductive reactance, 230
Instruments, multipliers, 199
  scales, a-c, 214
    current-squared, 215
    d-c, 195
  shunts, 197
Instrument types, 196
  D'Arsonval, 192
  electrodynamometer, 208
  hot-wire, 212
  iron-vane, 210
  rectifier, 214
  thermocouple, 213
Instruments, current measuring a-c, 208-214
  current measuring d-c, 195
  voltage measuring a-c, 208
  voltage measuring d-c, 195
Insulators, 10

Kirchhoff, Gustav Robert, 82
Kirchhoff's laws, 82, 83
  extended applications, 100
  simple applications, 94

Leakage flux, 227
Lenz's law, 218
Lines of force, electric, 237
  magnetic, 114
Load, 267
Logarithm, antilogarithm, 121
  bases, 117
  change of base, 124
  characteristic, 117
  cologarithm, 123

Logarithm (Continued)
  defined, 116
  mantissa, 117
  of a number, 119
  of a power, 123
  of a product, 121
  of a quotient, 122
  of a root, 123
  table, 118, 273–291

Magnet, 111
  permanent, 113
  temporary, 113
  uses, 116
Magnetic-circuit calculations, 149
Magnetic field, 111
  of magnets, 115
  around a coil, 128
  around a conductor, 126
Magnetic flux, 115
Magnetic flux density, 141
Magnetic force, 111
Magnetic lines of force, 114
  direction of, 114
  unit of, 115
Magnetic materials, 112
  Hypernick, 113
  Nicoloi, 113
  Permalloy, 112
  Perminvar, 113
Magnetic polarity, 114
Magnetic poles, 113
Magnetic properties of materials, 112
Magnetic shielding, 143
Magnetism, 111
  residual, 145
  theory of, 113
Magnetizing force, 135
Magnetomotive force, 133
  unit of, 133
Materials, magnetic properties of, 112
Mathematics, 2, 84
Maxwells, 115
Measurement, of current, 195–197
  of power, 206
  of resistance, 202–205
  of voltage, 195
Measurements, a-c, 206
Measurements, electrical, 192
Mho, 50
Microammeter, 195
Milliammeter, 195, 196
Minuend, 85
Molecule, 4
Multipliers, 199

Nicoli, 113
Numbers, 16
  laws of, 17

Ohm, 7
Ohm, defined, 21
Ohmmeter, 204
Ohm's law, 21
Ohm's law, a-c circuit, 179
Ohms-per-volt, 201

Permalloy, 112
Permeability, 129
  curves, 144
Perminvar, 113
Phase angle, defined, 169
Phase and a-c power, 187
Phase, for a condenser, 242
  for an inductance, 229
Pi, 166
Polynomial, 85
Power, 55
  a-c, 187
    apparent, 188
    average, 179
    in a capacitance, 244
    in an inductance, 232
    in a resistor, 179
    in a resistor and an inductance, 254
    maximum, 180
  d-c, 55, 56
  in-phase component, 188
  sources of d-c, 60
    internal resistance of, 66
  used by resistors, 58
Powers of ten, 31
Powers, table of, 294–311
Proton, 4

Radian, 165
Radicals, 28
  addition of, 28
  division of, 29
  subtraction of, 28
Radio, relation to electricity, 1
Reactance, capacitive, 243
  inductive, 230
Reciprocals, table of, 294–311
Rectifiers, 64
Relays, 132
Reluctance, 137
Residual magnetism, 145
Resistance, 6
  measurement of, 202–206
  unit of measure, 7

- Resistance devices, 11
- Resistivity, 6
- Resistors, connected in parallel, 43–45
  - variable, 59
- Resonance,
  - series, 256
    - effect of resistance, 257
  - parallel, 263
    - effect of resistance, 264
- Roots, table of, 294–311

- Series circuits, 40
- Series parallel circuits, 42
- Shielding, magnetic, 142
- Shunts, 197
- Signs, grouping, 24
  - law of, 24
- Simultaneous equations, 90
  - solution of, by
    - addition, 90
    - comparison, 92
    - determinants, 92
    - graphical, 90
    - substitution, 92
    - subtraction, 90
- Slide rule, 12
- Speakers, 131
- Subscripts, 16
- Subtrahend, 85
- Symbols, electrical, 12, 13
  - Greek, 272
  - mathematical, 15

- Thermocouple type instrument, 213
- Transformer, 224
  - losses, 227
  - properties, 224
  - step-down, 224
  - step-up, 224
  - turns ratio, 226
- Triangles, 155
  - adjacent side, 156
  - hypotenuse, 156
  - opposite side, 156
  - right, 155
- Trigonometric functions, 158
  - cosecant, 159
  - cosine, 159
- Trigonometric functions (Continued)
  - cotangent, 160
  - secant, 160
  - signs of, 164
  - sine, 159
  - table of, 292, 293
  - tangent, 159
- Trigonometry, 155

- Vector, 181
- Vector components, 184
  - addition of, 184
- Vectors, addition of, 182
  - current, 187
  - voltage, 187
- Vibrators, 63–65
- Volt, 6
- Voltage, a-c,
  - effective, 178
  - maximum, 178
  - rate of change, 242
  - root-mean-square, 178
  - vectors, 187
- Voltage drop, 38, 39
- Voltage regulation, 66
- Volt-ohmmeter, 206
  - picture of, 207
- Voltmeter, 192
  - multiplier, 199
  - ohms-per-volt rating, 201

- Wave, 168
  - a-c, 168
  - a-c voltage, 169
  - sine, 167
    - development of, 167
    - graph of, 167
  - additions, 170–174
  - complex, 174
  - power, 180, 232, 244, 254
- Wheatstone bridge, 203
- Wiring, form, 72
  - point-to-point, 68
  - receiver, 70, 71, 80
- Work, defined, 54

- Zero, division by, 17
  - exponent, 27

Class No. 621.3841
Book No. T79I
Author Turner
Title Basic Radio
Acc. No. 31485